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CALCULATION OF THE STEADY STATE DISCHARGE IN A GENERALLY ARRANGED PIPELINE SYSTEM BY THE METHOD OF CHARACTERISTICS

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Abstract: *The paper outlines the cause and effect of the hydraulic transients in a pipeline system in general. The emphasis shall be on understanding of the physics of transients and their propagation in a hydraulic system rather than an in-depth theoretical and mathematical analysis. Example of flow calculation is presented based on submitted theory.*

The pressure of a liquid in a conduit and its discharge are interdependent. Every change in discharge induces a corresponding change in pressure and vice versa. The changes in pressure caused by this dependence are called water hammer. The terms water hammer and hydraulic transients are used synonymously to describe an unsteady flow of fluids in pipelines, although the former term usually refers to water only. Different types of flow variation can contribute to water hammer, varying from a single identifiable alternation to an oscillating, periodic or pulsating disturbance. In power stations and water supply systems, transients are normally governed by a change in the operating status of the turbines/pumps or valves, by varying demands by the system. In this paper, this term water hammer is used, for the sake of simplicity. The changes of pressure at water hammer may be insignificant, but could be large, sometimes leading to the rupture of a pipeline, or of other devices forming part of the complex pipeline system. Another danger of water hammer is the difficulty of estimating in advance, without detail calculations, whether the water hammer imperils the system in a specific case or whether its effect can be neglected. Such calculations are laborious, especially for more complex hydraulic system. They require expert knowledge and such information about the system under consideration as may not always be available. The problem of water hammer may be important in the design of various types of hydraulic systems, such as water-supply network, penstocks of water machines etc. Unsteady flow develops in all these systems, at least during their opening and closing and is inevitably accompanied by water hammer.

1. INTRODUCTION

A change of the steady state operating conditions of a hydraulic system either unintentionally by means of closure of a valve or unplanned pump operation due to a system failure is communicated to the system by pressure waves travelling at approximately sonic velocity and propagating from the point in the system, at which the change in steady flow condition was imposed. As a pressure wave propagates along a pipeline it encounters junctions, changes in a pipeline inner diameter and/or pipeline wall material, boundary conditions such as a constant head reservoir etc., at which is transmitted and/or reflected giving rise to a complex pattern of pressure behaviour. Pressure wave is the propagation of energy, as in the transmission of sound, as a wave motion that is associated with the elastic deformation of the medium. The celerity of the waves in a rigid pipeline is given (1)

$$a = \sqrt{\frac{K}{\rho}} \tag{1}$$

where K = bulk modulus of elasticity ($K = 2.03 \cdot 10^9$ Pa for water 20^0C)
ρ = density of water ($\rho = 998\ kg \cdot m^{-3}$ for water at 20^0C)

The term of celerity is used to differentiate between velocity of moving water and the velocity of a pressure wave. Disturbances due to a sudden change in the water flow may be propagated at a celerity of approximately $a = 1426\ m \cdot s^{-1}$, which depends on the elasticity of the pipeline, whereas the fluid velocity is usually not more than $v = 6 m \cdot s^{-1}$. The propagation velocity of a pressure wave in pipeline containing water can be greatly reduced even if a very small amount of air present in the form of gas bubbles is dispersed throughout the water. Even small concentration of gas in flowing water can significantly reduce the wave velocity. If a small amount of air is present, the effect of the pipe wall elasticity becomes insignificant.

In fact, once the pressure wave has been generating, its transmission does not depend on the direction in which the water is moving or on whether is moving at all. The system attains a new state equilibrium, after some time, if the change has not reached destructive proportions.

The rate of change is of prime importance and governs the method to be employed in calculating the effect of pressure wave propagation.

2. THE BASIC THEORY OF TRANSIENT FLOWS

If the rate of change is slow, one can assume that the fluid is non-compressible, i.e. the change in flow condition is instantaneously transmitted through the system. The entire fluid column is accelerating and decelerating at the same value through out its length with the celerity being infinitely large. The analysis based on this assumption is referred to as the ***rigid column theory*** [2]. In the rigid column theory, the fluid in a pipeline is treated as an incompressible mass. A pressure difference applied across the ends of a water column produces an instantaneous acceleration or deceleration. The pipe wall lining is also assumed to be rigid.

2.1 The rigid column theory

If Newton's second law governing the flow in a pipeline with a cross-sectional area A[m^2] and a length L[m] is applied, the following relationship from which pressure fluctuations ΔH[m] can be determined for a valve closure at the end of the pipeline results:

$$\Delta H = \frac{L}{g} \cdot \frac{dv}{dt} \tag{2}$$

Where $\frac{dv}{dt}$ represent the change of flow velocity over certain period of time. This sometimes known as the *Allievi expression* for the maximum transient pressure generated for instantaneous valve closer. The usefulness of this approach is limited to cases where:

- The pressure and velocity changes are slow, such as those accruing between an upper reservoir and a surge tank and the elastic properties of the water column and the pipe material can be neglected without introducing a significant error.
- The time of operation of the hydraulic control is considerably greater than the time taken for a transient to pass through the water column. The theory might be suitable if the valve closure is very slow. The acceptable time of a valve closure $dt > \frac{L}{60}$ where dt equals the time-lapse in seconds between the two equilibrium stages and L [m] is the length of the pipeline.

2.2 The elastic pipeline theory

The elastic theory is based on the assumption that, wherever a disturbance occurs, the pressure wave that is created shall propagate along the pipeline at a rapid but nevertheless finite rate. This results in the wave moving through the system, reaching specific points after period of time (dependent on the wave celerity of the system and the location relative to the position where the disturbance was introduced), provided original steady flow conditions are experienced. It can be understood, therefore, that propagation of the pressure wave results in compression of the water and the deformation of the pipeline as the pressure wave moves through the system. Water is elastic (about 100 times more compressible than steel) and it behaves as a long spring when compressed in a pipeline.

The elasticity of the pipe wall increases the apparent compressibility of the water column. Since water and the pipe wall are elastic:

- The water does not decelerate/accelerate at the same rate instantaneously through the pipeline. During changing in flow and pressure the velocity of the water can vary from point to point at any instant. These changes are required to maintain continuity as a compression caused by pressure.
- It provides ‘storage space’ for the net inflow of the water and to satisfy the momentum relationship.
- In a pressure wave, kinetic energy may be converted to strain energy and vice versa. In practical situations, some energy is dissipated by friction so that the amplitude of the pressure wave gradually diminishes with the distance travelled.
- The behaviour of a pressure wave in a pipeline, the motion of a heavy spring, and the passage of sound waves in air, can be described by the differential equation for one-dimensional waves.

Whenever the pressure rises in a pipeline, some of the pressure energy is absorbed as an elastic energy by the pipe as it expands in response to the increase in pressure. The elastic nature of the pipe affects the wave speed. For a steel pipe the wave speed is given by (3)

$$a = \sqrt{\frac{K/\rho}{1+\left(K/E\right)\cdot\left(D/e\right)}} \tag{3}$$

Where: E - is the modulus of elasticity (E = 210 GPa for steel)
D – pipe inner diameter [m]
e – pipe wall thickness [m]

The celerity of the pressure wave is also influenced by the support conditions of the pipeline and this factor shall be taken into consideration [1] and [2].

3. THEORTICAL APPROACH TO THE PROBLEM

The fundamental equations of transients propagation, based on the elastic water column theory, can be established by solving two simultaneous equations based on the principle of continuity (4) and momentum (5) [1], [2].

3.1 The continuity equation for unsteady flow

The change in mass within a control volume of length ∂x must equal the difference between inflow and outflow through the surface of the control volume (4).

$$a^2\rho\frac{\partial v}{\partial x}+\frac{\partial p}{\partial t}=0 \tag{4}$$

The Equation (4) represents the continuity equation for unsteady flow. Or using the expressions of head H and discharge Q we can rewrite (4)

$$\frac{\partial H}{\partial t}+\frac{a^2}{gA}\frac{\partial Q}{\partial x}=0 \tag{5}$$

3.2 The momentum equation for unsteady flow

Applying the second Newton's law to an element of mass in a pipe (6)

$$\rho\frac{\partial v}{\partial t}+\frac{\lambda\rho}{2D}|v|v+\frac{\partial p}{\partial x}=0 \tag{6}$$

Relationship (6) can be rewritten using the same expressions as above (7)

$$\frac{\partial H}{\partial x}+\frac{1}{gA}\frac{\partial Q}{\partial t}+\frac{\lambda}{2gDA^2}|Q|Q=0 \tag{7}$$

The constants $\rho>0$, $\lambda\geq 0$, $D>0$, $a>0$ are the density of the water, the coefficient of friction, the internal pipeline diameter and the celerity of propagation of the pressure wave, respectively, x is the longitudinal coordinate along the pipeline axis, $v\ (x,t)$ is the mean cross sectional velocity of the water in the pipeline through a cross section with coordinate x at an instant t. It is positive when the liquid flows in the direction in which the coordinate x increases, $p(x,t)$ is the pressure of the water calculated with respect to the level of a reference plane according to equation (8)

$$p=p_r+\rho gh \tag{8}$$

Where $p_r(x,t)$ is the actual pressure in the pipeline and h is the elevation of the pipeline above the chosen horizontal reference plane.

3.3 Solution by the method of characteristics

The equations (5) and (7) represent the basic physics of a conservation of mass and conservation of a momentum. As partial differential equations they have a solution domain that is two dimensional, with one spatial dimension and one time dimension. Applying conventional solution to these equations often results in divergence due to the presence of the nonlinear terms. To solve the complete system of equations (5), (7) with two unknown functions – the pressure head function $H\ (x,t)$ and the flow rate function $Q(x,t)$, the numerical techniques are necessary. Applying of the method of characteristics (MOC) allows these equations to be converted to ordinary differential equations (ODE), dramatically simplifying the solution. However, simplifications can be made for approximate solution of which the best known is due to *Allievi* who derived the following:

- Continuity equation (assuming $v\dfrac{\partial H}{\partial x}$ is small compared to $\dfrac{\partial H}{\partial t}$).
- Momentum equation (assuming $v\dfrac{\partial v}{\partial x}$ is small compared to $\dfrac{\partial v}{\partial x}$).

In application to water hammer problem, it leads to describing motion of two waves traveling in opposite directions in a pipeline and they determine the pressure head H and discharge Q at any instant t and any point x in the pipeline. This method is often used in calculation of an unsteady flow in closed conduits of different hydraulic systems [1].

The basic equations (5) and (7) employed in this paper for the analysis of water hammer in a conduit were derived using many simplifying assumptions. Some of them have been mentioned, whereas others are so common that they are not mentioned. Let us point out some of the more important ones:

- The part of the conduit for calculation has a constant circular cross section. Its diameter is small comparing to the length of the conduit.
- The conduit is completely filled with homogenous water.
- The pressure in the conduit and the discharge of the water are functions of time and the longitudinal coordinate along the axis of the conduit.
- The density of the water and the cross sectional area of the conduit is linear functions of pressure. The elasticity of the conduit and the compressibility of the water are small.
- The pressure losses due to friction in the conduit are proportional to the square of the discharge. The quadratic relationship between pressure losses due to friction and discharge was found to hold for the steady turbulent flow of water. In the presented analysis, however, its validity is assumed also to hold for unsteady flow. Application of the quadratic relationship is satisfactory for cases of slow variation in discharge, where the distribution of velocity throughout the pipeline cross section is approximately the same as at steady flow. The assumption follows from lack of any universal theory describing hydraulic resistance in the unsteady turbulent flow. These considerations have to be taken into account in the interpretation of the calculated results.

3.4 Solution for the sections

The continuity and momentum equations (5) and (7) form a pair of quasi-linear hyperbolic partial differential equations in terms of two dependent variables, velocity and hydraulic-grade line elevation and two independent variables, distance along the pipeline x and time t. The equations are transformed into four ordinary differential equations by the characteristics method. The finite difference approximation based on the method of characteristics using a constant time step Δt and constant distance increment Δx takes the following form.

Using the substitution (9)

$$Q = \frac{\pi D^2}{4} \cdot v \tag{9}$$

We obtain the system of equations (5), (7) converted to the form

$$\frac{4a^2\rho}{\pi D^2}\frac{\partial Q}{\partial x} + \frac{\partial p}{\partial t} = 0 \tag{10}$$

$$\frac{4\rho}{\pi D^2}\frac{\partial Q}{\partial t} + \frac{8\lambda\rho}{\pi^2 D^5}|Q|Q + \frac{\partial p}{\partial x} = 0 \tag{11}$$

The system of equations is solved in the difference form by the method of characteristics.

Along the characteristics C^+

$$x - at = const \tag{12}$$

$$\Delta p + \frac{4a\rho}{\pi D^2}\Delta Q + \frac{8\lambda\rho}{\pi^2 D^5}|Q|Q\Delta x = 0 \tag{13}$$

Along the characteristics C^-

$$x + at = const \tag{14}$$

$$\Delta p - \frac{4a\rho}{\pi D^2}\Delta Q + \frac{8\lambda\rho}{\pi^2 D^5}|Q|Q\Delta x = 0 \tag{15}$$

The grid of characteristics for solving one section of the pipeline is shown in Fig. 1.

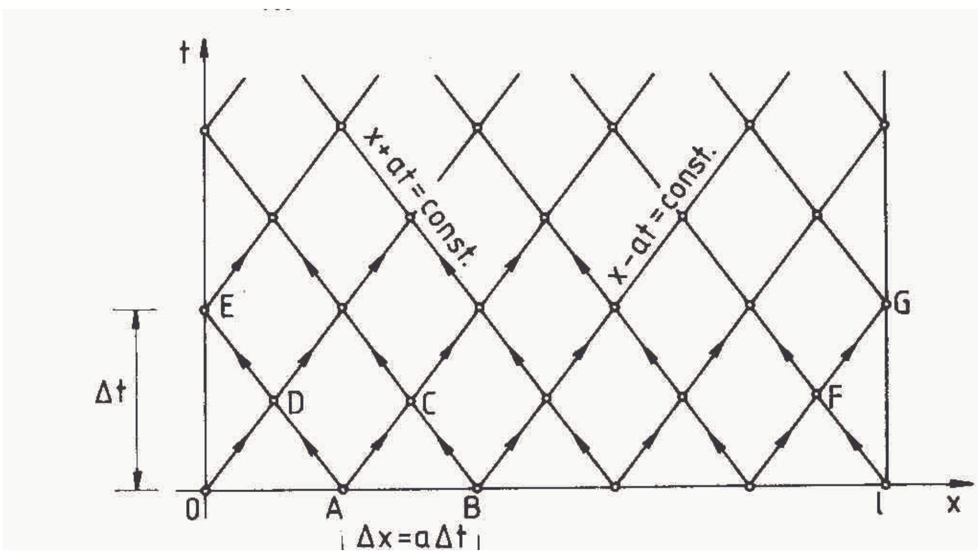

Fig. 1 – Grid of characteristics for the solution of a pipeline system

The grid is defined by the time interval Δt of the calculation, which is common to all sections, by the length l of the section and the celerity a. This also determines the longitudinal interval of the calculation for the section (16)

$$\Delta x = a \cdot \Delta t \tag{16}$$

The solution of the analysis of the section starts from the values p and Q for $t=0$, determined by the initial conditions. It is assumed that the initial flow in the individual section is steady, hence

$$\frac{\partial p}{\partial t} = \frac{\partial Q}{\partial t} = 0 \tag{17}$$

It follows from equations (10), (11) for the initial values that

$$Q(x,0) = Q_0 \tag{18}$$

$$p(x,0) = p(0,0) - \frac{8\lambda\rho|Q_0|Q_0}{\pi^2 D^5} \cdot x \tag{19}$$

Using equations (13), (15) the pressure and discharge at any point (at any time) may be determined from the known pressures at spatially adjacent points (at a previous time see Fig. 1). Thus, the solution may be marched forward through time from a known set of initial conditions.

4. VALIDATION OF THE COMPUTATIONAL METHODS

All the methods developed to calculate transient pressures are based on the equations of continuity and motion and derived laws of fluid mechanics discussed above. The methods are classified as follows:

- Arithmetic.
- Graphic.
- Impedance method.
- Characteristics.
- Others.

All these methods have been used, although the best known method is probably the method of characteristics. It provides a technique for solving transient flow equations and has many advantages:

- Stability criteria are firmly established.
- Boundary conditions are programmable.
- Minor loss terms may be retained if desired.

- Extremely complex systems may be handled.
- It is the most accurate of any of the finite difference methods.
- It allows detailed analyses of complex systems.

The only actual disadvantage of the method of characteristics, as compared with other methods, seems to be the fact, that there are quicker methods or methods with smaller demands on computer time for solving same water hammer problem. One of examples is represented by the solution for a periodically varying flow, when prevailing part of discharge is steady and the periodical variations are only small. Linearization of the pressure losses within the range of the variations of flow is admissible. A solution with the aid of the impedance method, and especially the determination of the natural frequencies of the system, shall be simpler then a solution applying the characteristics method although the later one is able to deal even with such calculations.

The WTHM (abbreviation for water hammer) [1] code has been used extensively in practice in designing of the pipelines and was subject to numerous validations in the past. The results of computations were repeatedly compared with data collected basing on our own and foreign experimental tests carried out under lab or field conditions. Example of steady state discharge calculation prior to emergency shut down from generating mode of Francis turbine operation of the Lipno Power Plant in the Czech Republic is presented below.

4.1 Calculation of discharge from pressure measurement

The WTHM program was used to calculate the discharge through a pipeline from the pressure curves measured during unsteady flow. The changes in pressure were measured between one measured section in front of spherical valve of the turbine referred to the surge tank free surface water level see Fig. 2. We used the pressures measured and calculated at points *1* and *2* during water hammer induced by emergency shutting down of the turbine and closing spherical valve in front of the turbine. Water hammer was calculated only in the pipeline between points *1* a *2*. Carry out the calculations required preparing a large set of data including among other:

- Geometry of the penstock (see Fig. 2 and Tab. 1).
- Steady state characteristics of the model turbine.
- Local head loss coefficients at penstock inlet section and at connections between penstock segments of different diameters.
- Relationship between position of the servomotor piston and opening of the guide vanes.
- The guide vanes and spherical valve closing laws.

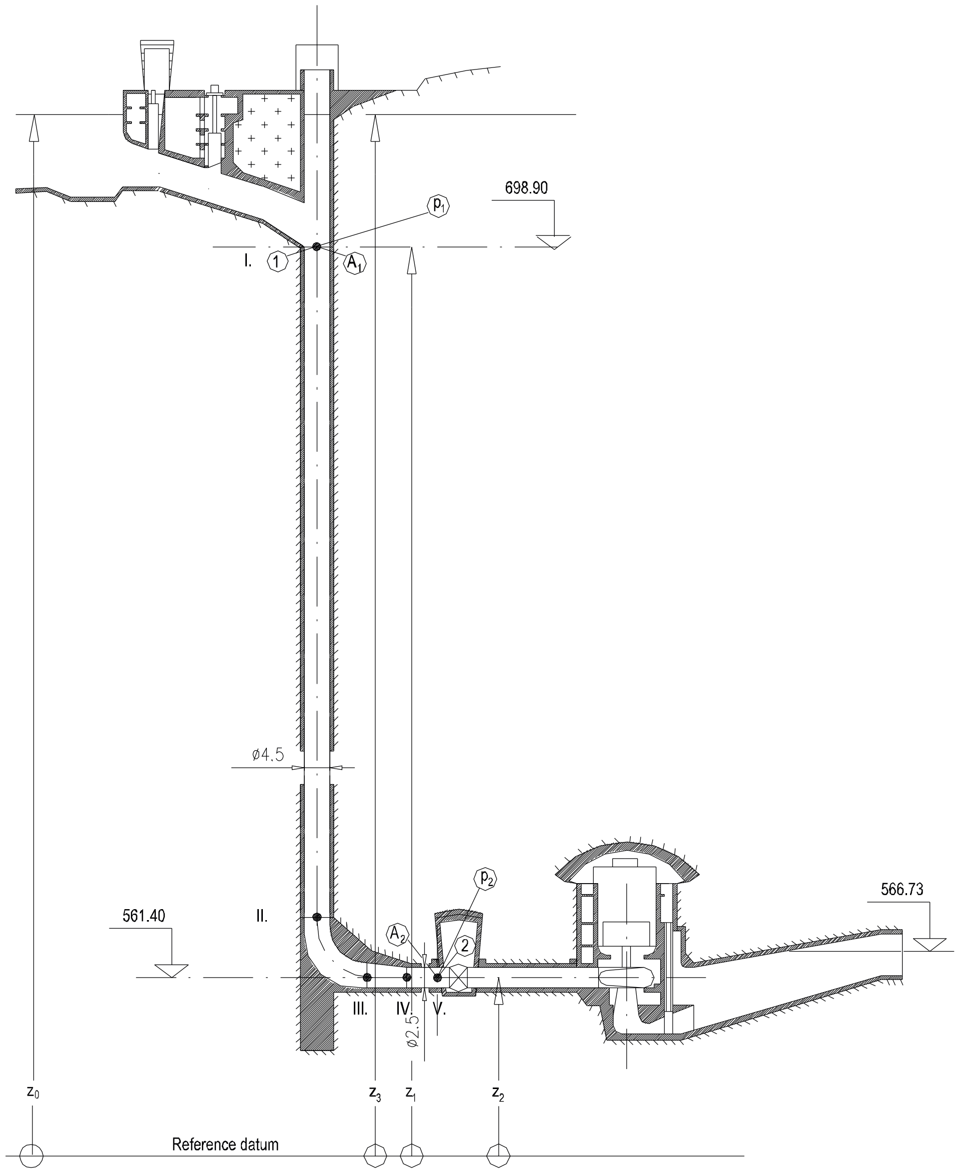

Fig. 2 – Geometry of the penstock in Lipno 1 Hydro Power Plant

The Lipno 1 Power Plant consists of two turbine-generating units. Seventeen fixed-blade Francis turbines are installed there. The water supply passage to each turbine consists of two short rectangular reinforced concrete supply intakes, surge chamber of diameter 5.2 m, vertical steel penstock with elbow, spherical valve of diameter 2.5 m and spiral casing. The draft tube enters to concrete tunnel of length 3.6 km.

The units operate at designed head of 156m and the head varies from minimum 148m to maximum 162m. The designed discharge of one unit is 63.5m^3/s.

Sections of the penstock	Axial length of the sections	Average inner diameters	Average cross section areas	Constant of the method
[-]	L_n [m]	D [m]	A [m^2]	C=L/A [m^{-1}]
Vertical section I-II	123.055	4.500	15.904	7.7374
Elbow II-III	16.199	4.500	15.904	1.0185
Reducing section III-IV	7.100	3.219	8.138	0.8725
Horizontal section IV-V	7.973	2.500	4.909	1.6242
	ΣL=154.327	-	-	ΣC=11.252

Tab. 1 – Dimensions of the penstock used for calculation

The pipeline diameter appears in basic equation (5) only in the term expressing the effect of the pressure losses. Hence, it would seem that its exact value is not very important. However, other terms of the basic equations also depend on its values may be seen in the modified forms of these equations (10), (11). Therefore, its value is significant for the calculations. If one section of the pipeline is made up to several parts of slightly differing diameter, the effective pipeline diameter D_{ef} in this section may be expressed by (20)

$$D_{ef} = \sqrt{\frac{\sum_i L_i}{\sum_i \frac{L_i}{D_i^2}}} \tag{20}$$

The propagation of transient waves in closed conduits can be illustrated by assuming a valve closure in a pipeline that links to the reservoir. At the calculated celerity, the wave travels through the pipeline to a boundary condition (reservoir) and back to the point of origin within a period of time (21) also referred as the pipeline period.

$$T = \frac{2L}{a} \tag{21}$$

At a time $T = \frac{4L}{a}$ when the wave reaches the closed valve, conditions are identical to those at $t=0$, i.e. the instant of valve closure. The wave velocity was calculated from a period of measured pressures $a=1390.06\ ms^{-1}$ which is much closed to the theoretical value $a_t=1400\ ms^{-1}$.

Using *energy equation* to describe flow conditions in the inlet section of the penstock (22)

$$z_0 = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + H_z^{'} + z_1 \tag{22}$$

Where $H_z^{'}$ is loss of head in the inlet parts of the penstock.
The coefficient of friction λ was determined from the difference in the pressure at the upstream and the downstream ends of the considered sections during initial steady state and from initial discharge.
Using Bernoulli equation for steady fluid flow (23)

$$\frac{p_{1s}}{\rho g} + \frac{Q^2}{2gA_1^2} = \frac{p_{2s}}{\rho g} + \frac{Q^2}{2gA_2^2} + \lambda \frac{L}{D_{ef}} \frac{Q^2}{2gA_{ef}^2} \tag{23}$$

$$\lambda = \frac{D_{ef}}{L} \left(\frac{p_{1s} - p_{2s}}{\rho \frac{Q^2}{2A_{ef}^2}} + \frac{A_{ef}^2}{A_1^2} - \frac{A_{ef}^2}{A_2^2} \right) \tag{24}$$

Where: $$A_{ef} = \frac{\pi D_{ef}^2}{4} \tag{25}$$

p_{1s} pressure at junction *1* during initial steady state recalculated to the reference datum (26)

$$p_{1s} = \left((z_3 - z_2) + \frac{1}{2g} \left(\frac{Q}{A_1} \right)^2 \right) \rho q \tag{26}$$

$$p_{2s} = \left(p_2 + \frac{1}{2g} \left(\frac{Q}{A_2} \right)^2 \right) \rho q \tag{27}$$

ξ_{ef1} coefficient of local losses at junction 1

$$\xi_{ef1} = 1 - \frac{A_{ef}^2}{A_1^2} \tag{28}$$

ξ_{ef2} coefficient of local losses at junction 2

$$\xi_{ef2} = \frac{A_{ef}^2}{A_2^2} - 1 \tag{29}$$

The time interval Δt of calculation was chosen to make the length of the section l equal to a whole multiple of the longitudinal interval Δx of calculation (16).

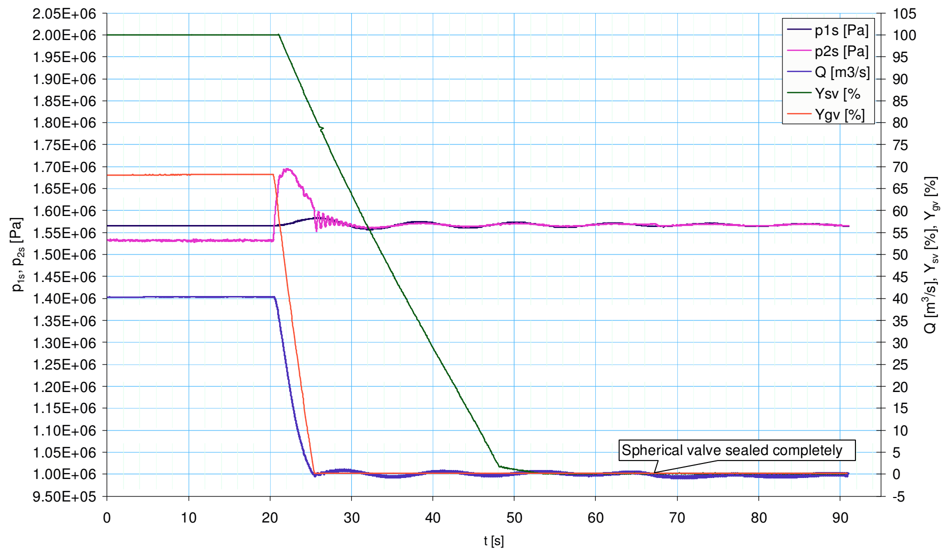

Fig. 3 – Measured pressure curves and calculated discharge by the method of characteristics

To obtain a clearer picture of the behaviour of the transients in the above hydraulic system consider course of pressures measured at the upper and lower ends of the high pressure penstock. The upper one represents pressure oscillations with long periods (proportional to the length of the low pressure intake part) in synchronization with the free water level variation in the surge tank. The lower one represents two pressure oscillations of quite different types. The first oscillation is one of very short periods (proportional to the length of the penstock). This is superimposed on the other, much longer, oscillation that is identical with the oscillation recorded at the upper end of the pressure penstock. The first oscillation of the longer period, is known as *mass oscillation,* and is caused by the water surging back and forth in the intake part between the reservoir and the surge tank. The shorter period oscillations are due to the reflection of the pressure waves in the penstock between the turbine and the surge tank. These pressure waves are the *water hammer or hydraulic transients.*

5. SUMMARY AND CONCLUSIONS

The method of characteristics has been chosen for the analysis of water hammer in this paper. It is the most universal of all the method considered, allowing us to analyse all cases which can be solved by any of the other methods, including very complex hydraulic systems.
The effect of various devices which form part of the hydraulic system, and of the pressure losses along the pipeline, which need not to be linearized in this case, can be concluded with comparative ease.
The method of characteristics employed solves the system of basic equations (4), (6) arranged in the form (10), (11) of two differential equations for two unknown functions $p(x,t)$, $Q(x,t)$ valid along the characteristics (12), (14). The wave velocity is assumed constant. This allows us to predetermine all the intersections of the employed grid of characteristics used, on which the values of the pressure p and the discharge Q shall be determined during calculations. The initial state of flow is entered. The calculation proceed step by step for multiplies of the chosen time interval Δt of the calculation. The boundary conditions for determining p and Q at the end points of the pipeline section are solved by interconnecting the sections and by the devices attached to the pipeline network.
Using this method, the development of an unsteady flow in a known pipeline system can be calculated from knowledge of the initial state of flow and a known adjustment regime of the devices affecting the flow. The results are values of discharge and pressure at all intersection points of the characteristics network considered and the variable parameters of all devices which form part of the network analysed.

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