Optimizing the acoustic discharge measurement for rectangular conduits

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Introduction

A central issue in the acoustic discharge measurement is the method on how to integrate the velocities measured on each acoustic path to determine the discharge. The IEC 60041 code [2] proposes the Gauss-Jacobi for circular and the Gauss-Legendre method for rectangular conduits. For the circular as well as the rectangular measuring section the positions of the acoustic paths and the weights for the velocities are specified. The integration methods given in IEC 60041 base on the assumption of a uniform velocity distribution in the measuring section. This clearly deviates from real flow situation, especially in the wall region. For the circular section an optimized weighted integration (OWICS) has been presented in several papers [1,7] and has proven to be more accurate in theoretical analysis and in measurements. OWICS bases on a more realistic velocity profile which is similar to a fully turbulent profile. The only difference to the corresponding method described in the IEC code is one constant which has been optimized.

The same optimization can be applied to the integration of the discharge in rectangular sections. A more realistic flow profile is chosen as the base of the integration method. The change of the new method OWIRS (optimized weighted integration for rectangular section) to the Gauss-Legendre method is also one constant which is optimized. The better the assumed velocity approximates the prevailing profile the more accurate the integration will be. For the turbulent flow in circular pipe good theoretical approaches are available. For the description of flow distribution in the rectangular pipe little literature is found. In the following CFD simulations are used of a wide range of Reynolds numbers and relative wall roughness to cover different flow profiles, which then can be used for the optimization of the integration.

Another advantage of the new theory is that the weights for the integration can be corrected if the acoustic paths are displaced from their intended position. Whereas the error can be large when using the standard method described in the IEC code, with the new theory the weights can be adapted to the actual, measured path positions and the integration error can in this way be reduced to a minimum.

1 Acoustic discharge measurement in rectangular sections

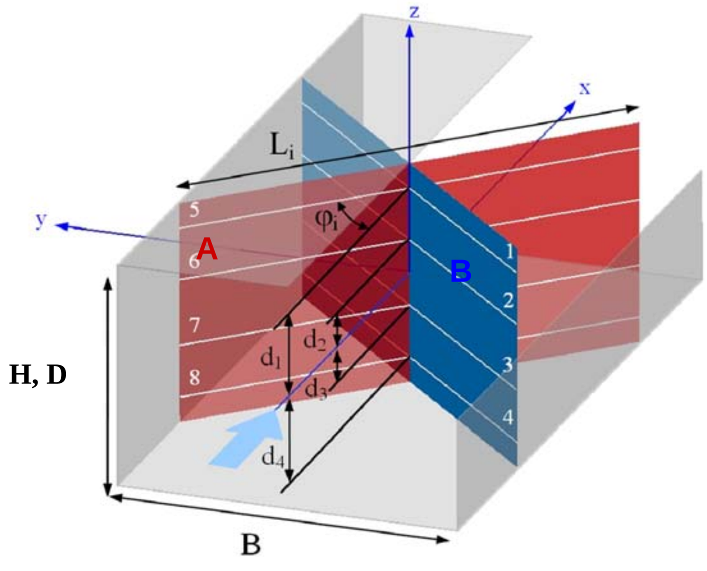


Fig. 1. Rectangular measuring section of a 2x4-path arrangement (two crossed acoustic planes)

Figure 1 shows a typical 2x4-path application in a closed rectangular conduit as it is described in the appendix J of the IEC 60041 code. In each of the acoustic planes A and B four paths are located. The two crossed planes are used to reduce the adverse effect of cross flow in the measuring section.

2 Theory of the numerical discharge integration

Using the transit time difference of an acoustic signal the averaged flow velocity projected on a path can be determined. From these measured velocities the axial velocity components averaged over the width $\bar{v}_{ax}$ can be estimated. To determine the discharge the function $F(z) = \bar{v}_{ax}(z) \cdot B$, the so called area flow has to be integrated.

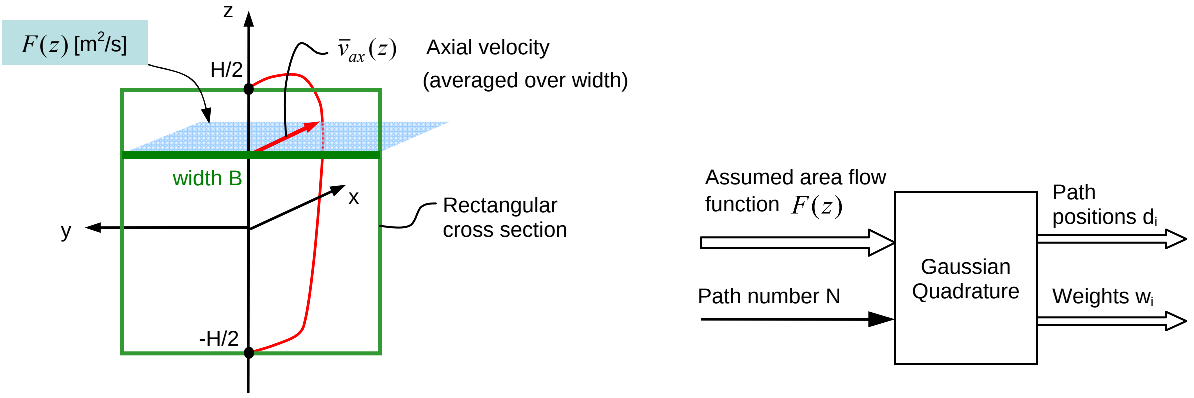


Fig. 2: Area flow function for the rectangular flow cross section (left) and a schematic of the Gaussian Quadrature (right)

The discharge Q results to

$$Q = \int_{-H/2}^{H/2} \bar{v}_{ax}(z) \cdot B \cdot dz = \int_{-H/2}^{H/2} F(z)dz \tag{1}$$

The velocity distribution and so the area flow function F(z) prevailing in the measuring section are unknown. Only at the N positions d_i where the acoustic paths are located the averaged path velocities $\bar{v}_{ax}(d_i)$ are known and the area flow function can be calculated.

$$Q_{real} = \int_{-H/2}^{H/2} F_{real}(z)dz = \int_{-H/2}^{H/2} F(z) \cdot R(z)\, dz \cong \frac{H}{2} \sum_{i=1}^{N} w_i \cdot F_{real}(d_i) = \frac{B \cdot H}{2} \cdot \sum_{i=1}^{N} w_i \cdot \bar{v}_{ax}(d_i) \tag{2}$$

The unknown real flow function $F_{real}(z)$ can be expressed with an assumed flow function F(z) multiplied by a residual term R(z). To determine the discharge numerically the measured velocities have to be weighted and summed up. The Gaussian Quadrature finds the path positions d_i and the weights w_i so that the numerical integration gets exact if R(z) is a polynomial of a degree less then 2N. For that the path number N and the assumed area flow function F(z) must be provided (see Figure 2, right). The corresponding theory of the Gaussian Quadrature applied to the acoustic discharge measurement is described by Tresch [1] and in more general terms by Stewart [3] and Press [4].

Two statements can be made at this point: If a larger number of acoustic paths are used R(z) will be a polynomial of a higher degree for an exact integration. Therefore the integration accuracy increases with a larger number of paths. On the other hand the better F(z) approximates $F_{real}(z)$, the more ineffectual R(z) becomes and therefore the more accurate the numerical integration gets.

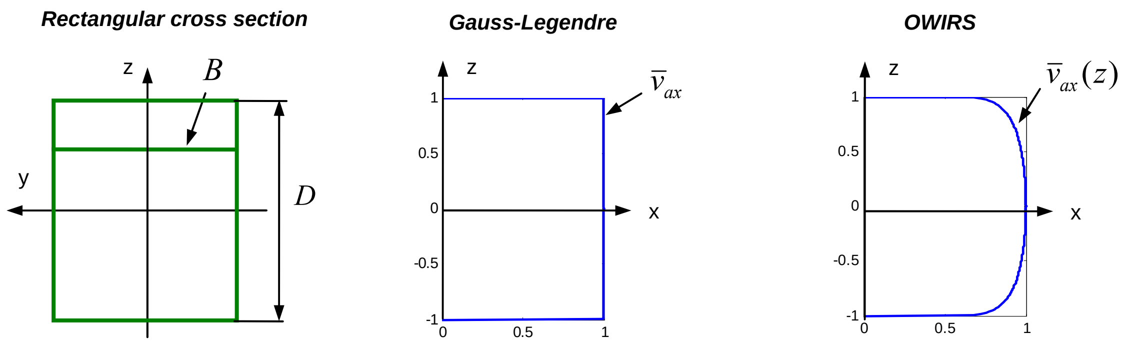
The Gauss-Legendre method described in the IEC 41 code bases on F = constant which stems from the fact that a uniform velocity profile is assumed. This does not reflect reality, especially close to the wall regions. The aim of this work is to find a better area flow function for rectangular flow cross sections which will lead to a more accurate integration. For circular cross section this has been realized by Voser [7] who introduced the OWICS method (Optimized Weighted Integration for Circular Sections).

Both methods for circular sections, the OWICS method and the Gauss-Jacobi method described in the IEC 60041 code, base on the concept of the following area flow function.

$$F(z) = C\left(1 - \frac{z^2}{(D/2)^2}\right)^{\kappa} \tag{3}$$

with κ = 0.5 for the Gauss-Jacobi and κ = 0.6 for the optimized method OWICS.

The same concept can also be used for the rectangular section as shown in Figure 3.



$$F(z) = \bar{v}_{ax} \cdot B$$
$$= \bar{v}_{ax} \cdot B \cdot \left(1 - \frac{z^2}{(D/2)^2}\right)^0$$

$$F(z) = \bar{v}_{ax}(z) \cdot B$$
$$= \hat{v}_{ax} \cdot B \cdot \left(1 - \frac{z^2}{(D/2)^2}\right)^{\kappa}$$

Fig. 3: Assumed area flow function of the Gauss-Legendre method based on a uniform velocity distribution and of the OWIRS method of a more realistic velocity distribution

For the Gauss-Legendre method the exponent κ of the standard area flow function (3) has to be set to zero for a uniform velocity distribution. For the OWIRS method κ has to be set to a value which best approximate the area flow function with the averaged velocity distributions $\bar{v}_{ax}(z)$ of fully developed turbulent flows. Figure 4 shows the area flow function for varying values of κ.

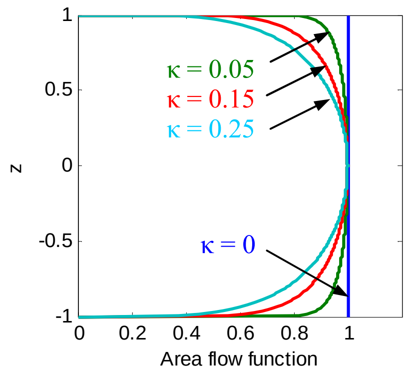


Fig. 4: Area flow functions for varying κ

The turbulent flow profile in conduits depends on the Reynolds number and the relative wall roughness. To choose an adequate value for κ the flow profiles in rectangular conduits have to be known. Whereas for circular cross sections Schlichting [5] gives analytical approaches for the profile, no such approaches could be found in literature for rectangular conduits. At this point it makes sense to use CFD simulations to predict the velocity profiles.

3 CFD simulations

As the flow profile changes with the Reynolds number and relative wall roughness, the range has to be identified for which κ has to be optimized. For conduits in hydro power plant the following ranges have been found representative: A range of 10^5 to 10^8 for the Reynolds number and a range of 10^{-5} to 10^{-3} for the relative wall roughness k_s/D [7]. To cover also the influence of the shape of the rectangular conduit a square section as well as a section with a width-to-height ratio of 3:1 were simulated.

The mesh used in the simulations was generated with the ICEM-CFD software. The computations were performed with ANSYS CFX 11.0, a commercial finite volume CFD code. The turbulence model used for all the computations is the Reynold stess model SSG. A translational periodical source term was applied to generate a fully developed flow profile in a short piece of a rectuangular conduit. For the walls a roughness was defined.

Figure 5 shows the flow profile as well as the secondary flow of the square conduit on the right side. The secondary flow is not relevant for the acoustic discharge measurement as the distribution is symmetric to the vertical axis, so that the secondary flow component is cancelled when measuring the average velocity along an acoustic path.

In Figure 6 the flow profile and the secondary flow are displayed for the rectangular conduit with a width-to-height ratio of 3:1.

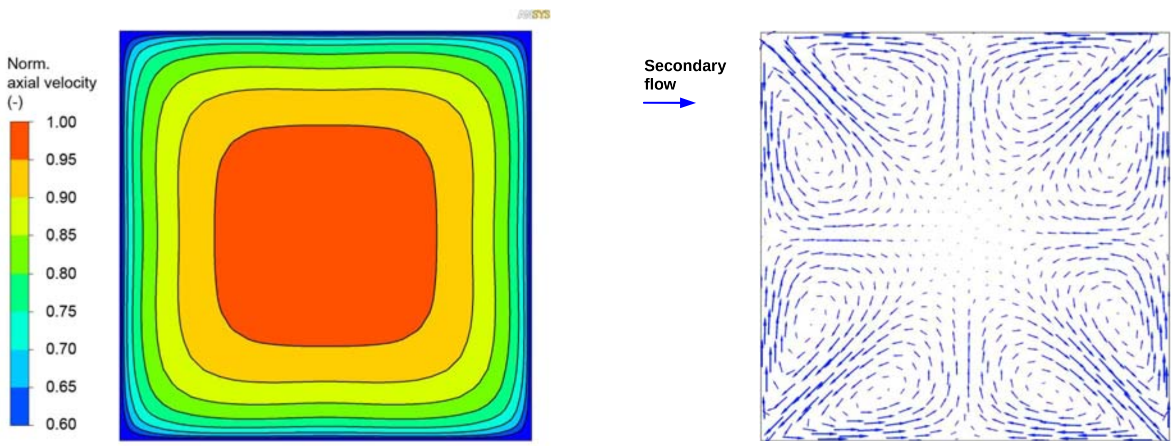


Fig. 5: Normalized axial velocity distribution (left) and secondary flow (right) in the squared conduit for Re = 10^7 and $k_s/D = 10^{-4}$

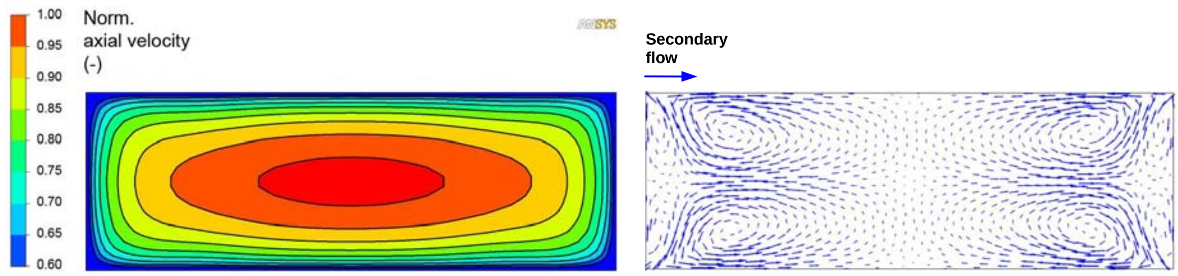


Fig. 6: Normalized axial velocity distribution and secondary flow in the rectangular conduit (B:H = 3:1) for Re = 10^5 and $k_s/D = 10^{-4}$

From the plane velocity distribution the area flow function was determined by averaging the velocity along the width. In Figure 7 the normalized area flow function is plotted versus the vertical axis z for different parameter settings. The area flow function changes slightly for different Reynolds numbers and relative wall roughness.

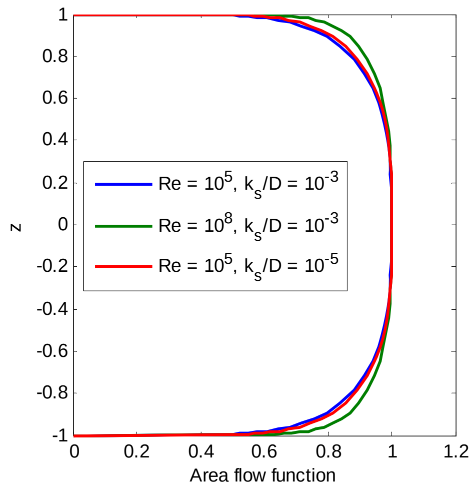


Fig. 7: Area flow function of the simulations for three different parameter settings

4 Optimization

In the process of optimization of the value of κ the discharge of an acoustic measurement was determined for all simulated profiles of the square conduit according to (4). Other path positions d_i and weights w_i are obtained for each value of κ. The discharge Q_{ADM} is compared to the reference discharge Q_{ref} extracted from the simulation. The integration error $e_{Integration}$ corresponds to the relative deviation. For this optimization always four path are applied (N = 4) as this is the proposed path number in the IEC 60041 code.

$$Q_{ADM} = \frac{B \cdot H}{2} \cdot \sum_{i=1}^{4} w_i \cdot \bar{v}_{ax}(d_i) \tag{4}$$

$$e_{Integration} = \frac{Q_{ADM} - Q_{ref}}{Q_{ref}} \tag{5}$$

Figure 8 shows the integration error for the Gauss-Legendre and the OWIRS method for the simulated cases. The mean integration error for each value of κ is plotted in the right graph. It can be seen that the mean integration error for κ = 0 is approximately 0.6%. The optimized value of κ is 0.15 for the simulated range since the mean integration error gets nearly 0% for this value.

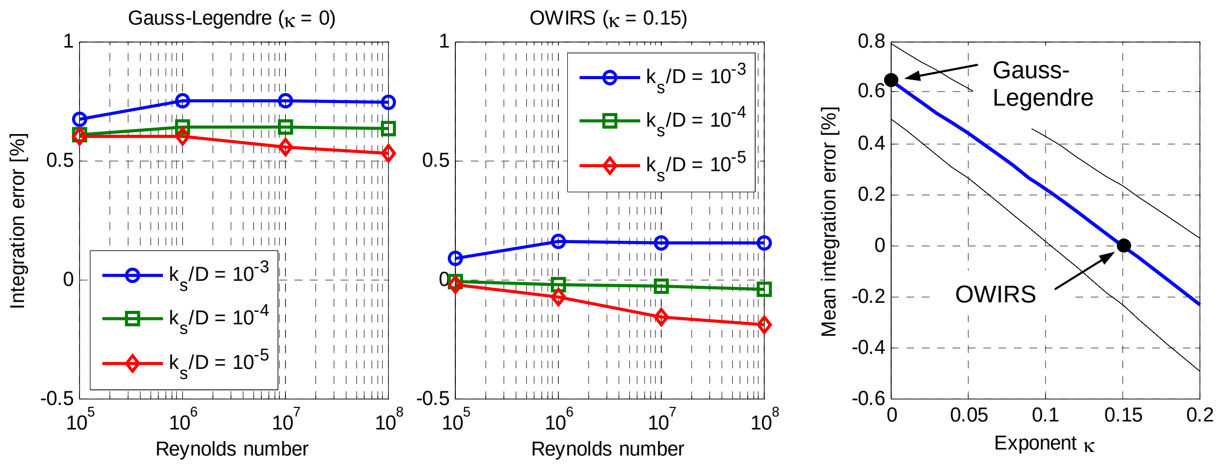


*Fig. 8: Integration errors for Gauss-Legendre (left) and for OWIRS (middle) and the mean integration error as a function of κ with thin black lines as ± 2*standard deviation σ (left)*

For the optimization only the velocity profiles of the squared conduit are considered. The reason is that the velocity profiles of rectangular conduits with a width-to-height ratio of less or larger than one are expected to be different. With a squared conduit the medial case is used to optimize the integration. The simulations of the rectangular conduit with B/H ≠ 1 is considered to check the robustness of the optimization when the cross section is not a square.

For the optimized value of κ optimized path positions and weights can be determined. For four paths these data are listed in Table 1. Of course, such path positions and weights can be calculated for any number of paths. In Tresch et al. [8] the values for a path number of 1 to 9 are presented.

	Gauss-Legendre method		**OWIRS method**	
	Paths 1 and 4	Path 2 and 3	Paths 1 and 4	Path 2 and 3
$d_i/(D/2)$	±0.861136	±0.339981	±0.844510	±0.329729
w_i	0.347855	0.652145	0.356143	0.634200

Tab. 1: Path positions $d_i/(D/2)$ and weights w_i for four paths for the Gauss-Legendre and OWIRS method

5 Analysis of the integration error

To prove that the integration error decreases with an increasing path number a simulation of the integration was performed for varying path numbers. The CFD-simulated flow profile of the square conduit at a Reynolds number of 10^7 and a relative wall roughness k_s/D of 10^{-4} was used for this path number variation. Figure 9 demonstrates the smaller error of the OWIRS method and the decreasing integration error with increasing path number.

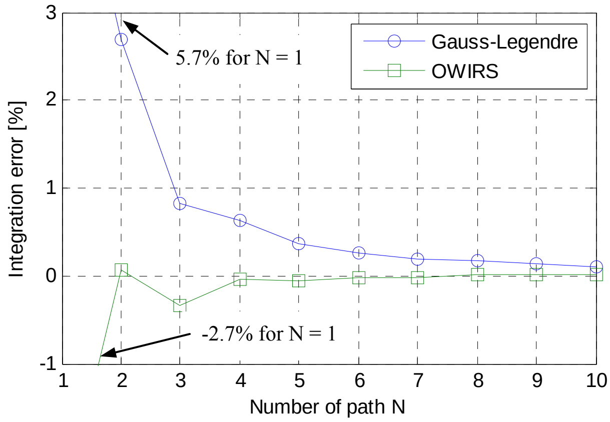
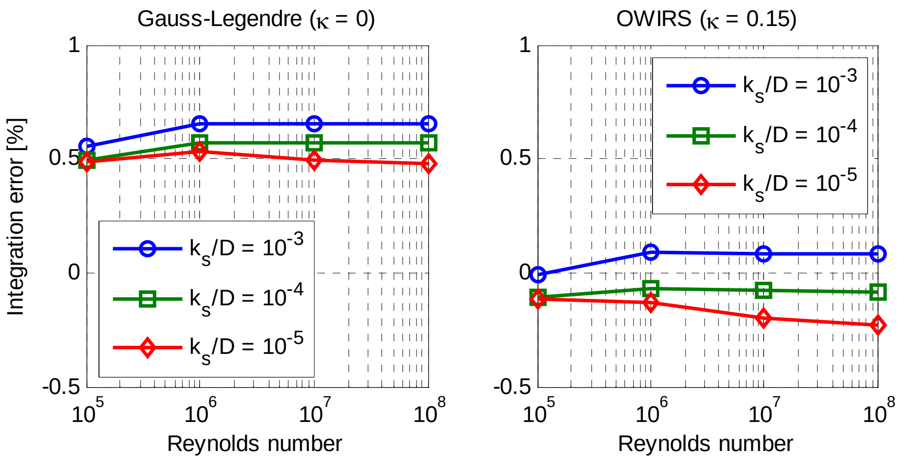


Fig. 9: Integration error as a function of the number of paths

As the flow profile in rectangular conduits also depends on the width-to-height ratio the integration errors of an ADM application in a section with a width-to-height ratio of 3:1 was analyzed. Four paths were arranged once in horizontal orientation and once in vertical orientation. The integration error was determined for the already mentioned range of the Reynolds number and relative wall roughness. Figure 10 shows the integration error for the Gauss-Legendre and the OWICS method for the simulated settings for the horizontal path orientation. The integration error for the vertical path orientation is displayed in Figure 11.



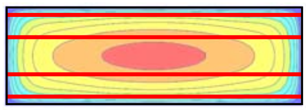
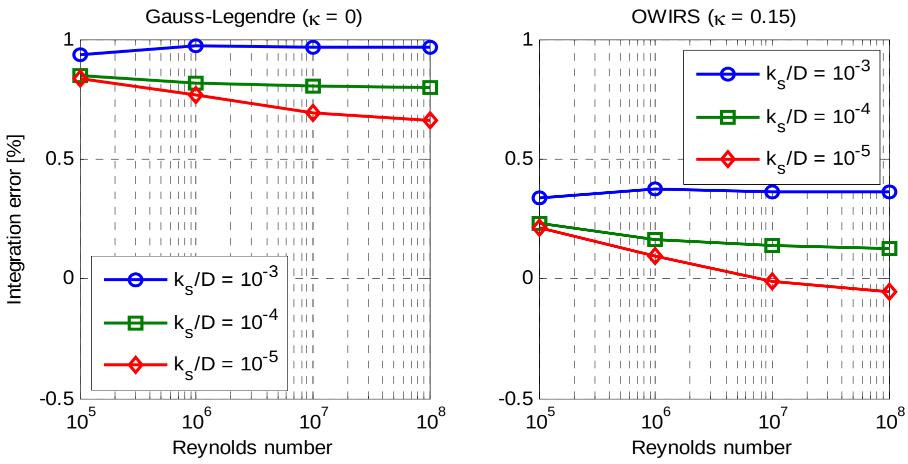


Fig. 10: Integration error for the Gauss-Legendre and the OWIRS method for a horizontal path orientation in the rectangular section 3:1



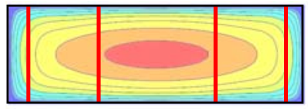


Fig. 11: Integration error for the Gauss-Legendre and the OWIRS method for a vertical path orientation in the rectangular section 3:1

From this case study it can be concluded that also for velocity distributions prevailing in a rectangular conduit with a width-to-height ratio of 3:1 the OWIRS method has much smaller integration errors compared to Gauss Legendre. The average integration error for the OWIRS method is 0.07% for a horizontal path orientation and 0.19% for a vertical path orientation. For the Gauss-Legendre method the average integration error is 0.56% and 0.84%, respectively.

6 Non-ideal path positions

For the circular section with four acoustic paths it was shown by Tresch et al. [1] that the weights can also be calculated by analytical equations using the actual path positions. Using these equations (6) the weights can be corrected in case of incorrect path positions and the integration error can be reduced relative to the proposed weights in the IEC 60041 code. The same can be done for the rectangular section. Only the presented coefficients g_1 and g_2 and the exponent κ had to be recalculated for the rectangular section.

$$w_1 = \frac{0.25 g_2 D^2 (d_3 + d_4 - d_2) - g_1 d_2 d_3 d_4}{(1 - 4 d_1^{\,2} / D^2)^{\kappa} (d_1 - d_2)(d_1 + d_3)(d_1 + d_4)} \qquad w_2 = \frac{0.25 g_2 D^2 (d_3 + d_4 - d_1) - g_1 d_1 d_3 d_4}{(1 - 4 d_2^{\,2} / D^2)^{\kappa} (d_2 - d_1)(d_2 + d_3)(d_2 + d_4)}$$

$$w_3 = \frac{0.25 g_2 D^2 (d_1 + d_2 - d_4) - g_1 d_1 d_2 d_4}{(1 - 4 d_3^{\,2} / D^2)^{\kappa} (d_3 - d_4)(d_1 + d_3)(d_2 + d_3)} \qquad w_4 = \frac{0.25 g_2 D^2 (d_1 + d_2 - d_3) - g_1 d_1 d_2 d_3}{(1 - 4 d_4^{\,2} / D^2)^{\kappa} (d_4 - d_3)(d_1 + d_4)(d_2 + d_4)} \tag{6}$$

Gauss-Legendre:	κ = 0	g_2 = 4*0.166667 = 2/3	g_1 = 2.000000
OWIRS:	κ = 0.15	g_2 = 4*0.139188 = 0.556753	g_1 = 1.837286

In this paper only the equations for four paths are listed. The adoption of the weights to the actual, measured position can be done for any number of paths. Equations for several path numbers are presented in Tresch et al. [8] for the different integration methods.

The integration error is investigated here for the case when one of the paths is positioned incorrectly. The flow profile of the square conduit at a Reynold number of 10^7 and a relative wall roughness k_s/D of 10^{-4} was used with a 4-path arrangement. Firstly the inner path was incorrectly positioned and secondly the outer path.

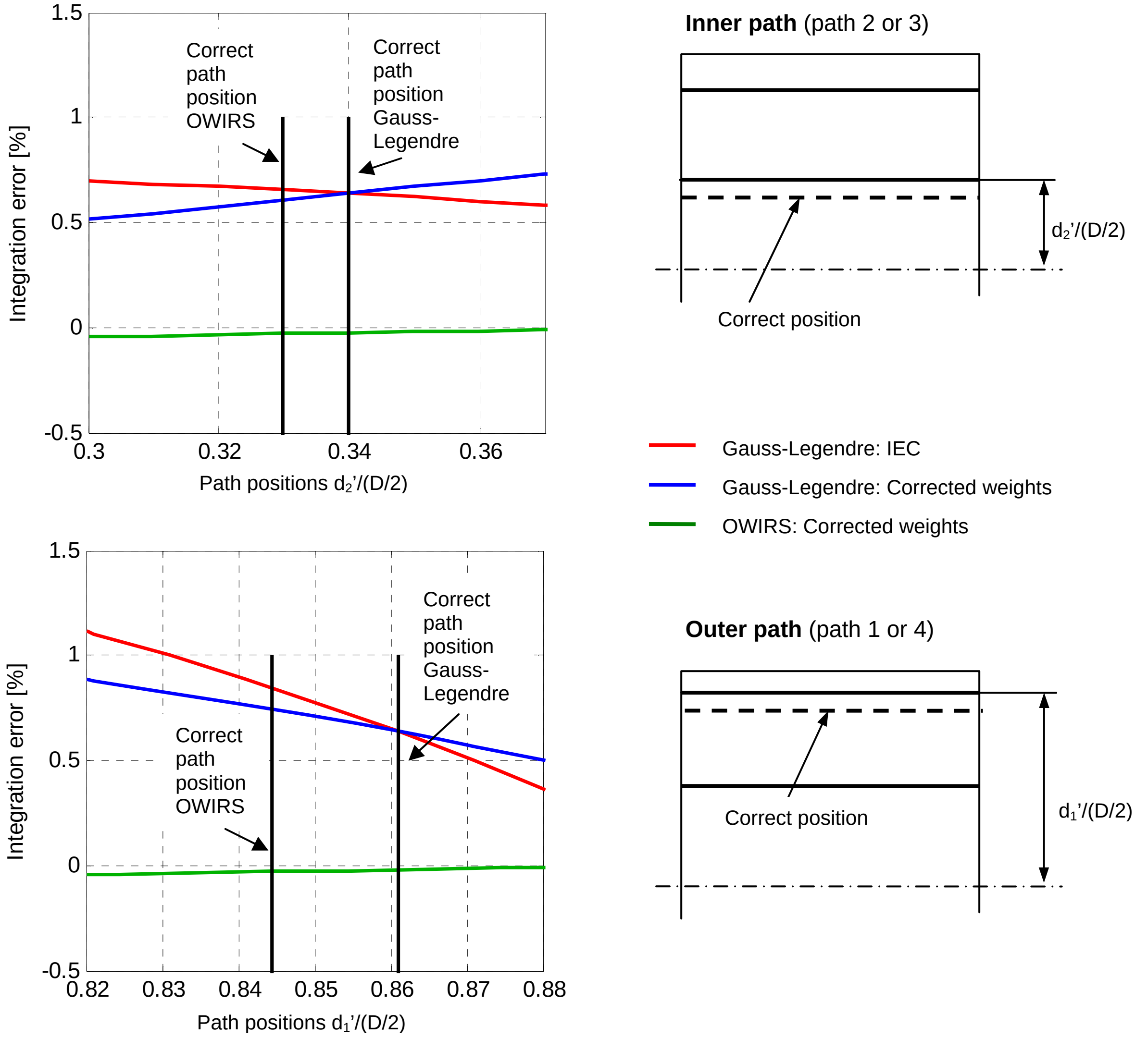


Fig. 12: Integration error as a function of incorrect positioned paths (correct positions are indicated on the left with the black lines)

Figure 12 shows the integration error for varying incorrect path positons d_1'/(D/2) and d_2'/(D/2) which deviate from the correct path position d_1/(D/2) and d_2/(D/2) listed in Table 1. The integration error of the Gauss-Legendre method (IEC) can be large in case of incorrect path positions even when the weights are corrected with the equations (6). For the OWIRS method, however, the integration error does practically not increase if the weights are corrected. If the paths are located according to the IEC code this can be interpreted as a displaced path position for the OWIRS method. The weights can be recalculated using (6) for the OWIRS method and the integration error remains small. This allows applying the optimized OWICS method to existing installations where the sensors were installed according to IEC 60041.

7 Conclusion

The introduced OWIRS method for the integration of the discharge for the acoustic discharge measurement in rectangular section corresponds to the approved optimized OWICS method for circular section. Basing on a turbulent flow profile the exponent κ was optimized (κ = 0.15). For this optimization CFD simulations of rectangular conduits were performed for a wide range of Reynolds numbers and relative wall roughness. Not only for a 4-path ADM but also for a path number between 1 and 9 the integration error could be reduced considerably compared to the Gauss-Legendre method described in the IEC 41 code. Also for the simulations of rectangular conduits with a height-to-width ratio of 3:1 the OWIRS method showed important improvements. An additional finding is that the integration errors observed in rectangular sections is generally larger than in circular sections.

In case of inexact path positions integration errors can get large. Equations are presented with which the weights can be calculated according to the actual path positions. The OWIRS method has to be used in such situations in order to keep the integration error small .

The difference between the optimized method OWIRS and the Gauss-Legendre method is only one constant. This results in other path positions and weights. Even for path installations which have been carried out in the past according to IEC 60041 the accuracy of the discharge measurement can be improved by recalculating the weights according to OWIRS.

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