Towards an Improved Model for Predicting Hydraulic Turbine Efficiency

Jessica Brandt
Engineer in Training
Manitoba Hydro

Dr. Jay Doering, P.Eng.
Civil Engineering
University of Manitoba

ABSTRACT

Field performance testing of hydraulic turbines is undertaken to define the head-power-discharge relationship, which identify the turbine's peak operating point. This relationship is essential for the efficient operation of a hydraulic turbine. Unfortunately, in some cases it is not feasible to field test turbines due to time, budgetary, or other constraints. Gordon (2001) proposed a method of mathematically simulating the performance curve for several types of turbines. However, a limited data set was available for the development of his model. Moreover, his model did not include a precise method of developing performance curves for rerunnered turbines.

Manitoba Hydro operates a large network of hydroelectric turbines, which are subject to periodic field performance testing. This provides a large data set with which to refine the model proposed by Gordon (2001). Furthermore, since Manitoba Hydro's data set includes rerunnered units, this provides an opportunity to include the effects of rerunnering in his model.

The purpose of this paper is to refine Gordon's model using Manitoba Hydro's data set and to include the effects of rerunnering in the model. Analysis shows that the accuracy of the refined model is within ±2% of the performance test results for an "old" turbine. For a newer turbine or a rerunnered turbine, the error is within ±1%. For both an "old" turbine and a rerunnered turbine, this indicates an accuracy improvement of 3% over the original method proposed by Gordon (2001).

1. INTRODUCTION

Mathematically modeling performance curves is an acceptable alternative when cost or time restraints prohibit field-testing. Gordon (2001) introduced a simple method of approximating the performance curves of various types of turbines. This method, as it applies to propeller type turbines, was evaluated against prototype test results obtained by Manitoba Hydro. Research indicates that some modifications to the mathematical method will improve the overall precision of the model.

This paper focuses on improving the mathematical method's peak efficiency calculation and creating a method of incorporating rerunnered unit data. For the purpose of fully demonstrating Gordon's method (2001), all equations required to plot the efficiency curve are also presented.

1.1. The Hydraulic Turbine

The design and applications of hydraulic turbines has evolved over time. Functionally, there are several different types of hydraulic turbines, each of which operates under a characteristic set of operating conditions. In Manitoba, most rivers have mild slopes that afford only a small operating head. Moreover, many generating stations were built in the beginning to middle of the last century. As a result, the most common type of turbine found in Manitoba is the axial-flow propeller turbine, as shown in Figure 1.

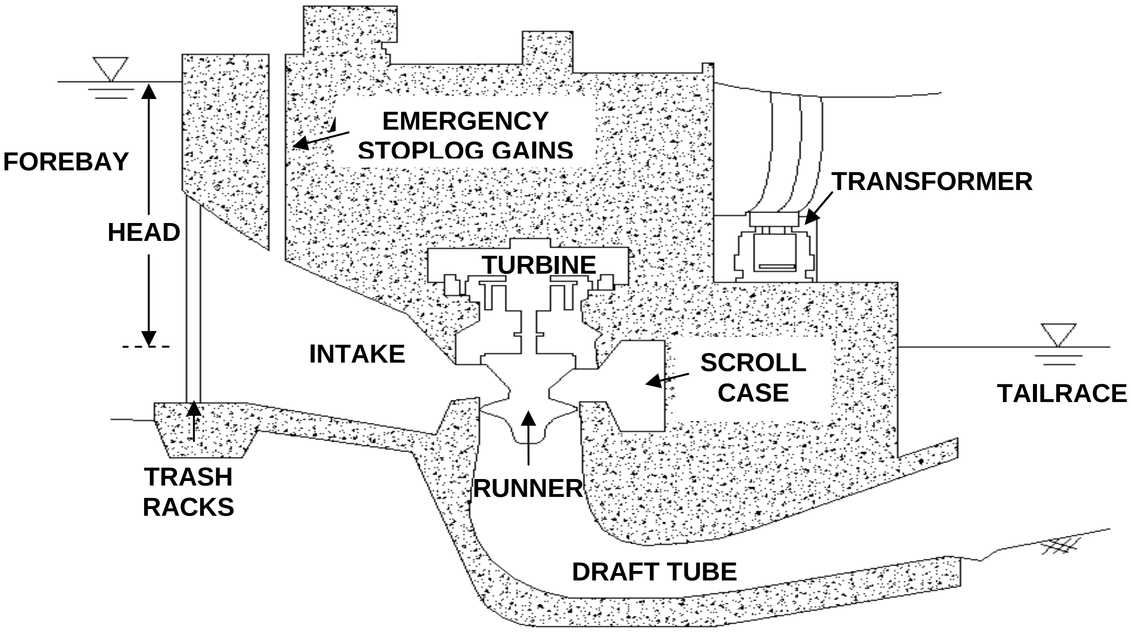

Figure 1. Axial Flow Turbine Diagram.

There are several key components of the turbine design that determine the power capabilities of the system. This includes the design head, the discharge, the runner throat diameter, and the runner rotational speed. The capabilities of these components are linked to the era in which the turbine was designed and how the system has degraded over time. The power capability of the unit is also limited by losses.

Due to the mechanics of the system, several types of losses are expected. First, frictional losses occur as water flows across the various surfaces of the system. Secondly, inlet and bend losses occur as the water is forced through the trash racks and through the geometry of the intake, scroll case, and draft tubes. And finally, the motion of the mechanical parts of the turbine results in mechanical losses. The sum of these losses plus the energy removed for power equals the head drop through the turbine unit.

1.2. Performance Curves

Performance curves are an excellent indicator of a turbine's power potential. Often these curves are employed by system controllers in order to maximize system capabilities and turbine efficiency. A typical efficiency curve is usually presented as efficiency versus discharge or efficiency versus power. The these two curves are related through the relationship

$$P = \frac{Q \times E \times H}{102.02} \tag{1}$$

where P is the power [MW], Q is the current operating discharge of the turbine, E is the efficiency of the system, H is the head, and 102.02 represents a system of constants for unit conversions.

A typical efficiency curve for a propeller turbine is shown in Figure 2. For a propeller turbine, the portion of the curve before peak efficiency is moderately steep and straight. After the peak, however, efficiency drops off more quickly.

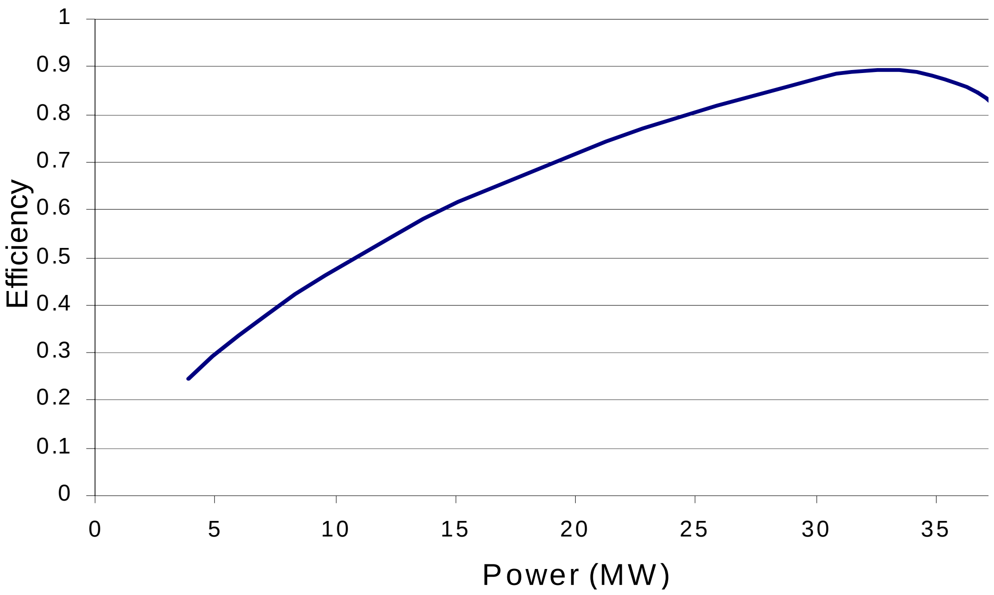

Figure 2. Propeller Turbine Performance Curve.

By today's standards, a performance curve is typically provided by a manufacturer in the design phase. Later, physical testing of the commissioned turbine verifies the manufacturer's projected performance curves. Future testing at regular intervals quantifies performance degradation and the effect of system changes.

2. DATABASE

Manitoba Hydro currently relies on field-testing results for the establishment and verification of performance curves. In this study, twenty-two field-tested propeller turbines were selected for analysis. These units vary in aspects such as age, size, and manufacturer. The range of variable is indicated in Table 1.

Table 1. Range of Design Characteristics.

Design Characteristic	Range	
	Lowest	Highest
Design Head (m)	17.1	27.6
Design Speed (rpm)	90	138.5
Runner diameter (m)	4.9	7.9
Installation Year	1926	1998

The Manitoba Hydro Performance Testing Group uses a measurement method that meets the requirements of ASME (1992). A German Test Code (1948) forms the basis for testing. This method has an expected accuracy of within ±2% of the true hydraulic performance of a unit.

In the performance testing, directional velocity meters are attached to a carriage assembly and lowered into the emergency stoplog gains (shown in Figure 3). There are between 7 and 11 Ott Meters per carriage (depending on the size of the stoplog opening). These are supported on aluminum arms, which are extended horizontally into the upstream flow during testing. Two to three carriages are assembled so that both (or all three) intake openings of the turbine unit can be tested simultaneously.

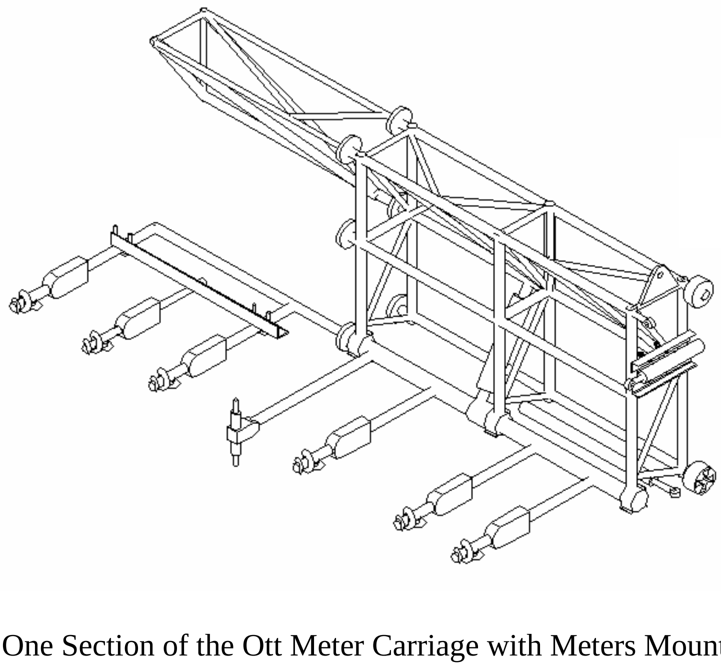

Figure 3. One Section of the Ott Meter Carriage with Meters Mounted.

A series of other monitoring systems are set up throughout the plant to evaluate other performance characteristics. These monitors includes water level probes to measure the forebay, headgate, and tailrace elevations; a Precision Watt Meter to measure the WATTS and VARS generated by the unit; a string gauge to measure wicket gate opening and blade angle (where required); and a differential pressure transducer to measure piezometer levels in the scroll case. The monitors are connected to a central data processing unit and collected by a data acquisition system.

The data gathered by the data acquisition system software is presented in text files that are interpreted by computer programs written by Manitoba Hydro. In this process, all flow data is adjusted to match the design head so that the measured performance results are representative of the design specifications.

Through careful analysis, a performance curve is generated for the tested unit. This curve is then issued for the turbine and a Performance Test Report follows. The Performance Test Report contains the turbine design characteristics and the adjusted flow data to re-create the performance curve.

3. Development of the Mathematical Approach

In 1992 James L. Gordon, a hydropower consultant residing in Quebec, devised a mathematical method for approximating hydraulic turbine efficiency curves for several types of turbines. He based this research on characteristics of the turbine design and age of the design technology. This mathematical approach was created to be especially useful for approximating gains through rerunnering, updating an existing performance curve, and creating a performance curve for a turbine that lacks a performance curve (very old turbines).

The method outlined by Gordon (2001) is a generic procedure, with calibration factors for different styles of turbines including the Francis, axial flow, and impulse turbines. In the development phase, 87 axial flow turbines were used to create axial flow turbine equations, however, only 3 of those were propeller turbines. The ensuing paragraphs describe the full derivation of the suggested approach, as they apply to the propeller turbine.

Using a spreadsheet simplifies the plotting of a smooth efficiency versus discharge curve. The resulting performance curve may then be compared to a manufacture’s performance curve or used for performance prediction purposes.

3.1. Efficiency Calculations

The mathematical method suggests that the efficiency of a turbine is dependent on the head, discharge, runner size, runner speed, and age of the turbine. In the ensuing formula for propeller turbines, the peak efficiency is assigned a starting value of 90.4%, and changes according to

$$\varepsilon_{peak} = A - \Delta\varepsilon_{year} - \Delta\varepsilon_{specific\,speed} + \Delta\varepsilon_{size} \;, \qquad (2)$$

where A is a constant equal to 0.904 for the propeller turbine, $\Delta\varepsilon_{year}$ is a function that considers the age of the runner, $\Delta\varepsilon_{specific\ speed}$ is a function that considers the design speed of the turbine, and $\Delta\varepsilon_{size}$ is a function that considers the radius of the runner. The successive paragraphs examine these dependent functions.

First to consider is the development of $\Delta\varepsilon_{year}$. Propeller turbines have been in existence for over 100 years. During this period, many design modifications created efficiency improvements over time. As technology advanced, efficiency gains progressively declined. This indicates that efficiency may be expressed as a function of age and the year of turbine design or turbine installation. Therefore, a function that considers the year of installation, $\Delta\varepsilon_{year}$, is expressed as

$$\Delta\varepsilon_{year} = \left(\frac{1998 - y}{B}\right)^{x} \;, \qquad (3)$$

where B is a constant equal to 252 for axial flow turbines, x is a constant equal to 2.03 for axial flow turbines, and y is the age of the runner (less than or equal to 1998).

Note that the value 1998 represents the cut off for efficiency improvement due to age. Gordon (2001) suggests that if a unit is newer than 1998, the year 1998 is to be used as y. This means the efficiency gains experienced due to technological improvement of new runners will not change much in the future. As well, this follows the development trend, where large improvements were initially occurring in the past, and only small, incremental improvements have been encountered in recent years, as shown in Figure 4.

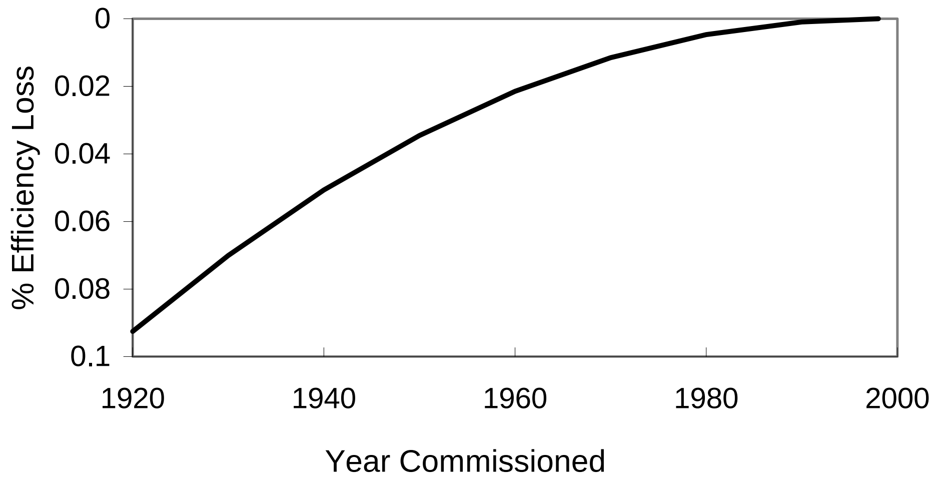

Figure 4. Peak Efficiency Improvements with Time.

The second function built into the efficiency equation is one for specific speed, $\Delta\varepsilon_{specific\ speed}$. Initially, specified speed information was based on ASME (1996) data. Gordon (2001) made slight modifications to the ASME to improve the function for the axial turbine case. The new function, shown as the parabolic form in Figure 5, demonstrates the loss in efficiency for the runner size as it deviates from the ideal case.

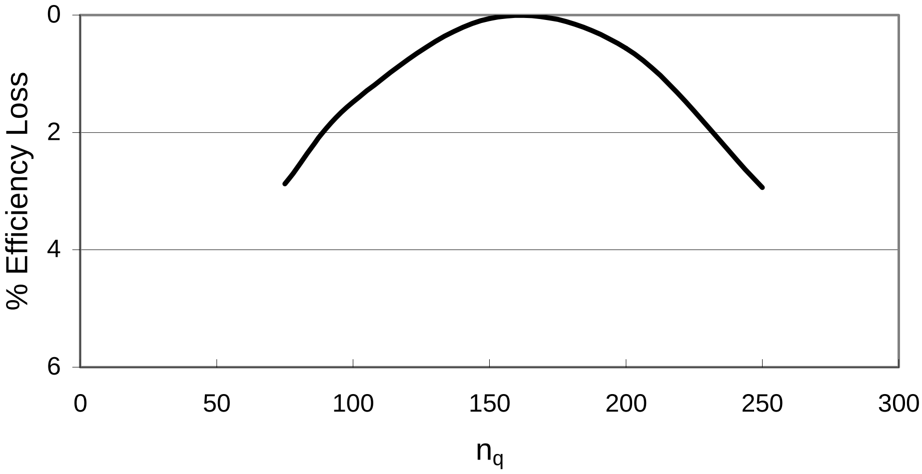

Figure 5. Efficiency Loss as a Function of Specific Speed.

The equation for specific speed is dependent on the relationship between turbine type and the specified speed. This function is described as

$$\Delta\varepsilon_{nq} = \left[\left(\frac{n_q - C}{D}\right)^2\right]^z \quad , \tag{4}$$

where C is a constant equal to 162 for axial turbines D is a constant equal to 533 for axial turbines, and Z is a modification to the exponent, equal to 0.979 for axial turbines. n_q is a function specifically describing the expected specific speed.

n_q is defined by

$$n_q = rpm\, Q_{rated}^{\;0.5}\, h_{rated}^{\;-0.75} \quad , \tag{5}$$

where rpm is the turbine synchronous speed, measured in revolutions per minute, Q_{rated} is the discharge at design head, measured in m^3/s, and h_{rated} is the design head, measured in meters.

The next function calculates efficiency loss due to the size of the runner, $\Delta\varepsilon_{size}$. This equation is derived from the Moody (1952) step up formula. The relationship is expressed as

$$\Delta\varepsilon_{size} = (1 - A + \Delta\varepsilon_{year} + \Delta\varepsilon_{nq})(1 - 0.798 d^{-0.2}) \quad , \tag{6}$$

where d is the runner throat diameter, measured in meters.

3.2. Discharge Calculations

The following equations work harmoniously with the efficiency equations, and form the x-coordinate axis of the graphed performance curve. Each equation is presented independently, and then combined with the efficiency equations presented in Section 3.3.

The first discharge equation expresses peak flow (Q_{peak}) in terms of the rated flow (Q_{rated}) and the year of commissioning. It is described as

$$\frac{Q_{peak}}{Q_{rated}} = \left(\frac{1998 - y}{325}\right)^2 + 0.945\,. \tag{7}$$

Here, the value 325 is based on mathematical model test results gathered by Gordon (2001). The value of 0.945 arises from efficiency gains over time, and the fact that new propeller turbines are expected to peak close to 95% efficiency.

The next function describes the effect of the synchronous–no–load discharge. This is the defining point at which the runner is spinning, but not quite fast enough to create power. It is represented by

$$\frac{Q_{snl}}{Q_{rated}} = \left(\frac{n_q}{350}\right)^2, \tag{8}$$

where Q_{snl} represents the synchronous-no-load discharge. Note that Q_{rated} and n_q were previously defined, and the value 350 was determined empirically by Gordon (2001).

The flow exponent, k, is directly related to the specific speed. k increases as specific speed decreases, thereby shaping the efficiency curve. The flow exponent is described by

$$k = 1.78 - \left(\frac{1998 - y}{92}\right)^2. \tag{9}$$

3.3. Plotting Efficiency versus Discharge

Two equations are required to define the efficiency portion of the mathematically modeled efficiency curve. Both equations are designed to consider the degradation of efficiency from the peak, but each one represents one side of the peak.

The equation of efficiency before the peak is represented by

$$\Delta\varepsilon_{peak} = \varepsilon_{peak}\left(1 - \frac{Q_{snl}}{Q_{peak}}\right)^{-k}\left(1 - \frac{Q}{Q_{peak}}\right)^{k}, \tag{10}$$

which is a relatively steep and straight curve, while the change in efficiency beyond peak is

$$\Delta\varepsilon_{peak} = \left[\left(\frac{1998 - y}{100}\right)^2 + 0.4\right]\left(\frac{Q}{Q_{peak}} - 1\right)^{1.5}, \tag{11}$$

which is a rapidly declining curve.

3.4. Rerunnered Units

The mathematical method does not have an exact way of incorporating the data from rerunnered units. Gordon (2001) suggests that because it is only the age of the technology that is changing, the same equations may be used with a different value for the year of installation. He suggests that the date of the old turbine installation be subtracted from the date of the new installation. The value for y then equals the original year plus two thirds of the difference between the dates.

For example, a unit that is constructed in 1930 is rerunnered in 1990. In this case the value of y would be

$$y = \frac{(1990 - 1930) \times 2}{3} + 1930$$
$$= 1970.$$

4. ANALYSIS

In this study, performance curves developed by the mathematical model are compared and modified to model performance curves developed by prototype testing. The differences at peak efficiency are minimized through adjustments of the mathematical model. Figure 8 through Figure 10 are curves that indicate the variability between prototype test results and the two versions of the mathematical model (original and modified).

To start, the peak efficiency of an older unit is examined. In this comparison, there was a general trend of overestimation on behalf of the mathematical model. The average error of the mathematical method was found to be ±5 percent of the prototype test results, which leads to a potential ±7% variance from the actual unit performance (testing method gives results within ±2 of actual performance).

The newer unit is fairly well represented by the mathematical method. In this case, the average difference between methods at peak efficiency is less than ±1%. The smaller error indicates that the mathematically based equations are more accurate for newer units than for older units. Moreover, this implies that the equations may not properly account for degradation due to age.

The performance of the rerunnered unit is also overestimated by the mathematical method. In this case, the mathematical method is within 4% of the prototype test results, or 6% of the actual peak efficiency.

4.1. Improving Modeled Peak Efficiency

Optimizing the variables improves the model's estimation of peak efficiency. Here, the peak efficiency derived through prototype testing is compared to the modeled peak efficiency for the 22 Manitoba Hydro turbines. A linear program minimizes the difference between the model and the test peak efficiency.

There are several variables that improve the modeled peak efficiency. They include constants *B* and *C*, and exponents *x* and *z*; components of the functions for degradation due to age and specific speed.

Table 2. The Modified Variables of the Mathematical Method.

Variable	Original	Suggestion
B	252	251
X	2.03	1.77
C	162	167
Z	0.98	0.51

Modifying the variables *B* and *x* affects several areas of the model. Primarily these variables are used in the computation of $\Delta\varepsilon_{year}$, the function for degradation due to age. This indicates that the technology of older propeller turbines is actually less efficient than what Gordon (2001) initially predicted. The effect of this modification is shown in Figure 6, where the top line represents the original shape, and the bottom line represents the new shape.

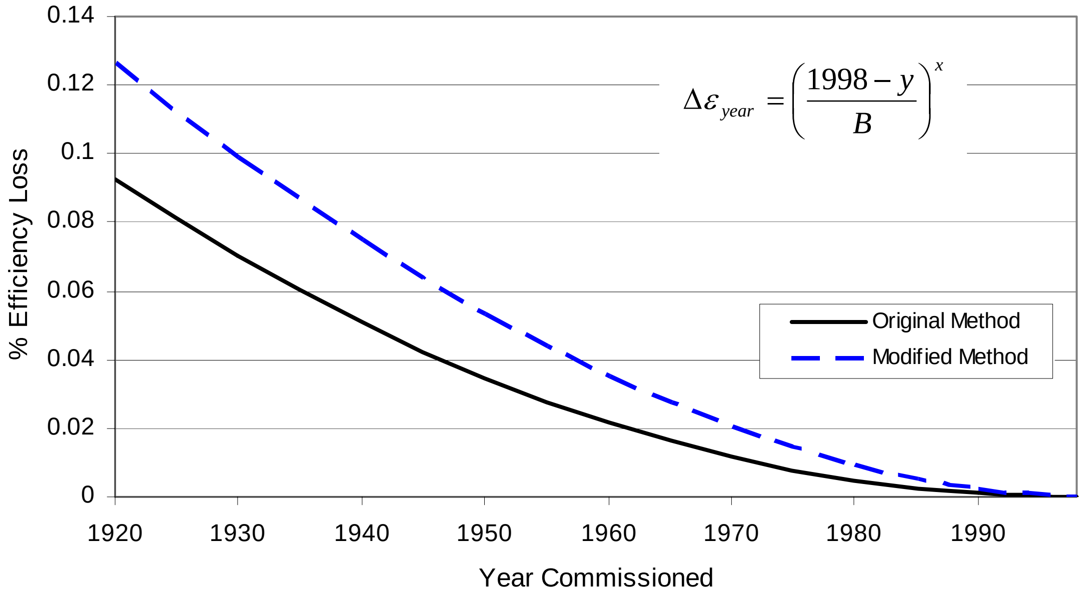

Figure 6. Suggested Modifications to the Variables B and *x*.

The modifications to variables C and z affect the calculation of $\Delta\varepsilon_{nq}$, the specific speed equation. Basically, these changes result in less efficiency loss due to the different specific speeds.

4.2. Improving Model for Rerunnered Units

The ensuing proposal integrates a new equation into the peak efficiency equation (Eq. (2)). This offers an improved method of handling rerunnered data resulting in the efficiency being directly related to the age of both the new and old technologies involved.

First, Eq. (2) is adjusted to account for the new unit age factor according to

$$\varepsilon_{peak} = A - \Delta\varepsilon_{year} - \Delta\varepsilon_{specific\,speed} + \Delta\varepsilon_{size} - \Delta\varepsilon_{unit} \quad , \qquad (12)$$

where $\Delta\varepsilon_{unit}$ is a function of the difference between unit age and the runner age.

Thus, the function for $\Delta\varepsilon_{unit}$ is defined by

$$\Delta\varepsilon_{unit} = \left(\frac{y_{unit} - y_{run}}{F}\right)^{G} \quad , \qquad (13)$$

where y_{unit} is the year of unit commissioning ($y_{unit} \leq 1998$), y_{run} is the year of rerunnering ($y_{run} \leq 1998$), F is a constant value, equal to 900, and G is a constant exponent, equal to 2.

The year is bounded to synchronize Eq. (13) with the previously defined age-related equations. Furthermore, this equation was developed in a manner applicable to rerunnered units and non-rerunnered units for simplicity. In the latter case, the effect of $\Delta\varepsilon_{unit}$ will be negated.

The function $\Delta\varepsilon_{unit}$ may be presented graphically, as shown in Figure 7. The relationship is parabolic, i.e., an increasing difference in technologies results in an increasingly large loss in efficiency.

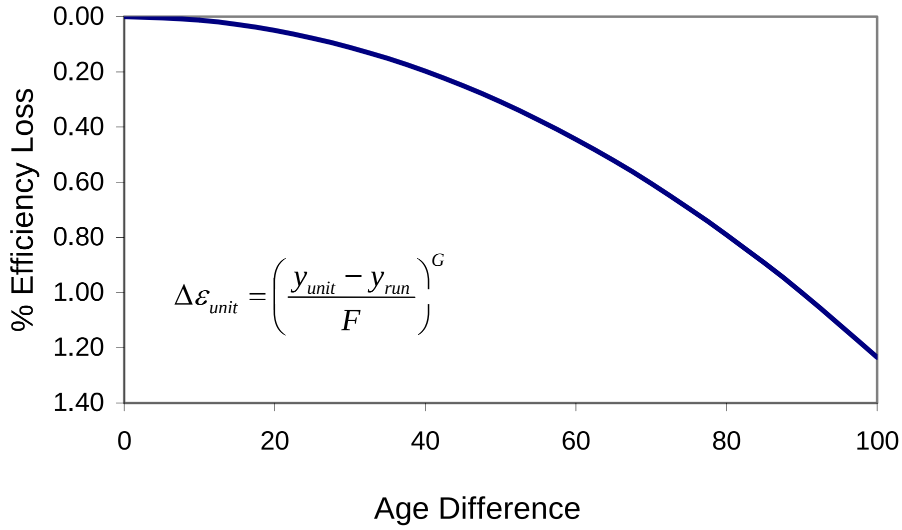

Figure 7. Relationship between Age and Efficiency

Incorporating the modification factor for rerunnering improved the results of the mathematical method in this situation. In this case, errors were reduced from 4% (the accuracy of the original model for rerunnered units with respect to performance tests) to less than 1%.

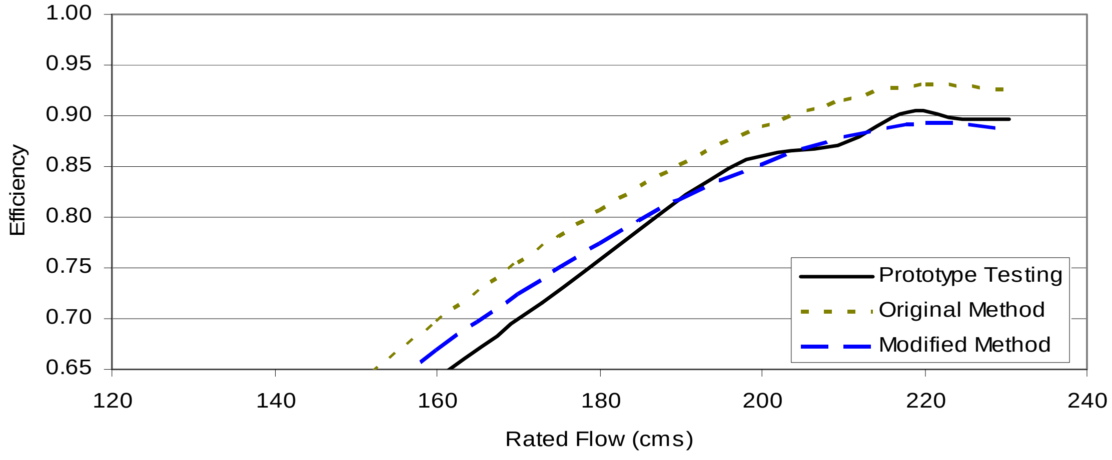

Figure 8. “Old” Propeller Turbine Performance Curves.

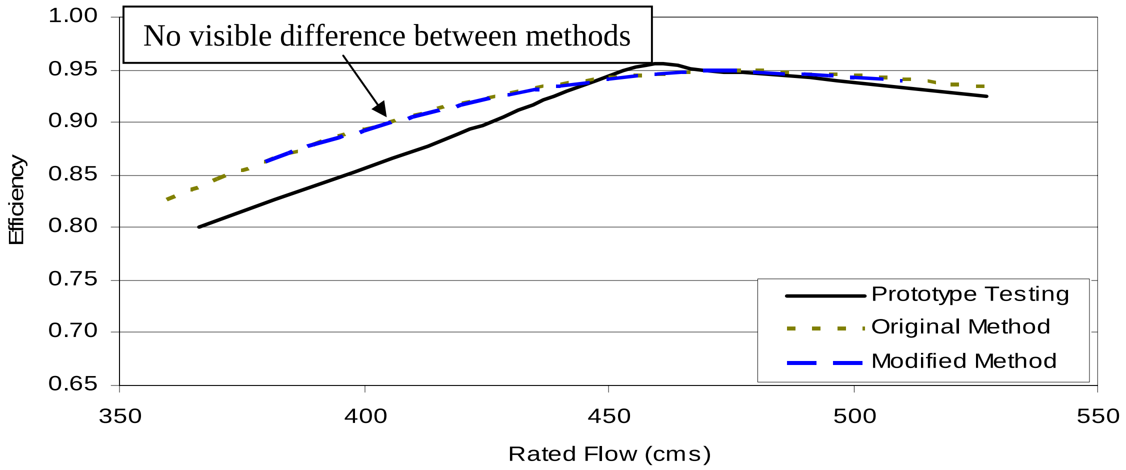

Figure 9. “New” Propeller Turbine Performance Curves.

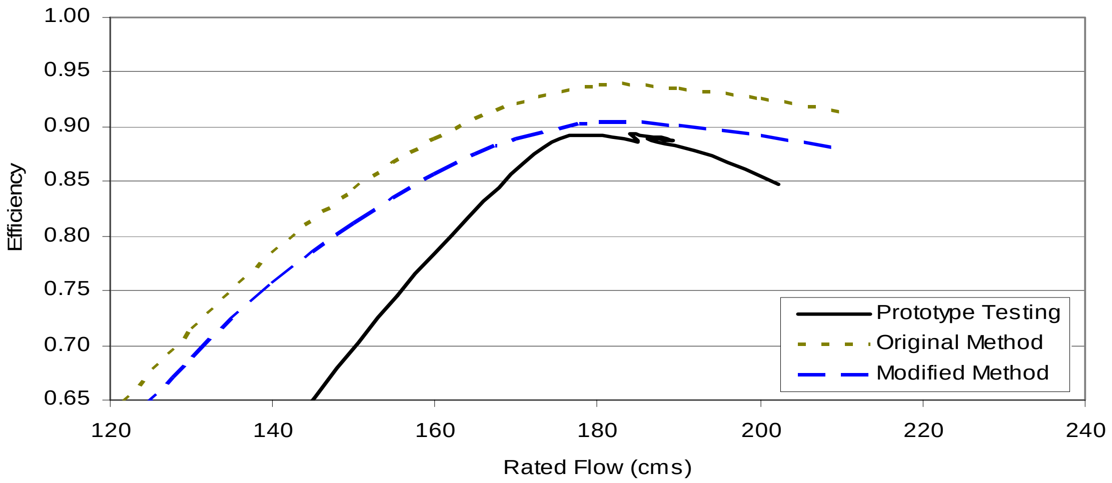

Figure 10. Rerunnered Propeller Turbine Performance Curves.

4.3. Conclusion

Where time, money, or other factors prohibit the practice of field-testing a propeller turbine, mathematical modeling may be a viable alternative. This report evaluates a mathematical modeling method at peak efficiency and suggests ways to refine the method. This study is based on the analysis of 22 propeller turbines in the Manitoba Hydro system.

Through incorporation of the suggested modifications to the mathematical model, it is possible to improve the model as follows, where the percent error shown indicates a variance from the actual turbine performance.

Table 3. Summary of Model Improvements

Turbine Case	Original Model	Modified Model
Older Turbine	7%	4%
Newer Turbine	6%	3%
Rerunnered Turbine	6%	3%

Overall, this method of mathematically modeling peak efficiency shows an accuracy of within 4% of the actual turbine peak performance, making it a viable option in select situations.

List of Symbols

Symbol	Definition	Symbol	Definition
A	constant (0.904)	k	discharge exponent
B	constant (252)	n_q	turbine specific speed
C	constant (162)	P	power
D	constant (533)	Q	turbine discharge
D	turbine throat diameter	Q_{peak}	turbine discharge at peak efficiency and rated head
$\Delta\varepsilon_{nq}$	specific speed adjustment for efficiency	Q_{rated}	turbine discharge at rated head and rated load
ε_{peak}	peak efficiency	Q_{snl}	turbine discharge at speed-no-load and rated head
$\Delta\varepsilon_{size}$	runner size adjustment factor for efficiency	rpm	turbine synchronous speed
$\Delta\varepsilon_{specific\ speed}$	specific speed adjustment factor for efficiency	y	year of turbine design (less than 1998)
$\Delta\varepsilon_{unit}$	efficiency adjustment factor that accounts for new runner technology in an old turbine casing	y_{unit}	year of original turbine runner design
$\Delta\varepsilon_{year}$	age adjustment factor for efficiency	y_{run}	year of new runner turbine design in rerunnered unit
F	constant (900)	x	constant (2.03)
G	constant (2)	z	constant (0.979)
H	head		

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