UNCERTAINTY ANALYSIS OF THE ACOUSTIC METHOD WITH RESPECT TO ABSOLUTE TRAVEL TIME AND TIME DIFFERENCE DETERMINATION

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ABSTRACT

A broadly used type of ultrasonic flowmeters is based on the measurement of the transit time of a acoustic pulse against the flow and in direction of the flow. To determine the mean flow velocity or the flow rate the absolute time of each puls and the difference time between two opposite pulses of the same acoustic path are required.

The most difficult problem is to reach a sufficient time resolution because the difference transit time can reach small values (dependent on the diameter of the section and the flow velocity).

This paper presents the effects of uncertainty due to the measurement of the absolute and difference transit times.

1. INTRODUCTION

The acoustic discharge measurement method ADM is based on the superposition of the propagation velocity of a transmitted acoustic pulse (figure 1) with the flow velocity. To determine the mean velocity of the flow, the transit times t_u and t_d of an upstream and a downstream signal are needed.

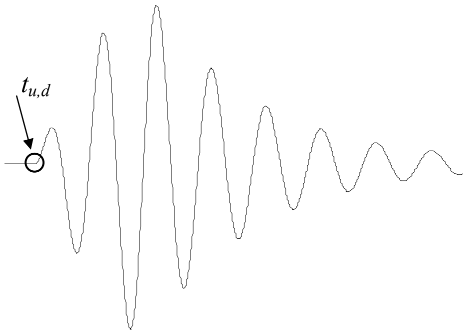

Figure 1- A typical recieved acoustic signal where $t_{u,d}$ denotes the transit time

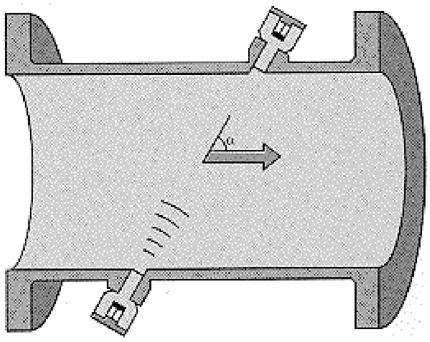

Figure 2- Arrangement of two acoustic sensors in a circular section

The acoustic sensors are mounted at the pipe wall with an angle α to the mean flow (fig. 2). The mean axial velocity is computed by the equation (see [1])

$$\bar{v}_{ax} = \frac{L}{2\cdot\cos\alpha}\cdot\left(\frac{1}{t_d}-\frac{1}{t_u}\right) = \frac{L}{2\cdot\cos\alpha}\cdot\left(\frac{\delta t}{(t_u-\delta t)\cdot t_u}\right) \quad (1)$$

where L is the length of the acoustic path (distance between the two transducers) and δt denotes the transit time difference $t_u – t_d$. The actual acoustic velocity c in water is not needed for the determination of the path velocity as can be seen from equation (1).
The most difficult part of the measurement is the determination of the two transit times $t_{u,d}$ with an appropriate algorithm, where the signal is digitized with an A/D converter. Special care must be given to an accurate determination of the transit time difference.

In the following two approaches for the influence of transit time measurement errors on the determination of the path velocity v_{ax} are presented.

- The first approach assumes the measurement of the absolute transit times of an up- and downstream sonic pulse (t_u and t_d) of the same acoustic path,

- the other just one absolute (t_u or t_d) and the difference transit time δt measurement.

It is shown that the accuracy of the second approach is approximately twice the accuracy of the first one.

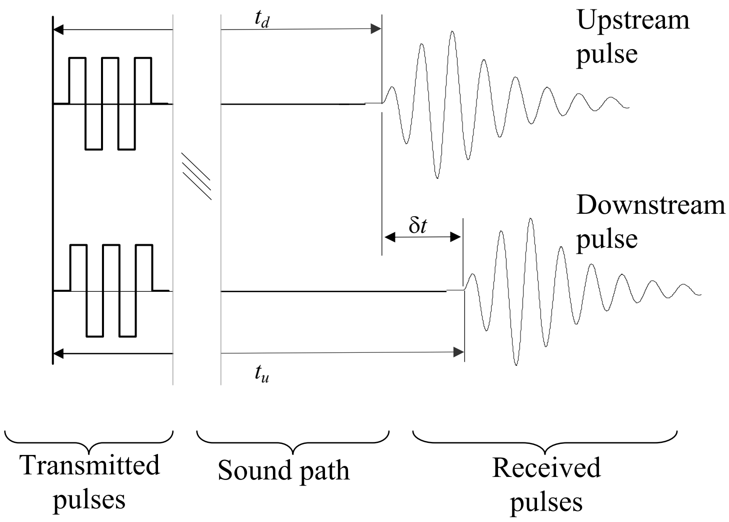

Figure 3

2. TYPE OF ERRORS

There exist mainly two types of errors that occur in a measurement system, random and systematical errors.

2.1 Random errors

The characteristic of these errors is that they are stochastical. In most cases one assumes that the errors of consecutive measurements are independent and thus are also uncorrelated. The errors are specified by a probability density function. Two different probability density functions are important in a typical data aquisition measurement chain:

Normal distribution (Gaussian): The normal probability distribution is characterized by a mean μ_n and a variance σ_n^2. This distribution is used for all kinds of unknown uncertainties. It is also a fact that the sum of n independent random variables which are not normal distributed, converges to a normal distribution for large n.

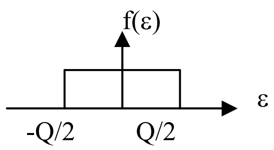

Figure 4: probability density of an uniformly distributed error variable

Uniform distribution: This distribution is often used in digital signal processing for A/D conversion effects and finite precision effects in computation. The error is uniformly distributed with a height of *1/Q* over a finite interval of length *Q* as shown in Fig. 3.

The mean of this distribution is μ_u is zero and the variance σ_u^2 is given by (see [2]):

$$\sigma_u^2 = \int_{-\infty}^{\infty} \varepsilon^2 f(\varepsilon) d\varepsilon = \frac{1}{Q} \int_{-Q/2}^{Q/2} \varepsilon^2 d\varepsilon = \frac{1}{Q} \frac{\varepsilon^3}{3} \Bigg|_{-Q/2}^{Q/2} = \frac{Q^2}{12}$$

The parameter Q is typically the quantization step in finite precision arithmetic or in A/D conversion. For N consecutive measurements of statistical independent data the variance of the error of the averaged results goes down as:

$$\sigma_{averaged}^2 = \frac{\sigma_{not\_averaged}^2}{N}$$

2.2 Systematic errors

Systematic errors are unknown but bounded, highly dependent and correlated. For example these errors are caused by deviations from device dimensions. Systematic errors can not be detected by repeated measurements . This means that successive measurements cannot average out such errors. They can be compensated if they are known. Otherwise worst case assumptions have to be applied for the determination of its effect.

For a single measurement it makes little difference whether the error is systematic or statistical. For systematic errors the worst bounds have to be used, while for statistical errors a worst case bound has to be specified if the distribution is gaussian (e.g. 3 times the standard deviation).

2.3 Signal processing chain

At different stages in the measurement chain from the physical phenomena to be measured to the transit time errors occur. In Fig. 5 six error locations are introduced.

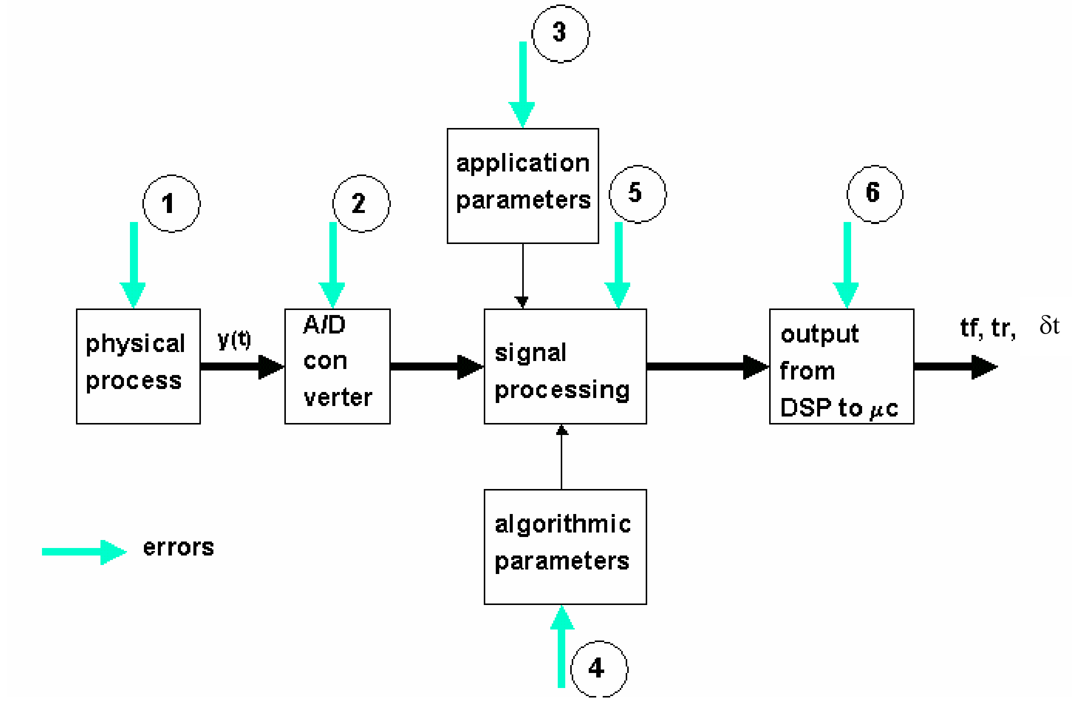

Figure 5- Occurence of errors in the measuring chain

These locations are the following:

1) *Physical process*: The hydraulic conditions determine the type and magnitude of errors or uncertainties. The signal varies due to various physical causes. The nature of these effects are of the statistical type with the simplified assumption of a normal distribution.

2) *A/D converter*: The A/D converter digitizes the incoming analog signal. The conversion to e.g. 12 bits generates with not to stringent assumptions a quantization noise which is uniformly distributed.
3) *Application parameters*: These parameters are given by the configuration and can be determined to a certain accuracy. They concern mainly geometrical data, e.g. the path length L and angle α. The errors generated here are of the systematic type.
4) *Algorithm*: The structure and the type of algorithm determine the existence and magnitude of systematic errors. The number of taps chosen, limit for instance the performance of a FIR low pass filter.
5) *Signal processing*: This part processes the incoming data with a finite arithmetic. Therefore quantization errors occur with the quantization of the coefficients and at performing arithmetic operations. The coefficient quantization is a systematic error while the arithmetic errors can be considered statistical with a uniform distribution.
6) *Output from DSP*: This output quantization can be viewed like an A/D conversion. Either the results are quantized as fine as the intermediate results of the signal processing path or they are quantized more coarsely. This again is considered a statistical error.

After the determination of t_d ,t_u and δt equation (1) is applied to obtain v_{ax}. For the error analysis uncertainties or inaccuracies in L, α, t_d ,t_u and δt are of importance. Now we consider the following simplifications:

1) We consider only errors of the transit time determination and not of the geometrical parameters. If the geometrical parameters L and α play a role in the error analysis, then its effect will be felt twice: In the signal processing chain from the signal input *y(t)* to the transient times and from the transient times to the path velocities.

2) For the worst case analysis we restrict ourselves to the output quantization error ε_Q 6) only, which gives bounds to minimal and maximal errors. It must be noted that an unknown systematic offset in both absolut transit times t_u and t_d, do not affect δt, because of the subtraction of the absolute times for the determination of δt. This offset can therefore be of an order of magnitude larger than the output quantization. Only if the path length L (<0.5m) and the path velocity v_{ax} (<0.5m/s) are very small, this offset can no longer be neglected. If systematic errors in t_u and t_d do not cancel they have to be introduced as a separate error source ε_s, which can be incorporated into the output quantization source 6. The result will be a much coarser quantization, but the analysis will remain the same.

3) The statistical uncertainties of the entire signal processing chain (except the quantization 6) from the physical process to the transit times is summarized in a single noise source ε_s with normal distribution with mean μ_s and variance σ_s^2. The output quantization ε_Q with uniform noise distribution 6) is treated seperately and is considered uncorrelated to the noise source ε_s.and thus can mean and variances can be added to a single mean and variance

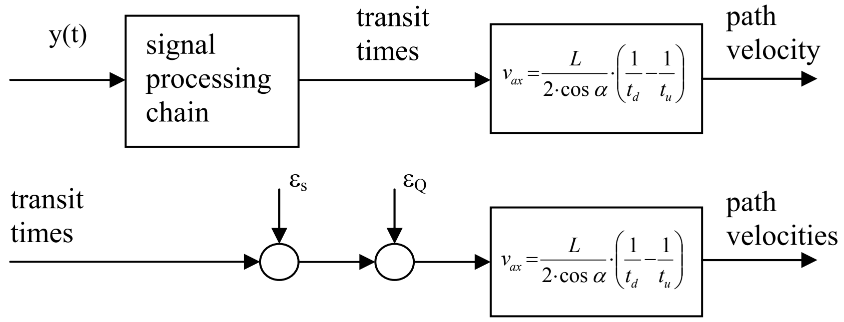

Figure 6 - Error model for error propagation for path velocities determination

3. PATH VELOCITY ERROR FROM UP- AND DOWNSTREAM ABSOLUTE TRANSIT TIME MEASUREMENTS

3.1 Systematic errors

Systematic errors of a function $F(x_1, x_2,..., x_n)$ with regard to errors in $x_1, x_2,..., x_n$ can be determined in a first approximation by linearization (see [3]):

$$\Delta F = F - F_0 = \sum_{k=1}^{n} \left(\frac{\partial F}{\partial x_k} \right)_{F_0} \Delta x_k$$

$$x_k = x_{k,0} + \Delta x_k \qquad \text{k} = 1,..,\text{n}$$

$$\text{relative systematic error } e_{\text{rel}} = \frac{\Delta F}{F_0} = \frac{F - F_0}{F_0} = \frac{1}{F_0} \sum_{k=1}^{n} \left(\frac{\partial F}{\partial x_k} \right)_{F_0} \Delta x_k \qquad (4)$$

For worst case analysis all terms have to be taken positive and for the Δx_k's the maximal possible value have to be choosen. For simplicity reason the index 0 is omitted in the forthcoming analysis.

If the two absolute transit times are measured one obtains for the path velocity from equation (1)

$$\bar{v}_{ax} = \frac{L}{2\cos\alpha} \left(\frac{1}{t_d} - \frac{1}{t_u} \right) = \bar{v}_{ax}(L, \alpha, t_d, t_u)$$

With equation (4) it follows:

$$\frac{\Delta \bar{v}_{ax}}{\bar{v}_{ax}} = \frac{\frac{L}{2\cos\alpha} \left(\frac{1}{L} \Delta L - \tan\alpha_0 \Delta\alpha - \frac{1}{t_d^2} \Delta t_d + \frac{1}{t_u^2} \Delta t_u \right)}{\bar{v}_{ax}}$$

$$\frac{\Delta \bar{v}_{ax}}{\bar{v}_{ax}} = \frac{\left(\frac{1}{L} \Delta L - \tan\alpha \; \Delta\alpha - \frac{1}{t_d^2} \Delta t_d + \frac{1}{t_u^2} \Delta t_u \right)}{\left(\frac{1}{t_d} - \frac{1}{t_u} \right)} \qquad (5)$$

With the simplification $\Delta L=0$ and $\Delta\alpha=0$ we obtain:

$$relative \;\; error = \frac{\Delta \bar{v}_{ax}}{\bar{v}_{ax}} = \frac{\left(-\frac{1}{t_d^2} \Delta t_d + \frac{1}{t_u^2} \Delta t_u \right)}{\left(\frac{1}{t_d} - \frac{1}{t_u} \right)} \qquad (6)$$

The resolution Q in absolute time is in the order of *ns*. If we assume a resolution of $Q=1ns$, then the maximum quantization error for the rounding operation is $+/-Q/2=+/-0.5ns$. For the worst case both errors Δt_u and Δt_d have a maximal value of $Q/2=0.5$ns. Equation (6) reduces to

$$relative\ error = \frac{\Delta \bar{v}_{ax}}{\bar{v}_{ax}} = \frac{\left(\frac{1}{t_d^2} + \frac{1}{t_u^2}\right)\frac{Q}{2}}{\left(\frac{1}{t_d} - \frac{1}{t_u}\right)} = \frac{1}{\delta t}\frac{\left(t_d^2 + t_u^2\right)}{t_u \cdot t_d}\frac{Q}{2} \qquad (7)$$

$$\delta t = t_u - t_d$$

Figures 7,8 and 9 show the relative error as a function of path length, path velocity and time resolution Q. Fig. 9 is a zoom of Fig. 8 for small velocities.

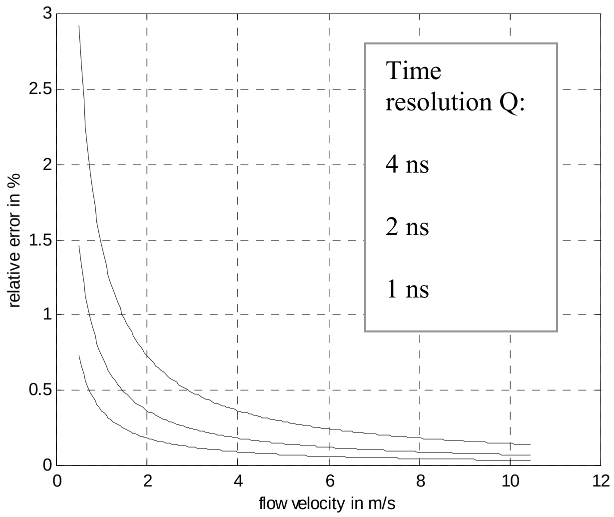

Figure 7 - Relative error in velocity as a function of velocity, absolute time resolution *Q* and fixed path length (diameter D=0.3m)

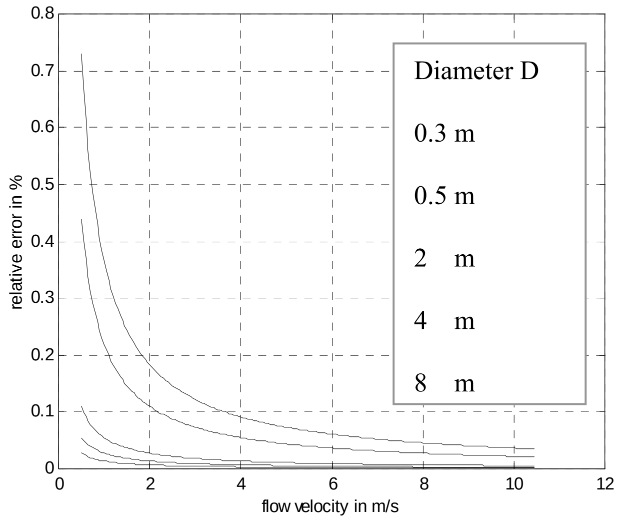

Figure 8 - Relative error in velocity as a function of velocity, path length (diameter *D*) and fixed absolute time resolution *Q* of 1ns

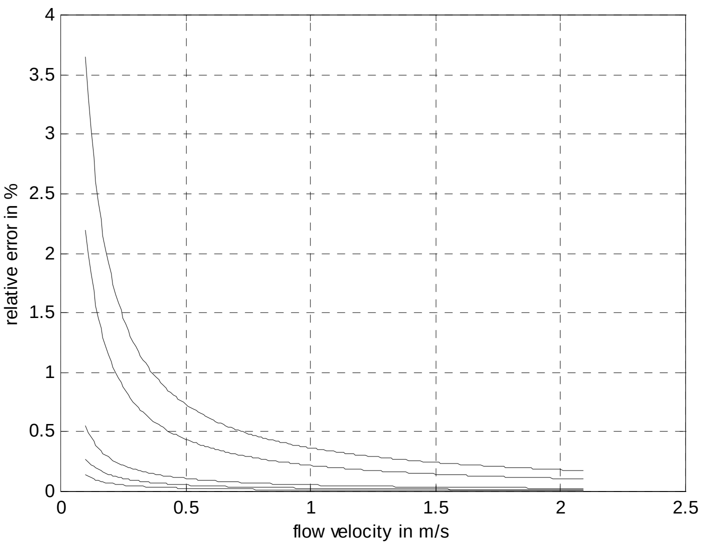

Figure 9 - Zoom of figure 8 for small velocities (0.1..2.1m/s)

The curves show all the same trends. For small velocities the relative error increases hyperbolically. Small time resolutions Q and large diameters reduce the relative error substantially.
If we assume $t_u \simeq t_d$, one gets from equation (7) the simple result:

$$relative\ error \cong \frac{\Delta \bar{v}_{ax}}{\bar{v}_{ax}} = \frac{Q}{\delta t} \qquad (8)$$

The statistical error of a function $F(x_1, x_2,..., x_n)$ with regard to errors in $x_1, x_2,..., x_n$ are described by the Gaussian error propagation law. The variances of each error source are added weighted with some factors.

$$\sigma_F^2 = \sum_{k=1}^{n} \left(\frac{\partial F}{\partial x_k} \right)_{F_0}^2 \sigma_{x_k}^2$$

$$x_k = x_{k,0} + \Delta x_k \qquad \sigma_{x_k}^2 = E\left[\left(x_k - E(x_k) \right)^2 \right] = E(x_k)^2 - (x_{k,0})^2$$

Assumptions : Δx_k uncorrelated, small with respect to $x_{k,0}$, $E(\Delta x_k) = 0$

(Where *E(x)* denotes the *expected value*.) The relative variance is the given by

$$\frac{\sigma_F^2}{F_0^2} = \frac{1}{F_0^2} \sum_{k=1}^{n} \left(\frac{\partial F}{\partial x_k} \right)_{F_0}^2 \sigma_{x_k}^2$$

and the relative standard deviation

$$\sigma_{F,rel} = \sqrt{\frac{\sigma_F^2}{F_0^2}} = \frac{1}{F_0} \sqrt{\sum_{k=1}^{n} \left(\frac{\partial F}{\partial x_k} \right)_{F_0}^2 \sigma_{x_k}^2} \qquad (9)$$

For the path velocity determination we assume statistical errors in the transit times only and not in the geometrical parameters. Thus one obtains the following relative standard deviation for the path velocity

$$\frac{s_v}{\bar{v}_{ax}} = \frac{\sqrt{\left(\left(\frac{1}{t_u^2} \right)^2 \sigma_u^2 + \left(\frac{1}{t_d^2} \right)^2 \sigma_d^2 \right)}}{\left(\frac{1}{t_d} - \frac{1}{t_u} \right)} = \frac{\sqrt{\left(\left(\frac{1}{t_u^2} \right)^2 \sigma_u^2 + \left(\frac{1}{t_d^2} \right)^2 \sigma_d^2 \right)}}{\left(\frac{\delta t}{t_d \cdot t_u} \right)} \qquad (10)$$

Formula (9) can be simplified by the following assumptions:

- Both errors in the absolute transit time determination have the same statistics
- Both absolute transit times are nearly equal

Then we get for (10):

$$\frac{s_v}{\bar{v}_{ax}} \cong \frac{\sqrt{2}\, \sigma_{t_d}}{\delta t} \qquad (11)$$

When comparing equation (11) with equation (8), one can recognise the same dependency on δt. The expression in the denominators differs, but are a mere scaling of the same curve. That means the curves of the systematic error can be used for the statistical error too.
With the assumption of the error model shown in Fig.6, the variance of σ_d^2 resp. σ_u^2 is given by:

$$\sigma_d^2 = \sigma_{\varepsilon_Q}^2 + \sigma_{\varepsilon_s}^2 \qquad (12)$$

4. PATH VELOCITY ERROR FROM ONE UP- OR DOWNSTREAM ABSOLUTE TRANSIT TIME AND THE TRANSIT TIME DIFFERENCE MEASUREMENT

We start from the same basic equation (1) and assume a measurement t_d in the downstream path and a measurement of the transit time difference δt with corresponding measurement errors Δt_d and $\Delta\delta t$:

$$v = \frac{L}{2\cos\alpha}\cdot\left(\frac{1}{t_d}-\frac{1}{t_u}\right) = \frac{L}{2\cos\alpha}\cdot\left(\frac{1}{t_d}-\frac{1}{t_d+\delta t}\right) = \frac{L}{2\cos\alpha}\cdot\left(\frac{\delta t}{t_d(t_d+\delta t)}\right) = \bar{v}_{ax}(L,\alpha,\delta t,t_d)$$

The same analysis as before yields with ΔL and $\Delta\alpha = 0$

$$\boxed{\frac{\Delta\bar{v}_{ax}}{\bar{v}_{ax}} = \frac{t_d}{(t_d+\delta t)}\frac{1}{\delta t}\Delta\delta t - \frac{(2t_d+\delta t)}{t_d(t_d+\delta t)}\Delta t_d} \qquad (13)$$

If we assume that t_d is much larger than δt (similar to $t_d \simeq t_u$) , then equation (13) reduces to :

$$\boxed{\frac{\Delta\bar{v}_{ax}}{\bar{v}_{ax}} \cong \frac{1}{\delta t}\Delta\delta t - \frac{2}{t_d}\Delta t_d} \qquad (14)$$

If we compare this result with equation (8), we identify the first term of the right hand side of equation (14) with the right hand side of equation (8), if $\Delta\delta t$ is equal to Q. This means if the time resolution is *2ns* the rounding error for δt is ±1ns. So the second term of the right hand side of equation (14) makes the difference between the two measurement methods. But in most cases this term is negligible compared to the first term.

The factor 2 in the second term means that an error in the absolute transit time has a double weight compared to an error in the transit time difference. But as already mentioned above $t_d \gg \delta t$, the second term is of no importance. As an example, the absolute transit times of the installation (with a diameter of 0.5m) at the HTA in Lucerne are of approximately 260µs for the short paths and 450µs for the long paths and the transit time differences are for v=0,5...8m/s approximately between 0.1...2µs. Put in equation (14) yields for the most pessimistic case :

$$\frac{\Delta\bar{v}_{ax}}{\bar{v}_{ax}} \cong \frac{1}{0.1\mu s}\Delta\delta t - \frac{2}{450\mu s}\Delta t_d$$

That means in this case an error in δt is about 2300 times worse than the same error in t_d.

Usually the diameter of such a conduit is larger than 0.5m. In this case the error due to the measurement of the absolut transit time is really negligible.
Figures 10 and 11 show the the dependency of the relative error with respect to the transit time

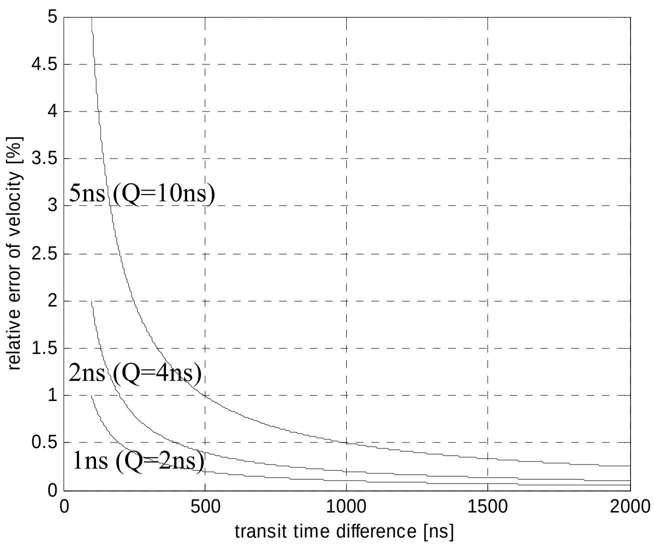

Figure 10 - Relative velocity error in function of the *transit time difference* for different rounding errors in ns with the corresponding time resolution Q in brackets.

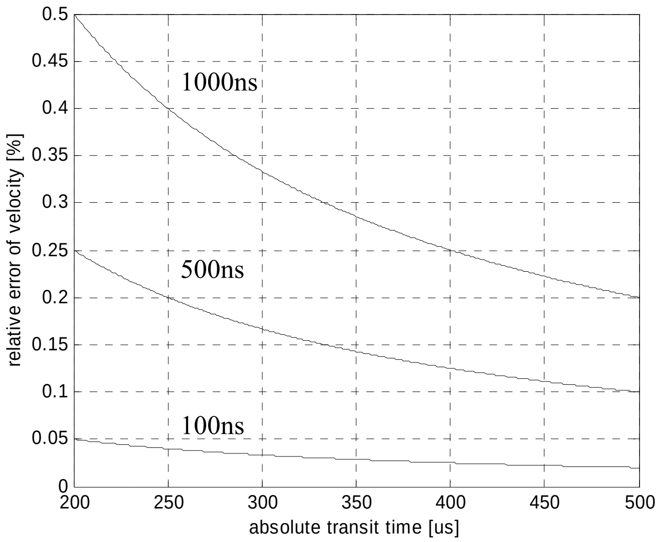

Figure 11 - Relative velocity error in function of the *absolut transit time* (in µs) for different time resolution (errors) in ns..

5. CONCLUSIONS

If the relative velocity error has to be kept below 0.5% (for velocities larger than 1m/s and a diameter of 0.5m), then the transit time difference has to be determined with a time resolution of Q=1ns (Fig. 12) in the worst case for a single measurement. Successive measure- ment decreases the error according to the "*root-law*" of statistics (see [3]).

In the first case of two absolute transit time measurements, both measurements should not have a maximal rounding error of larger then ±Q/2.
In the case of measuring the difference time and one absolute transit time the maximal rounding error could reach ±Q (Fig. 10), whereas the time resolution for the absolute transit time measurement has to be only in the order of *100ns* (Fig. 11).

Hence the accuracy of the second approach is twice the accuracy of the first one.

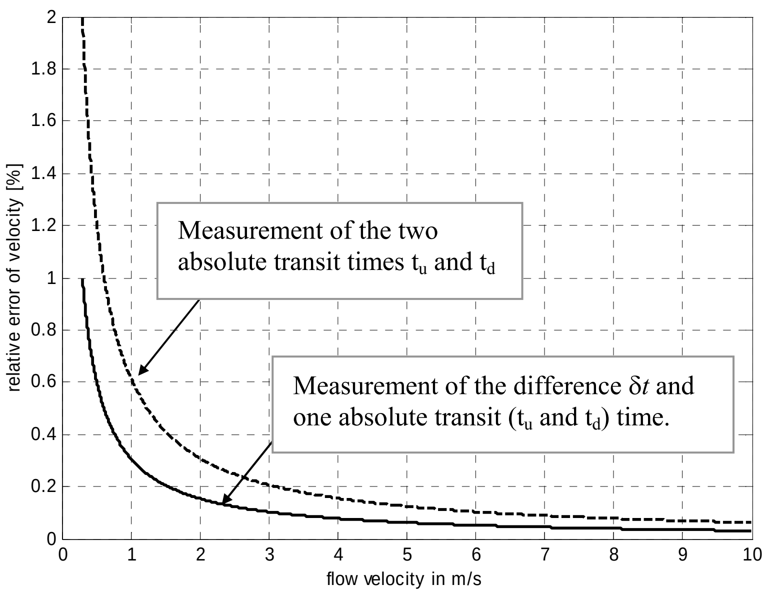

Figure 12 - Comparison between the two different approaches for a fixed time resolution (Q=1ns) and diameter (0.5m).

ACKNOWLEDGEMENT

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