Uncertainty of the Hydrometrical Methods

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Abstract

The current requirements for optimal hydro systems operation and waters quality protection have led to the wide development of methods and devices for discharge measurements. The legislation of the EU countries stipulates these methods and devices under stringent metrological control for providing the needed accuracy and comparability of the results. According to the recommendation of the International Organization for Standardization "uncertainty" is adopted as criterion for evaluation the measuring methods, which by definition represents the half of the confidence interval within the true value of measured quantity can be expected to lie with stated probability.
These estimations in Bulgaria meet many practical difficulties related mainly to the lack of systematic data concerning some of the parameters in the uncertainty calculation.
In this paper the results of one generalized investigation of the hydrometrical methods uncertainty which are applied in the Bulgarian water economy are presented. A brief theoretical description of selected methodology is made and the results obtained from mathematical modeling of unknown parameters as well as the probable values of uncertainty of dilution and tracer velocity-area methods are presented. The accomplished comparative analysis facilitates the choice of suitable measuring method under the conditions and goals of the measurement undertaken in any specific case.

Key words: *Uncertainty, Uncertainty Type B, Errors, Uncertainty modeling, Dilution Method Uncertainty, Tracer Velocity-Area Method Uncertainty.*

Introduction

The reliable and rational hydro systems planning, organization, management and control are based on the authentic and accurate primary data related to the water sources and water demand regimes. Taking into consideration that all the data obtained from direct measurements and observations a common conclusion could be made that they are affected by the personal skills and used techniques of the measuring team. In order to unify the measurement results in a unity system for an explicit interpretation in accordance with the metrological standards [1, 2, 3, 4, 5] it is adopted a two components presentation, reported as:

OBSERVED VALUE $\pm$ MEASUREMENT UNCERTAINTY

The measurement uncertainty gives the range of values within with the true value of measurand is expected to lie with a stated coverage probability. It represents the dispersion of the observed value and can be defined as a description of the measurement error. In metrology it is adopted as the positive or negative half-width of a symmetric coverage interval, centered around the estimate of a quantity with a specified coverage probability. It is associated with the mathematical confidence interval that represents values for the population parameter as the difference between the parameter and the observed estimate is not statistically significant at the certain confidence coefficient α :

$$P\left(\left|\bar{x} - m_x\right| < \delta\right) = p \qquad (1)$$

Where m_x is associated with the true value of the measurand;

$\bar{x}$ is the observed estimate, treated as a random variable;

$p = 1 - \alpha$ is the chosen confidence probability (confidence level).

Theoretical basis

The uncertainty of the result of measurement generally consists of several components which may be grouped into two categories according to the method used to estimate their numerical values:

- Uncertainty type A – based on any valid statistical method for data treating;

- Uncertainty type B – based on the scientific judgment adapting all the relevant available information – previous measurement data, knowledge and experience of the behavior of investigated value, characteristics of measurement instruments, manufacturer's specifications and uncertainties assigned to the relevant standards and handbooks.

In this meaning the **Uncertainty type A** evaluates measuring data and the results are expression of the specific measurement conditions. This approach to the problem is known and theoretically well-founded and the final results are representative for the observed case. In accordance with the standards it is recommended that each user must determine independently the values of the uncertainties for any particular measurement.

From metrological point of view more applicable is the **Uncertainty type B**. It is usable for methods assessment and the obtained evaluation is useful for comparative analysis of the separate method applicability. It could be used as a representative estimation of the method accuracy.

It is not usual this classification to correspond with the uncertainty considered with random and systematical errors that is correct to be defined as:
- Uncertainty components arising from random effects;
- Uncertainty components arising from systematic effects.

The representing of each uncertainty component that contributes to the uncertainty of a measurement result by a statistically estimated standard deviation termed standard uncertainty with suggested symbol u_i and is represented by a statistically estimated variance $S'_{\bar{x}}$.
When the statistically estimated variance is given with a stated coverage factor, giving a coverage interval intended to contain the value of the measurement with high probability it is used the term expanded uncertainty:

$$U = k_p S'_{\bar{x}} \tag{2}$$

Where k_p denotes the coverage factor by the state confidence probability.

Type A evaluation of uncertainty
Evaluation Type A is based on any well known statistical methods for evaluation of the measurement results by calculation of the standard deviation of the mean. Practically the results of a series of independent observations are taken as a sample for the calculation of the statistics:
- sample mean:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{3}$$

- sample standard deviation:

$$S = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n-1}} \tag{4}$$

- standard uncertainty associated with the standard deviation of the mean:

$$S'_{\bar{x}} = \frac{S}{\sqrt{n}} = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n(n-1)}} \tag{5}$$

And by (2) is calculated the expended uncertainty.

In case of sample with generated normal distribution (based on the Central Limit Theorem) the coverage factor k_p is chosen by the $3\sigma, 2\sigma, \sigma$ rules taken for confidence probability 99,74%, 95,44% and 68,26% got from the basic normal distribution equation:

$$P\left[\left(m_x - \delta\right) < x < \left(m_x + \delta\right)\right] = \phi\left(\frac{\delta}{\sigma}\right) - \phi\left(-\frac{\delta}{\sigma}\right) = 2\phi\left(\frac{\delta}{\sigma}\right) - 1 \tag{6}$$

Where $\delta = z\sigma$ denotes the maximum standard deviation.

When the population standard deviation is unknown and has to be estimated from data the Student`s t-distribution is applicable. The *t*-factor is taken as a coverage factor. It depends on the state confidence level and on the sample degrees of freedom $\nu = 1 - n$. The overall shape of the probability density function of the t-distribution resembles the bell shape of a normally distributed variable with mean 0 and variance 1. In case of increasing of ν values it becomes closer to the normal distribution and for $n \geq 30$ in case of mean evaluation it is almost the same as the normal distribution [6]. This gives opportunity to make a parallel with the normal distribution about the coverage factor for degrees of freedom $\nu = \infty$ [2, 3] presented in *Table 1*:

Table 1 Value of t_p from the t-distribution for degrees of freedom $\nu = \infty$ that defines An interval – $t_p(v)$ to + $t_p(v)$ that encompasses the fraction p of the distribution

p, %	68,27	90	95	95,45	99	99,73
k_p	1	1,64	1,96	2,0	2,58	3,00

At the recommendation of the International Standards [1,2,3,4,5] the 95% confidence level must be taken and the coverage factor is denoted as *k* as implicit with value 2. In case of other solution the state of the confidence level must be denoted in the lower part as for example: k_{68}, k_{90}, k_{99} corresponded to 68%, 90% and 99%.

Type B Evaluation of Uncertainty

The evaluation type B as associated with the application of specific scientific and practical knowledge. Most common it is used for evaluation in pre-studying cases, research of the material behavior and characteristics, calibration data and specification of the measurement equipments data and its transformation in standard uncertainty based on the probability theory, mathematical modeling and the methods if the mathematical statistics. Thus from the primary available information it is calculated the standard deviation and the standard and expanded uncertainty.

Combined Uncertainty

The main specificity of discharge measurement methods is that the measurand is not measured directly, but is determined as a result of functional related measurements. Their relation should express not simply a physical law but a measurement process, and in particular, it should contain all quantities that can contribute a significant uncertainty to the measurement result.

If $f_R(R_1, R_2, ..., R_n)$ is the function that gives the relations between the independent non-linear combination measurement results their propagation must be linearlized by approximation to a first order Maclaurin series expansion – a Taylor series with zero derivative:

$$f_R \approx f_R^0 + \sum_1^n \frac{\partial f_R}{\partial R} R_i + \sum_i^n \sum_{j(i \neq j)}^n \frac{\partial f_R}{\partial R_i} \frac{\partial f_R}{\partial R_j} R_i R_j \qquad (7)$$

Where $\frac{\partial f_R}{\partial R_i}$ denotes the partial derivate of f_R with respect to the *i*-th variable. In the metrology it is known as influence (dimensional sensitivity) coefficient of the quantity R_i : $\theta_i = \frac{\partial f_R}{\partial R_i}$.

Then the propagation of error follows the linear case:

$$\sigma_R^2 = \sum_{i=1}^n \left(\frac{\partial f_R}{\partial R_i} \right)^2 \sigma_R^2 \qquad (8)$$

Thus the measurement values are different by type and their investigation may be rendered dimensionless by writing

$$\theta_i' = \frac{\partial f_R / f_R}{\partial R_i / R_i} \qquad (9)$$

and

$$S_R' = \sqrt{\sum_1^n \left(\theta_i' S_{R_i} \right)^2} \qquad (10)$$

Where $S_{R_i} = \frac{S'_{\overline{R_i}}}{\overline{R_i}}$ is the relative standard deviation of the mean.

The expanded standard uncertainty can be denoted as:

$$U = kS'_R \qquad (11)$$

From practical point of view this expression denoted the measuring method uncertainty.

The uncertainties of the measurement instruments may be taken as a systematic uncertainty. Then it may be calculated as:

$$B_R = \sqrt{\sum_{1}^{m} B_i} \qquad (12)$$

According to the standards two types of combination are possible [4,5]:
Linear edition:

$$U_{ADD} = B + k_p S'_R \qquad (13)$$

Root-sum-square combination:

$$U_{RSS} = \pm\sqrt{B_R^2 + k_p^2 S'^2_R} \qquad (14)$$

But more representative is the separated presentation as follow:

Measurement result $\overline{X}$
Measurement method uncertainty $U = kS'_R$
Uncertainty arising from systematic effects: B_R

MATHEMATICAL MODELING OF THE HYDROMETRICAL MEASURING METHODS UNCERTAINY

The practical determination of the uncertainty value for each measuring method is difficult for realization because of the restricted practical opportunities for implementation of sufficiently measurement series in the possible applied range. More successful is the implementation of the mathematical modeling. Then preceding an evaluation type B it is possible to become a theoretically well sustained, reliable and stabile result about the uncertainty.
Taking into consideration that the measuring methods uncertainty comprises a lot of components such as measuring characteristics definition; nature, principle and procedure of the measurements; instruments and working group precision, software and calculation components, constants each measurement method needs an individual approach according to the specific circumstances. But it is not directly related with the individual measurement result. Moreover it is possible to be calculated without the presence of any available measurement results as it is taken the expected by a state probability standard deviation even before its realization.
The specified methods in addition to the scientific knowledge, practical experience, data provided by previous measurement, calibration and manufacturer specifications and uncertainties assigned to handbooks and standards by a reasonable mathematical model can adapt the nature of each measurement method. Joined in an unity system they can be used for evaluation of the uncertainty in the sequence:

1. Adduce the equations for calculation of the variety hydrometrical methods in general form;
2. Specify the measurement parameters from the computation composition;
3. Calculation of the relative influence coefficient for each parameter;
4. Representation of the uncertainty equation in expedient form;
5. Specify of a realistic variants for the probability investigation, assuming wide range for variation of the measurement values and expert evaluation of the possible maximum field errors;
6. Calculation the rate S' of the parameters used in the estimation equation;
7. Calculation of the result uncertainties for the investigated hydrometrical method;
8. Analysis of the investigation results and a recommendation for the appliance of the method.

The calculation of the rate S'_R for every particular parameter is made by mathematical modeling with the help of the random variables and on a random principle the possible values of the statistic are generated in a sample by the theoretical treatment that with an expert established absolute maximum error Δx (with confidence probability 100%) the measurand is expected to be in the interval:

$$\bar{x} - \Delta x \le x_i \le \bar{x} + \Delta x \tag{15}$$

Then taking into consideration the absolutely random nature of x_i :

$$x_i = \bar{x} - \Delta x + 2RND.\Delta x = \bar{x} + \Delta x(2RND - 1) \tag{16}$$

Where RND is a random number within the interval $0 \le RND \le 1$.

This model is realized by numerical experiments with a program SIGMA where was settled that sufficient number of the sample is $n = 1000$ to become a stabile S' .

For illustration of the practical potential of this methodology the results of implementation of the uncertainty modeling for two practically applied methods are presented. In the calculation is accepted confidence level 95% and coverage factor $k = 2$ corresponded to $\nu = n - 1 = \infty$.

DILUTION METHOD

The dilution method equation for discharge is:

$$QC_0 + qC_1 = (Q + q)C_2 \tag{17}$$

where Q is the discharge being measured;

q - discharge of the strong solution injected into the flow;

C_0 - natural background of the tracer of the flow;

C_1 - concentration of the strong injected tracer solution;

C_2 - concentration of tracer after full mixing at the sampling station, including the background concentration of the stream;

As $C_1, C_2 >> C_0$ equation (17) can be modified as:

$$Q = q\frac{C_1}{C_2} \tag{18}$$

Measurand values: $q,\ C_1,\ C_2$.

Uncertainty equation: $U = \pm 100.t_{95}\sqrt{S_q'^2\,\theta_q^2 + S_{C_1}'^2\,\theta_{C_1}^2 + S_{C_2}'^2\,\theta_{C_2}^2}$ % (19)

Relative influence coefficient: $\theta_q = 1;\ \theta_{C_1} = 1;\ \theta_{C_2} = -1$

Investigated variants and final results:

With variation of the participated parameters and expert established maximum errors eight boundary variants are investigated:

Discharge of the strong solution injected into the flow q and Δq: 0,002/3.10^{-5} и 0,0010/3.10^{-5} m^3/s

Concentration of the strong injected tracer solution C_1 and ΔC_1: 10/10^{-5} и 30/10^{-5} %

Homogeny concentration of tracer in the stream C_2 and ΔC: 0,010/10^{-5} и 0,001/10^{-5} %

The modeling results are presented in Table 2.

Table 2 Uncertainty of the dilution method ± *U*, %

	C_1 = 10 %		**C_1 = 30 %**	
q \ C_2	**0,010**	**0,001**	**0,010**	**0,001**
0,0002	2,60	2,70	2,64	2,74
0,0010	0,55	0,58	0,53	0,59

Analysis of the investigation results:

The method uncertainty is within the interval 0,6 – 3% according to the measurement conditions.

The accuracy rises with increasing of the discharge of the strong solution injected into the flow.

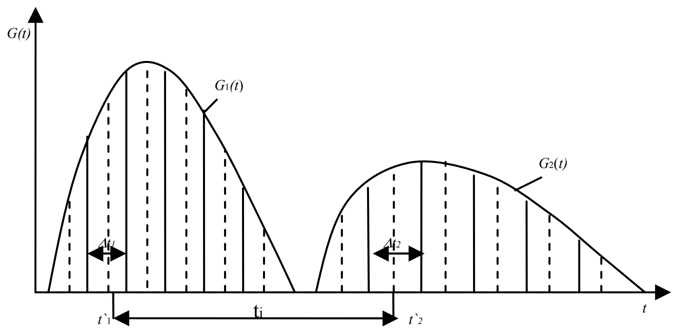
The accuracy rises with increasing of the concentration of the strong injected tracer solution.

The method accuracy does not depend on the proportion C_2/C_1.

Recommendation for the appliance of the method:
Better accuracy of the method is possible to be obtained with injection of tracer solution with bigger discharge and bigger concentration of the strong injected tracer solution

TRACER-VELOSITY-AREA METHOD
This theoretical investigation is made for IRAF "tracer-velocity-area-method", developed and applied in practice by prof. Evelin Monev developed for electrolyte tracer (1990)[7, 8]. Its measuring procedure is identical as this used by Allen (1923) [9] and by Thomas and Dexter (1959) [10] but the difference is in the result interpretation. In the Thomas and Dexter, and in Allen`s case the time of cloud travel between electrodes is measured on the chart time scale between the center of mass of the two plotted conductivity cloud areas above the background conductivity level [10]. In the Monev`s methodology this time is determined as the mean time interval between the corresponding vertical elementary sectors of the two plotted conductivity cloud areas $\overline{\left(\frac{1}{t}\right)}$ (fig. 1)

Figure 1.



Its basic equation is:

$$\overline{\left(\frac{1}{t}\right)} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{t_i} \qquad (20)$$

The discharge equation is:

$$Q = Fv = FL\overline{\left(\frac{1}{t}\right)} \qquad (21)$$

where F is the cross section area;
v - the mean flow velocity;
L - the length of the measurement base (distance between the measuring electrodes);
n - number of the corresponded elementary vertical sectors on the plotted conductivity cloud areas.

This method is well theoretically based and gives more accurate practical results by comparison with the other two methods especially in case of small distance between the measuring electrode base or in case of covering of the two plotted conductivity cloud areas.

Of course all mentioned methods can be applied with different tracers – salt, temperature tracers, radioisotopes, etc. and the only difference is that different detection equipment is needed.

Discharge equation: $$Q = \frac{FL}{n}\sum_{i=1}^{i=n}\frac{1}{t_i} \qquad (22)$$

Measurand values: F, L, t

Uncertainty equation: $$U = 100.t_{95}\sqrt{S_F'^2\theta_F^2 + S_L'^2\theta_L^2 + S_t'^2\theta_t^2} \quad \% \qquad (23)$$

Relative influence coefficient: $\theta_F = 1;\ \theta_L = 1;\ \theta_t = -1$

Investigated variants and final results:

As the mean velocity of the open channel flow practically are within the interval *0,2-4 m/s* and the length of the measuring base usually is chosen between *10* to *50 m.* Then the corresponding to them time periods *t* between the corresponding vertical elementary sectors of the two plotted conductivity cloud areas are limited as follows:

In case of: $L = 10$ *m.*, $\Delta L = 0,02$ *m* *t* is within: $t = 50$ *s* to $t = 2,5$s , $\Delta t = 0,5$ *s*

$L = 50$ *m.*, $\Delta L = 0,05$ *m* $t = 100$ *s* to $t = 12,5$ *сек.* , $\Delta t = 0,5$ *s*

Taking into consideration this limitation that put the investigation in realistic condition 12 variants are investigated, respectably for cross section areas: $F = 1\ m^2$, $\Delta F = 0,01\ m^2$

$F = 20\ m^2$, $\Delta F = 0,7\ m^2$

$F = 100\ m^2$ *u* $\Delta F = 2,7\ m^2$.

The modeling results are presented in Table 2.

Table 2 Uncertainty of the tracer velocity-area method ± *U*, %

F	**L = 10 m**		**L = 50 m**	
m^2	**t=50 s**	**t=2,5 s**	**t=100 s**	**t=12,5 s**
1	0,30	4,22	0,24	0,88
20	0,78	4,37	0,77	1,15
100	0,63	4,42	0,59	1,05

Analysis of the investigated results:

The method uncertainty is within with the interval 0,3 – 4,5% according to the measurement conditions.

The accuracy rises with increasing the length of the measurement base that increases the measuring time period.

The accuracy on a small scale depends on the cross section area. It is mentioned small tendency for increasing in case of small cross sections.

Recommendation for the application of the method:

The method gives very good accuracy $U \leq \pm 1\ \%$, if the base length causes measuring time period $t \geq 15 \div 20$ *s*.

In this meaning it is recommended in case of high velocity to be selected longer measurement basis. If this is practically unacceptable the uncertainty rises and in case of length base L=10m and $t \leq 3$ s. the uncertainty is possible to increase ± 5 %.

Conclusion

In accordance with the International Standards every measurement result must be accomplished with information about its uncertainty – accuracy evaluation without which the measurement result becomes meaningless.

The uncertainty is able to present any particular measurement result or measuring method accuracy. In the second case the obtained evaluation is possible to be applied as information or as a measurement accuracy criterion for measuring method selection. Thus integrated the knowledge of mathematical statistics and mathematical modeling it is possible to obtain a versatile result about the applicability of each measurement method in hydrotechnical practice facilitating the specialists in selection of appropriate solution at any specific case. The mentioned in this paper investigations are an example about the uncertainty measuring method implementation and can be used as an evaluation of any possible practical occasions.

References

[1] ISO VIM (DGUIDE 99999), *International vocabulary of basic and general terms in metrology (VIM),* International Organization for Standardization, 2004

[2] M3003, *The expression of Uncertainty and Confidence in Measurement*, United Kingdom Accreditation Service, edition 2, Jan. 2007

[3] NIST Technical Note 1297, *Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results*, NIST, 1994 edition

[4] ISO/TR 5168:1998(E), *Measurement of fluid flow – Evaluation of uncertainties*, International Organization for Standardization, Geneva, Switzerland, 1993

[5] ISO 748, *Measurement of liquid flow in open channels – Velocity-area methods*, International Organization for Standardization, Geneva, Switzerland, 1993

[6] Gateva K., D. Koseva, A. Spasov, *Obsta teorija na statistikata*, Nauka I izkustvo, Sofia, 1991

[7] Monev E., *Radioizotopna hidrometria*, BAS, Sofia, 1990

[8] Chilikova-Lubomirova M., *Investigation of the Electrolyte Solution Effectiveness into the Hydrometrical Practice*, Water Problems, Vol. 36, 2007, p.39-45
[8] Allen, C. M., E. A. Taylor, *The Salt Velocity Method of Water Flow Measurement,* Trans American Soc. Of Mech. Eng., 1923, №45
[9] Water Measurement Manual, A guide to effective water measurement practices for better water management, U.S. Department of Agriculture, Bureau of Reclamation, Washington, 2001