ACCURACY OF AN ABSOLUTE METHOD USED FOR RELATIVE MEASUREMENTS: STUDY OF AN ACOUSTIC TRANSIT TIME METHOD

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SYNOPSIS

Accurate measurements of relative discharge are important for evaluating the change in electricity production from hydro power units in different conditions, e.g., before and after upgrades. National stimulation initiatives to increase environmentally friendly production further underline the importance of these measurements. The purpose of this work is to provide an example of a field study where the analysis of discharge measurements heavily affects the accuracy of determining the improvement in efficiency.

Here we study two sets of field measurements using an acoustic transit time method (Accusonic) recorded in 1990 and 2008. By carefully cancelling terms of the error analysis, the error in the *before and after* discharge measurement can be reduced from 0.97% to 0.31%. However, there is some uncertainty in the effect of the change in flow profile on the error analysis.

Also, we consider a simplified version of an acoustic method using single paths, which shows errors in the 0.5% to 1.0% range when used as an index method. However, its use in evaluating changes in efficiency is limited.

The main conclusion of this study is that additional information about the flow profile, which can be obtained using an 8-path method, is needed to obtain high relative accuracy. Without the additional information, undetected changes in the flow profile may introduce large errors in any method that physically covers a smaller part of the flow cross section.

1. INTRODUCTION

Hydro power in Sweden, as in most of Europe, is undergoing significant refurbishment with upgrades of existing power plants. New equipments such as runner and guide vanes are being installed on the existing designs. To evaluate the status of these power plants before and after a major upgrade, robust and reliable field measurement techniques are needed.

When evaluating the investments needed to upgrade a unit, it is actually the relative increase in performance that is of interest. This is true for both large refurbishment projects that include a new runner and possible guide vanes and small upgrades such as modifications of the spiral casing or the draft tube, which yield efficiency increases in the range of 0.2% to 0.5% [1, 2].

In many parts of the world significant efforts are being made to increase the production of green CO_2-neutral energy through subsidies from the government or through certificate systems. In Sweden, investments in environmentally friendly production are stimulated by a market-based electricity certificate system that is designed to assist the increase of electricity production from renewable energy sources such as hydro power. In hydro power refurbishment projects, the increase in production is regarded as new renewable energy produced. Currently, approximately 30% of the value of the production increase can be contributed to certificates, which plays a crucial role in determining the feasibility of a reinvestment. Verification of the relative improvement in the plant’s energy production is crucial to determine the correct number of certificates [3].

These two factors, especially the certificate system, make it necessary to accurately measure the actual improvement in efficiency due to the upgrade. In this case a comparison of the plant’s discharge is sufficient to determine the improvement. The goal of this study is to examine and discuss whether the *before and after* measurements can be used to improve the accuracy of the discharge measurement.

A majority of the turbines in Sweden have low heads. Therefore, the use of field measurement techniques such as the Gibson and the Thermodynamic method to determine discharge is somewhat limited. Further, low head turbines typically have short and curved intakes that make the flow pattern complex, which increases the challenge in obtaining accurate results. Installing advanced flow measurement equipment such as acoustic flow meters is relatively expensive and complex, and a large number of acoustic paths are needed to obtain sufficient accuracy.

Therefore it is of interest to investigate whether the *before and after* measurements can be used to evaluate both the relative and the absolute accuracy. The goal of this study is to clarify the requirements for accurate measurements of relative discharge and include these requirements in future versions of test standards such as IEC and ASME.

2. DESCRIPTION OF FIELD STUDY

The hydro power unit Gejmån G1, which is situated in northern Sweden, was inaugurated in 1970. The runner in this unit is of Francis type, and the penstock is approximately 6 km long. The penstock ends with a steel tube that has a nominal radius of 1.35 m, and there is a straight section that measures approximately 33 m (12 diameters) in length where a well-defined flow profile can be expected. The maximum discharge of the Gejmån plant is approximately 27 m^3/s.

An 8-path acoustic transit time equipment unit (Accusonic) was mounted and used in 1990. In 2008, after a refurbishment project that included replacement of the runner and generator, a new measurement was obtained. The transducers were placed approximately 24 m (9 diameters) downstream of a stone pocket and approximately 6 m (2 diameters) upstream of a converging bend before the valve (see Figure 1). Figure 2 illustrates the enumeration of the acoustic paths. The repositioning of the transducers in 2008 could be repeated with high accuracy due to the steel plates with fittings for transducer holders that were welded to the inside of the penstock in 1990.

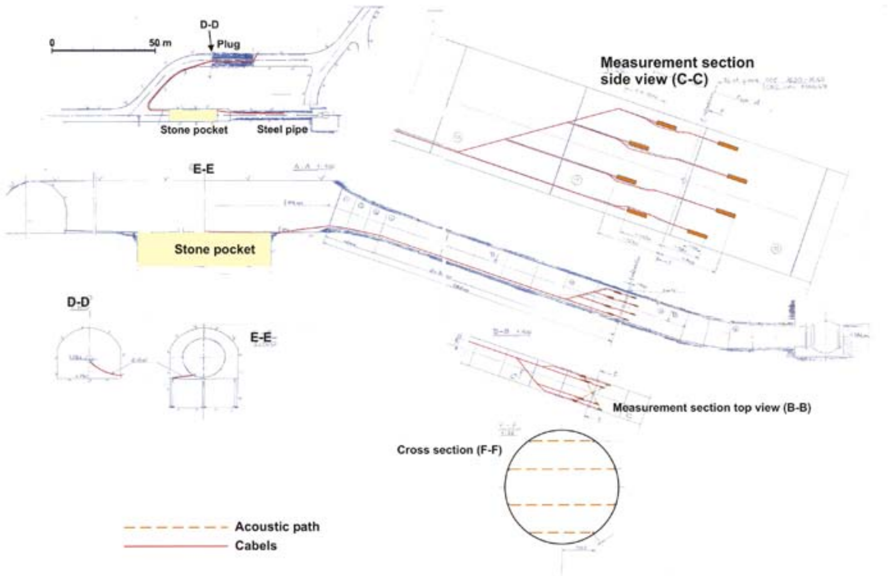


Figure 1: Detail from drawing of penstock with stone pocket and measurement section. The valve before the spiral casing can be seen on the right.

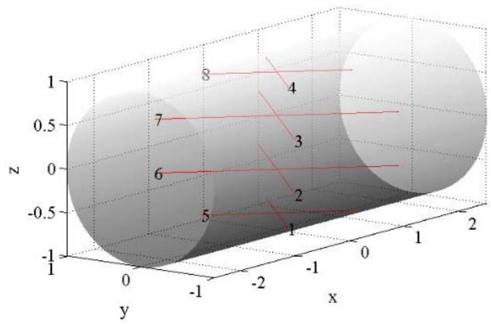


Figure 2 : Enumeration of acoustic paths. The flow is along the positive direction of the *x-axis*, and the *y-axis* is horizontal.

3. CALCULATION AND ERROR ANALYSIS

This section shows results from the error analysis process using ordinary propagation of uncertainty described in [4]. All errors are given with 95% confidence intervals.

Velocities can be computed from time measurements via

$$v_i = \frac{\Delta t}{\bar{t}^2} \frac{L}{2\cos\theta}, \tag{1}$$

where

$$\Delta t = t_{reverse} - t_{forward} \text{ and } \bar{t} = (t_{forward} + t_{reverse})/2. \tag{2}$$

The discharge is computed from velocities according to

$$Q = 2r^2 \sum_{i=1}^{8} v_i w_i k_i \ , \tag{3}$$

where r is the radius of the penstock, w_i are quadrature weights, and k_i are constants used for cross-flow correction. The weights are

$$\begin{aligned} w_i &= 0.217079, \quad i = 1,4,5,8, \\ w_i &= 0.568319, \quad i = 2,3,6,7, \end{aligned} \tag{4}$$

where i = 1, 4, 5, 8 correspond to outer paths and i = 2, 3, 6, 7 correspond to inner paths, see Fig. 2.

3.1 Error analysis of absolute measurements

The total error in an absolute measurement of discharge is composed of systematic and random parts as shown below.

$$\left(\frac{\Delta Q}{Q}\right)_{tot} = \left(\left(\frac{\Delta Q}{Q}\right)_{syst}^2 + \left(\frac{\Delta Q}{Q}\right)_{rand}^2\right)^{1/2}. \tag{5}$$

The systematic part is determined from systematic errors of the constituents of equations (1) and (3) using ordinary error propagation. The resulting formula is

$$\left(\frac{\Delta Q}{Q}\right)_{syst} = \left(c_{vw}\left(\left(2\frac{\Delta\bar{T}}{\bar{T}}\right)^2 + \left(\frac{\Delta L}{L}\right)^2 + \left(\tan(\theta)\Delta\theta\right)^2\right) + \left(2\frac{\Delta R}{R}\right)^2 + \left(\frac{\Delta Q}{Q}\right)_{\text{int}}^2 + \left(\frac{\Delta Q}{Q}\right)_{prof}^2\right)^{1/2}, \tag{6}$$

where the terms are explained in Table 1. Equation (6) is explained by the following short comments:

- The constant c_{vw} is due to the squared sum of quadrature weights and velocities.
- The brackets multiplying c_{vw} contain the systematic error of the velocity.
- The factors of 2 originate from the squares in equations (1) and (3).
- Note that the θ-term is an absolute error term whereas all other terms are relative to the value of the quantity.

Table 1 Explanation of the terms of the error analysis.

c_{vw}	Constant due to squared sum of v and w. When all v_i are equal, c_{vw} = 0.387.
$\Delta\overline{T}/\overline{T}$	Error in the mean of time measurements (forward and reverse).
$\Delta L/L$	Error in measurements of face-to-face distance between transducers.
$\Delta\theta$	Error in measurement of the angle between acoustic paths and the centre line of the penstock expressed in radians.
$\Delta R/R$	Error in measurements of the radius of the penstock.
$(\Delta Q/Q)_{int}$	Error in Q due to the inability of the Chebyshev-Gauss quadrature formula in integrating an ideal flow profile exactly.
$(\Delta Q/Q)_{prof}$	Error in Q due to the flow profile not being ideal.
$(\Delta Q/Q)_{syst}$	Systematic error of Q.
$(\Delta Q/Q)_{rand}$	Random error of Q.

The above analysis does not take into account the protrusion effect or deviations in actual transducer placements. Although there is some disagreement on how the protrusion effect must be treated, we omit it here because systematic errors are not the main focus of this manuscript.

3.2 Applied error analysis of absolute and relative measurements

Let us assume that one is interested in the error of the difference between two measurements at the same operating point. The error of the difference is

$$\left(\frac{\Delta(Q_2-Q_1)}{Q_2}\right)=\left(\left(\frac{\Delta Q_1}{Q_1}\right)^2+\left(\frac{\Delta Q_2}{Q_2}\right)^2\right)^{1/2}, \tag{7}$$

If two different absolute methods are used, then one should use their total errors in equation (7). However, if some parts of the methods are equal, then some terms of the systematic error may possibly cancel. Table 2 shows the relevant values that are needed to apply the error analysis process to the Gejmån plant along with a few short comments. The underlying measurements were performed in 2008.

Table 2: Values needed to apply the error analysis to the Gejmån power plant.

Row	Property	Value	Comment
0	c_{vw}	0.392	Computed from one velocity distribution.
1	$\Delta\overline{T}/\overline{T}$	±0.01%	Specified by Accusonic.
2	$\Delta L/L$	±0.03%	Variations in measurements that should be equal.
3	tan(θ)	1	A 45° arrangement is used.
4	$\Delta\theta$	±0.0019 rad	Variations in measurements that should be equal.
5	$\Delta R/R$	±0.24%	Variations in measurements that should be equal if the penstock cross section are absolutely circular.
6	$(\Delta Q/Q)_{int}$	±0.12%	From [5], with a 1/7-exponent turbulent profile.
7	$(\Delta Q/Q)_{prof}$	±0.40%	A rough estimate, see below.
8	$(\Delta Q/Q)_{syst}$	±0.651%	Computed from values above using equation (6).
9	$(\Delta Q/Q)_{rand}$	±0.22%	Determined from variations in actual measurements, see section 4.1.

No modification of the penstock was performed between the measurements. Hence, systematic errors in *T*, *L*, θ, and, *R* are equal and can be cancelled. The Q_{int}-error can also be cancelled. The remaining parts are the random error and the integration error due to the flow profile (Q_{prof}) being non-ideal. Although the latter term may cancel completely or in part, there is significant uncertainty in the effect of flow profile differences on the quadrature error. The value 0.40% used in Table 2 on row 7 is a rough estimate that is on the conservative side. In [5], a value of 0.3% is obtained as the integration error that includes both Q_{int} and Q_{prof} for a heavily distorted profile, and in [6], a corresponding value of 0.145% is obtained. Table 3 summarises the total errors obtained in the different settings. The middle column shows the errors obtained when an acoustic measurement is used to evaluate a discharge measurement at the prototype on a single occasion, both as an absolute measuring method and as a relative, i.e., index method. The right column shows the corresponding errors for the comparison of two consecutive measurements, i.e., a *before and after* setting.

Table 3 Total error of discharge measurement with and without systematic parts.

Situation	**Single measurement**	***Before and after* measurement**
Standard absolute measurements (all rows of Table 2)	0. 69%	0.97%
Index measurement including quadrature error (rows 7 and 9 of Table 2)	0.45%	0.64%
Index measurement excluding quadrature error (row 9 of Table 2)	0.22%	0.31%

4. INVESTIGATION OF 1-PATH METHODS

If acoustic methods are used in a *before and after* setting, there might be an interest in simplifying their use. One possibility, which is investigated in the following section, is to use single acoustic paths. To deal with measurements and errors in a structured manner, we introduce the following model:

$$Q_i = c_i A v_i , \tag{8}$$

where index i refers to the path number. The factor c_i would be 1 for a completely uniform profile and can be used for comparisons of the similarity of the profiles between two 8-path measurements.

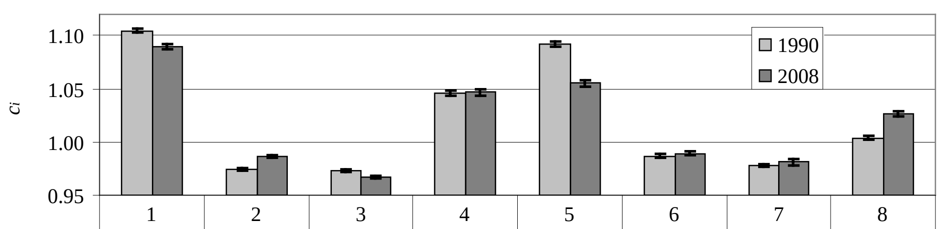
The relative accuracy of a 1-path method via equation (8) is determined by

$$\left(\frac{\Delta Q_i}{Q_i}\right)_{rel} = \left(\left(\frac{\Delta c_i}{c_i}\right)_{rand}^2 + \left(\frac{\Delta c_i}{c_i}\right)_{other}^2 + \left(\frac{\Delta v_i}{v_i}\right)_{rand}^2\right)^{1/2} . \tag{9}$$

The error in c_i is influenced both by a random error in measurement and by a change in the conditions when it is used for a repeated measurement. Such changed conditions can be any conditions that affect the flow profile, e.g., head variations. Additionally, the flow profile and hence c_i can be affected by small changes in surrounding conditions, e.g., a different amount of debris in the stone pocket (see Figure (1)).

The error in c_i is investigated using Gejmån recordings from 1990 and 2008. For each path and each point of operation, c_i is computed according to equation (8) using the mean values of Q and v_i. The results with error intervals are displayed in Figure 3.

Because each c_i corresponds to an average over the discharge range, the random error in Figure 3 also includes possibly systematic discharge dependant errors. The random part of the error is often visibly smaller than the actual change between the years. The change is often larger for outer paths; for i=5 it is -3.4%. However, the inner path i=2 is also subject to a significant change of 1.3%. Hence, the flow profile cannot be considered to be unaffected between the years.



	1	2	3	4	5	6	7	8
1990	1.105	0.974	0.973	1.046	1.092	0.987	0.978	1.004
2008	1.090	0.987	0.967	1.047	1.055	0.989	0.981	1.026

Path # (*i*)

Figure 3: Coefficients c_i with random errors measured in 1990 and 2008.

Although the random error in velocities v_i should be determined using repeated measurements at a fixed point of operation, no such determination was performed here. Considering only the variations in single 3-minute time series underestimates the error because there are long time scales that are not captured. As a substitute for the desired repeated measurements, we study the spread in the discharges computed using the 1-path methods shown in Figure 4. Here each c_i is chosen so that the mean error is zero. Hence, there are no random errors in c_i that can affect the errors displayed in the figure. Although there could still be other c_i-errors, no error trends are immediately visible in Figure 4, and therefore, we assume that the errors mainly consist of $\Delta v_i/v_i$.

To estimate the random error, we consider the 11 points of operation where Q>20 m^3/s. Figure 5 shows both the mean of uncertainties for single time series and the spread between the errors of all 11 time series, i.e., the spread for each path in Figure 4.

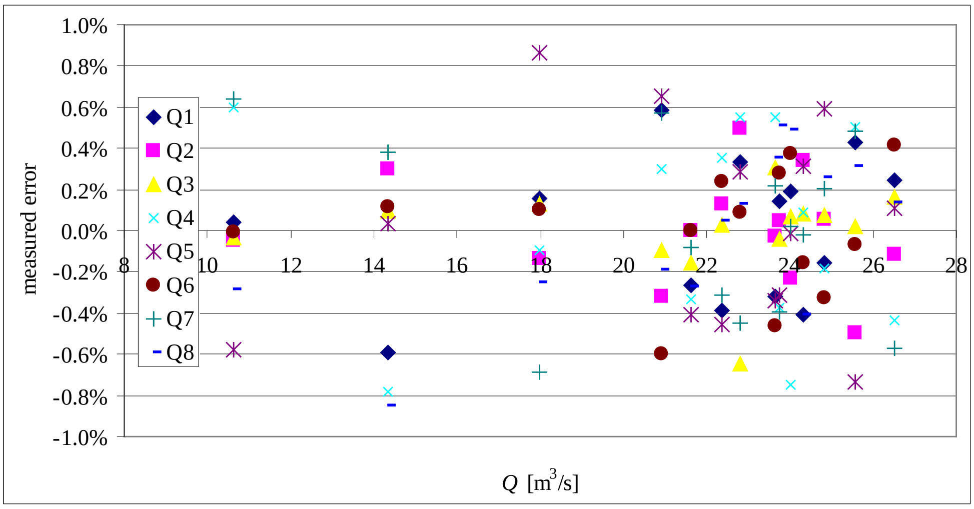
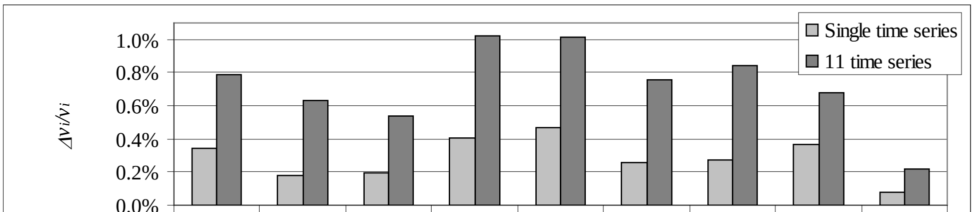


Figure 4: Errors in Q_i for single time series.

The random error of the full 8-path measurement is shown on the right in Figure 5 and is denoted by "8-path". Also, the mean spread of 0.081% in these single 3-minute series underestimates the random error, and we do not have measurements to directly determine a more accurate value. The solution is to scale the mean spread value guided by the relation between the single time series and spread of the 11 time series. The mean proportion is 2.65, and the dark grey bar over "8-path" is simply this factor multiplied by 0.081%, yielding $(\Delta Q/Q)_{rand}$ = 0.22%. This value is probably a "worst case" scenario, because the factor 2.65 includes unknown errors apart from v_i.



	1	2	3	4	5	6	7	8	8-path
Single time series	0.34%	0.18%	0.19%	0.40%	0.47%	0.26%	0.28%	0.37%	0.081%
11 time series	0.79%	0.63%	0.54%	1.02%	1.02%	0.76%	0.84%	0.68%	0.216%

Path # (i)

Figure 5: Random errors for velocities measured in 2008 showing the mean errors of the single time series and the spread of the 11 time series.

There are various ways to implement a 1-path method in terms of choosing the calibration constants c_i. Let us assume that there is an old 1-path measurement and that transducers are remounted later at the same positions for a new measurement. This case can be examined with c_i from Gejmån 1990 to compute the discharge using velocity measurements from 2008. However, the large differences in c_i between the years (up to 3.4%) already suggest that this approach requires some luck. Accurate results can be obtained only if c_i has not changed considerably, which is impossible to determine without additional information about the flow profile. Such information can be obtained by other means, e.g., using an 8-path method. The following question then arises: Why should we use only one path for discharge measurement?

An alternative setting is that a valid and trustworthy discharge measurement is available. However, this trustworthy measurement would have to be obtained by some other method, which diminishes the usability of the 1-path method in a *before and after* setting. The 1-path method could still be used as an index method with random errors as displayed in Figure 5.

6. DISCUSSION AND CONCLUSIONS

A full 8-path method can measure the difference in discharge with an inaccuracy of 0.31% if the flow profile is *sufficiently similar* between the measured cases. However, what is meant by *sufficiently similar* is not yet clear.

It may be possible to reduce the random errors through longer measurement series. This will improve the accuracy of the *before and after* measurement 8-path method, but only if the flow profile is *sufficiently similar.*

The error due to flow profile ($(\Delta Q/Q)_{prof} = 0.4\%$) is a rough estimate and is difficult to determine more accurately. This is unfortunate because it constitutes a large portion of the systematic error and does not necessarily cancel in the *before and after* measurements. From a given 8-path measurement, we cannot yet estimate the integration error. However, the use of 8 paths provides information about the size and nature of the change in the flow profile.

When measuring velocities, the inner paths perform better than the outer paths. A carefully set 1-path method should be able to achieve inaccuracy of approximately 0.5% by reducing random errors with longer time series or by placing the measurement path through the centre of the penstock. However, the use of the 1-path method in a *before and after* setting cannot be recommended.

The *before and after* perspective is important for evaluating upgrades and receiving financial support for making investments in green energy. Large upgrades such as runner replacement will still be controlled by guarantees at an absolute level. In the *before and after* perspective, the before measurement can be used to verify the implementation of the test equipment if the test accuracy is good enough.

The potential of the *before and after* perspective is to obtain greater accuracy by neglecting some systematic errors. Standardised criteria are needed for this procedure.

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