SELECTED PROBLEMS IN CALCULATION PROCEDURES FOR THE GIBSON DISCHARGE MEASUREMENT METHOD

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ABSTRACT

The pressure-time (Gibson) method belongs to the primary methods of discharge measurement through hydraulic machinery. The method consists in determining the flow rate by the integration of recorded time course of pressure difference variations between two cross-sections of the hydropower plant penstock. The paper presents two selected problems related to calculation procedure of discharge measurement by means of Gibson method. The first one is connected to the calculation of hydraulic losses in a penstock measurement section while determining flow rate by means of Gibson method. The modification of procedures usually utilized to calculate these losses was proposed and the experimental results that verify its importance, basically in case of applying Gibson method for flow rate measurement in investigated hydraulic machinery under the pump regime, were presented.

Second problem regards the estimation of upper integration limit of pressure gradient time-course recorded when using considered measurement method. To determine this limit, the free oscillations characteristic of recorded pressure difference occuring after shutting-off the liquid flow in a pipeline was applied. Two methods of eliminating the effect of these fluctuations on determined volumetric flow rate value were presented. They differ from the testing procedure described by the International Standard IEC 41.

1. INTRODUCTION

The pressure-time method, commonly known as Gibson method, is one of the primary methods of discharge measurement through hydraulic machinery [1-4]. The method is recommended by the International Standard IEC 41:1991 as well as its European equivalent EN 60041 [1]. The discharge is determined by integrating the static pressure difference between two penstock sections as recorded during a swift shut-off of the liquid flow. The accuracy of discharge measurement using this method depends on a number of factors that mainly are related to the pressure measurement process [5-7] and calculation aspects [1, 8, 9].

Calculation of the pressure drop due to hydraulic friction in a penstock segment between two hydrometric cross-sections utilized for the Gibson method purposes is the first problem considered in the paper. Under some conditions, particularly during the pump-turbine tests, sudden, temporary changes in the flow direction occur. The commonly utilized discharge measurement procedures assume friction related pressure drop depending on discharge square, with no account taken for the possible flow reversion, which implies, that the calculations of the viscous pressure drop show always the same sign, irrespectively of the flow direction. The proposed changes in calculation procedure enable considering the actual flow direction in the friction loss calculation, thus increasing accuracy of the discharge measurement, particularly when using the Gibson method for the pump mode of operation with considerable fluctuations in flow direction taking place. The authors' experience indicates that application of the modified calculation procedure enables using the Gibson method for pump mode of operation (so far the recommendations for Gibson method assume its utilization only in case of the turbine mode of operation [1]).

The theoretical analysis has been supported by test results, in which the discharge was determined both by means of the standard calculation procedure (IEC 41) and the extended procedure accounting for temporary changes in the flow direction.

The second problem discussed in the paper is associated with defining the upper integration limit of the recorded time course of pressure difference fluctuations between two hydrometric sections of the penstock. The natural, free oscillations of the pressure difference (occurring after the flow has been cut off) are utilized to determine the integration limit. The analysis of free oscillation influence on the discharge measurement results and two proposals of eliminating these oscillations are included in the paper. The proposed procedures are different from those presented in the International Standard IEC 41 [1].

2. MATHEMATICAL RELATIONSHIPS OF THE GIBSON METHOD

In order to derive a relationship for computing the volumetric flow rate *Q* let us consider a pipeline with the flow section area *A* that may change along its length – figure 1. Let us assume that the water stream is stopped by a cut-off device. Taking into account one pipe segment of length *L*, limited between cross-sections 1-1 and 2-2, we assume that the velocity and pressure distributions in cross-sections of this segment are uniform. It is also assumed that the fluid density and the flow section area do not change due to the water hammer effect.

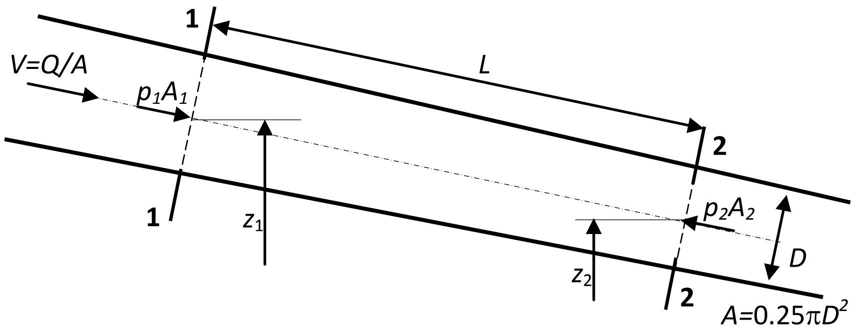

Fig. 1. Segment of a pipeline with marks needed to explain the theoretical basis of the pressure-time method.

According to these assumptions, the relationship between the parameters of one-dimensional unsteady flow in two selected cross-sections of a pipeline can be described by energy balance equation which’s well known from literature [10]:

$$\alpha_1 \frac{\rho Q^2}{2A_1^2} + p_1 + \rho g z_1 = \alpha_2 \frac{\rho Q^2}{2A_2^2} + p_2 + \rho g z_2 + \Delta P_f + \rho \frac{dQ}{dt} \int_0^L \frac{dx}{A(x)} \tag{1}$$

where ρ denotes liquid density, p_1 and p_2 present static pressures in pipeline sections 1-1 and 2-2, respectively (see Fig. 1), z_1 and z_2 are elevations of 1-1 and 2-2 hydrometric pipeline section weight centres, α_1, α_2 are the Coriolis coefficients[1] (kinetic energy correction coefficients) for 1-1 and 2-2 sections, respectively, *Q* is the flow rate (discharge), *g* means gravity acceleration and, finally, ΔP_f is the pressure drop caused by friction losses between 1-1 and 2-2 sections.

Let us introduce the following quantities to equation 1:

- Static pressure difference between 2-2 and 1-1 pipeline sections related to the reference level:

$$\Delta p = p_2 + \rho g \;{}_2 \rlap{-}z \; p_1 - \rho g \;{}_1 , \tag{2}$$

- Dynamic pressure difference between 2-2 and 1-1 pipeline sections:

$$\Delta p_d = \alpha_2 \frac{\rho Q^2}{2A_2^2} - \alpha_1 \frac{\rho Q^2}{2A_1^2} , \tag{3}$$

- Geometrical factor of the examined pipeline segment of *L* length:

$$F = \int_0^L \frac{d\,x}{A(x)} . \tag{4}$$

Then, we get the differential equation in the form:

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[1] The value of the Coriolis coefficient for fully developed turbulent flow in the pipeline is within the limits from 1.04 to 1.11 [10].

$$\rho F \frac{dQ}{dt} = -\Delta p - \Delta p_d - \Delta P_f \tag{5}$$

The left hand side term[2] of equation (5) is the unsteady term which takes into account the history of the volumetric flow rate variation $Q = V\,A$, recorded during the flow transients. This term represents the effect of fluid inertia in the considered pipeline segment.

After integrating equation (5) over the time interval (t_0, t_k), in which the flow conditions change from initial to the final ones, we obtain the flow rate difference between these conditions. If we assume that the flow rate value in the final conditions (q_k), i.e. after the cut-off device has been closed, is known, we get the following formula for the volumetric flow rate under initial conditions (before the water flow stoppage was initiated):

$$Q_0 = \frac{1}{\rho F} \int_{t_0}^{t_k} \left(\Delta p(t) + \Delta p_d(t) + \Delta P_f(t) \right) dt + q_k \tag{6}$$

The flow rate in the final conditions (q_k), if different from zero due to leakage in the closing device, has to be measured or assessed using a separate method [11,12].

3. HYDRAULIC LOSSES CALCULATION IN CASE OF VARIABLE FLOW DIRECTION

Presented above theoretical description of Gibson method is valid for both, turbine and pump modes, of hydraulic machinery. Nevertheless, the IEC 41 standard recommends to apply the mentioned method only in case of turbine mode. Authors' experimental results reported below indicate on the possibility of utilization the method also in case of pumping mode after the proper modification of calculation procedures which is introduced. In order to determine the flow rate basing on equation (6), it is necessary to estimate the pressure drop caused by the friction loss between the measurement cross sections in a pipeline. Typical calculation procedures (including the one presented in IEC 41 standard) assume the hydraulic losses to be dependent on the square of flow rate, given in form:

$$\Delta P_f = kQ^2 \tag{7}$$

where k is a constant coefficient experimentally determined.

The hydraulic losses calculated in accordance with relationship (7) do not depend on flow direction (both are always of the same sign). Following this type of procedure may lead to the generation of additional error of discharge value determination. It results from the fact that under some conditions, particularly in case of pump-turbine tests, the significant temporary change of liquid flow direction takes place. Therefore, it is proposed to substitute formula (7) by the following:

$$\Delta P_f = kQ|Q| \tag{8}$$

The proposed modification of calculation procedure enables to account for actual flow direction and to increase measurement accuracy, particularly under pump mode operating conditions for which, in general, the temporary flow inversion during the pump shut-off required by the method occurs.

In order to verify the rightness of considered approach and to estimate the influence of temporary flow direction change on the discharge measurement, the dedicated research in pumping system in the IMP PAN laboratory were conducted. The pressure difference variations between the hydrometric cross sections in the penstock for various pump stopping procedures with controlled liquid flow cut-off course were measured in the system. Afterwards, basing on recorded differential pressure courses, the flow rate was calculated according to procedure presented in IEC 41 standard and with regard to modification of friction loss calculation proposed by authors. Determined flow rate values were compared to the measurement result obtained using ultrasonic flowmeter. Differential pressure records and determined time-discharge curves are presented in figure 2.

Obtained results indicate on the validity of introducing the modification of discharge calculation procedure that accounts for the sign of hydraulic loss (friction) value with regard to flow direction. In

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[2] For steady flows this term is equal to zero and then equation (1) takes the form of the Bernoulli equation for the real liquid flow.

performed investigation, the disagreement between the flow rate determined accordingly to calculation procedure provided by IEC 41 and the one measured by ultrasonic flowmeter is less than 1.5%. In the worst case the discrepancy arrives at 7.5% - see figure 2. It is worth to mention that the difference between flow rate determined according to IEC 41 and its value measured with the use of ultrasonic flowmeter multiplies as the reverse flow increases. On modifying the calculation procedure (consisting in regarding the flow direction), the differences between Gibson method in pump regime and ultrasonic flowmeter measurement were at the similar level and were contained in the range 0.3÷0.7%. It can thus be stated that introduction of described calculation procedure improvement increases the accuracy of hydraulic machinery tests in pump mode carried out by means of considered method under instantaneous flow direction change.

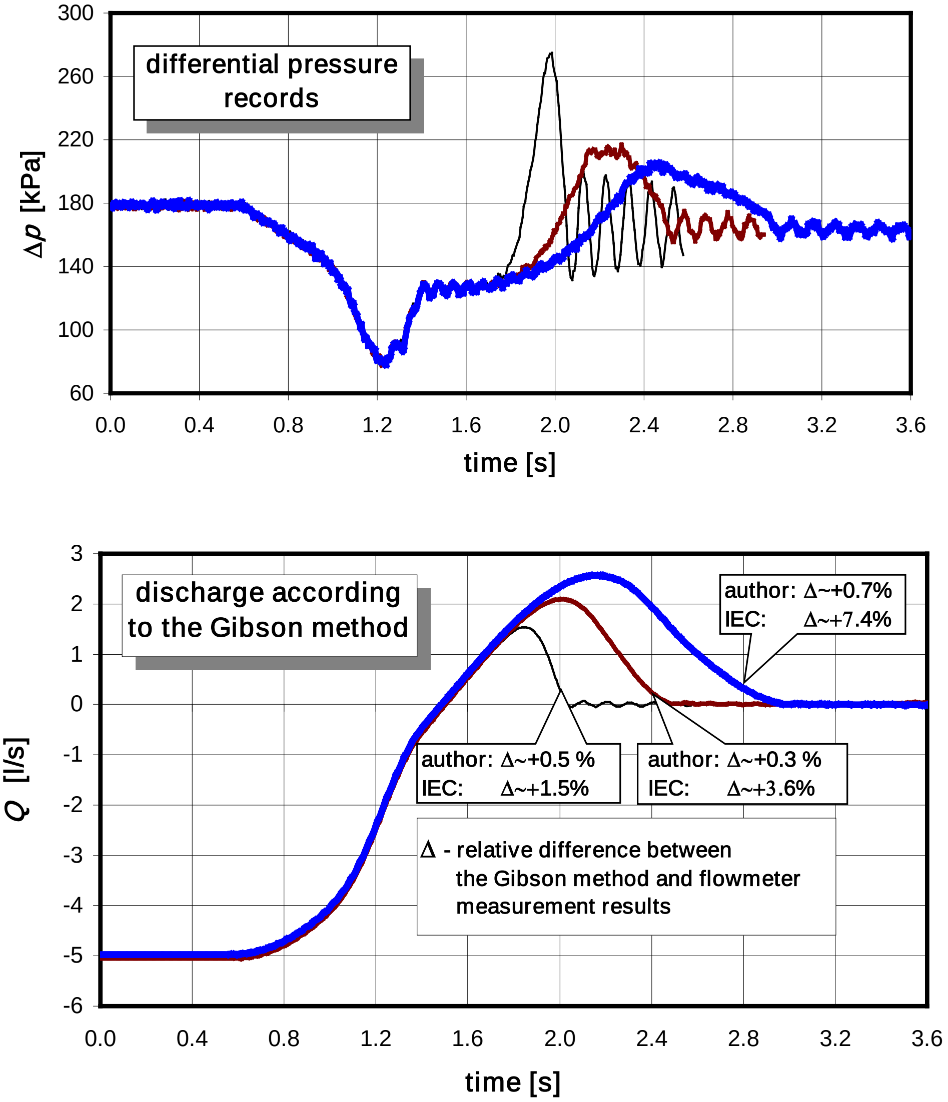

Fig. 2. Laboratory test results. Records of differential pressure in the conduit and the time-discharge curves determined by means of the Gibson method

The significance of proposed modification is depicted in figure 3, which represents the flow measurement results of pump-turbine testing at its pump mode applying Gibson method. In examined case the difference between flow rate determined in compliance with IEC 41 and after introducing the correction accounting for the friction losses sign during temporary flow direction change was estimated at 1%.

In conclusion, it can be noted that the foregoing theoretical studies as well as experimental measurements on laboratory and field scale, justify the possibility to utilize pressure-time method also in case of pump operation mode of hydraulic machinery.

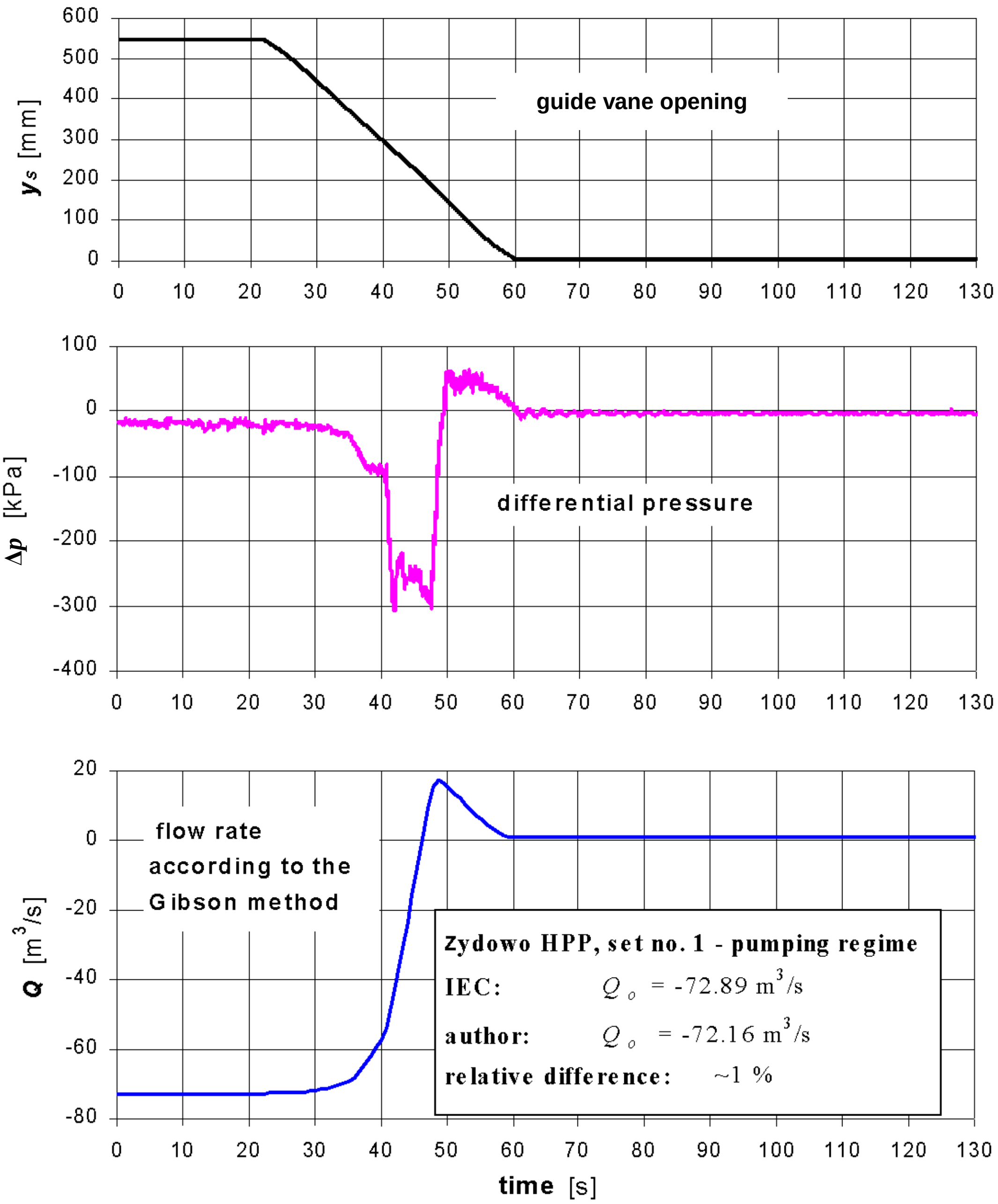

Fig. 3. Zydowo HPP. Discharge measurement by means of Gibson method in the pumping regime.

4. DETERMINATION OF THE UPPER INTEGRATION LIMIT OF THE PRESSURE-TIME CURVE

Equation (6) that enable to calculate initial flow rate requires to specify integration limits – time interval in which the flow has been cut-off. Contrary to t_0 time (lower limit of integration), the determination of t_k time (upper limit of integration) presents difficulties. Even precise synchronization of measurement of flow-closing device course and the variation of pressure does not ensure the exact time determination. The reason for this is the lack of relation between the shutting-off time moment of closing device and flow cut-off time moment (in some cases despite the flow cut-off procedure termination the closing-device is still in motion, e.g. in result of elastic strain). Therefore, to determine the upper integration limit the character of free pressure oscillations is presumed figure 4. These fluctuations remain in the pipeline after the flow has been cut-off as a result of interaction between inertial effects and effects associated with liquid compressibility and deformability of pipeline walls [8]. One of the procedures of upper integration limit calculation in Gibson method is given in IEC 41 standard. However it includes mathematical inaccuracy – it does not ensure to set a zero-value integral of free pressure difference oscillations with intent to eliminate their influence on the flow rate measurement. The consideration regarding the procedure improvement is presented below.

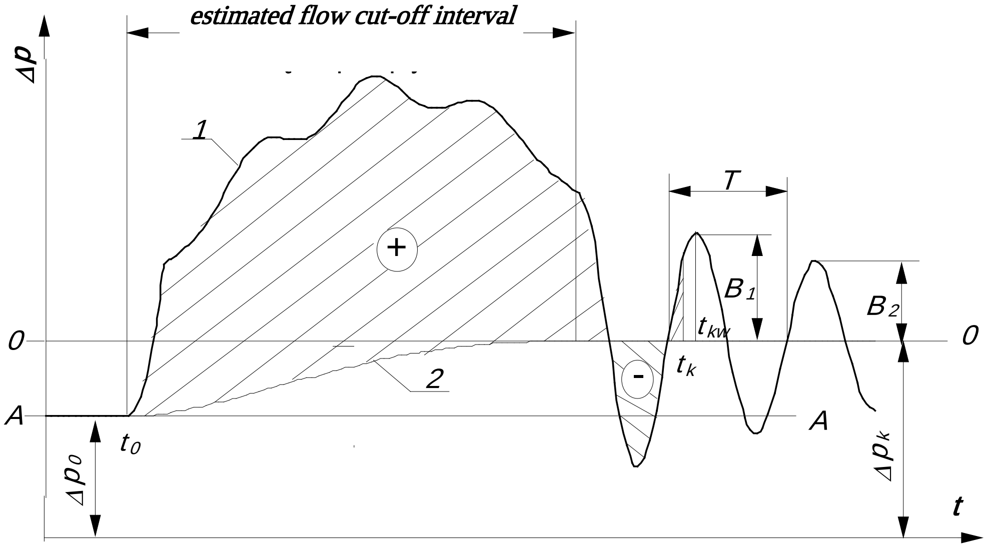

Fig. 4. Character of pressure difference transients between the measurement cross-sections during flow stoppage.

Let us assume that free pressure oscillations after the flow cut-off may be described by function (figure 5):

$$\Delta p(t) = B_o e^{-ht} \cos(\omega t) \tag{9}$$

with $\omega = 2\pi/T$ denoting the circumferential wave frequency, $h = (1/T)\ln(B_i / B_{i+1})$ – oscillation damping coefficient (reciprocal of the relaxation time), $\ln(B_i / B_{i+1})$ – logarithmic damping decrement, T – pressure wave period.

Figure 5 presents free pressure oscillations with the notation applied.

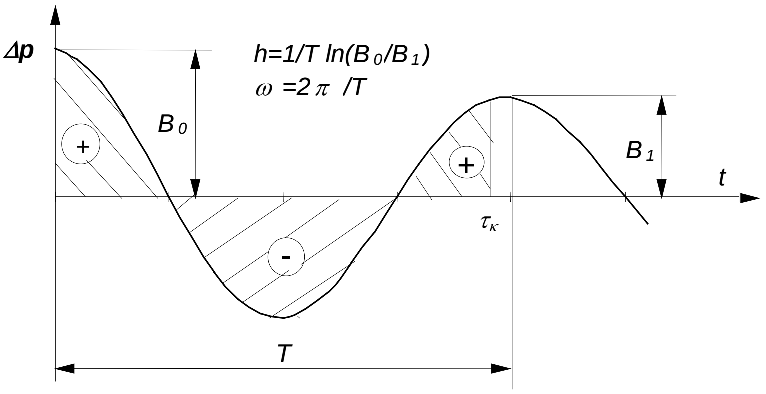

Fig. 5 Free pressure oscillation course including the notation applied

In order to avoid the influence of free pressure oscillations on the discharge value to be determined, the time point τ_k, fulfilling the condition

$$\int_0^{\tau_k} B_o e^{-ht} \cos(\omega t)\,dt = 0 \tag{10}$$

is sought for. The above equation is equivalent to the condition of equal total fields (areas) defined by the damped pressure wave curve below and over the time axis – figure 5.

Basing on the analysis performed, it has been stated that the procedure of determining the τ_k time, as presented in the IEC 41 standard, is improper – it does not lead to a strict solution of equation (10).

It can be proved that this procedure follows from the solution of an equation based on the indefinite integral of equation (10) which can be determined analytically. By solving equation

$$\int B_o e^{-ht}\cos(\omega t)dt = B_o \frac{e^{-ht}}{h^2+\omega^2}\left[-h\cos(\omega t)+\omega\sin(\omega t)\right]=0\,, \tag{11}$$

in respect to time *t*, one derives an analytical expression which is presented in the IEC 41 standard and used to determine the end of the integration interval.

The precise (strict) solution should be based on the definite integral of equation (10) which can be written down in the following form:

$$\int_0^{\tau} B_o e^{-ht}\cos(\omega t)dt = \frac{B_o}{h^2+\omega^2}\left\{e^{-h\tau}\left[-h\cos(\omega\tau)+\omega\sin(\omega\tau)\right]+h\right\}=0\ . \tag{12}$$

Equation (12) cannot be solved analytically.

By comparing equations (11) and (12) it can be seen that the term

$$\text{Eq(12) - Eq(11)} = B_o h/(\omega^2 + h^2) \tag{13}$$

has not been accounted when formulating the IEC 41 standard procedure.

It follows from the above that the integral of free pressure fluctuations does not equal zero in the operating points determined according to the IEC 41standard.

It is evident from the considerations that one of the possibilities to eliminate the influence of free pressure oscillations on the determined flow rate is to subtract the term $B_o h/(\omega^2 + h^2)$ from the integral calculated basing on the recorded pressure difference course in accordance to procedure presented in IEC 41 standard. This value can be easily calculated from the recorded pressure diagram.

Another approach aimed at eliminating the effect of free pressure fluctuations is the numerical determination of roots of integral given by equation (10), [6].

The influence of free pressure oscillations generated in the pipeline after the flow cut-off on the discharge measurement by means of Gibson method increase as the fluctuation amplitude increase. It appears from the authors' experience that in some cases it may arrive at 0.5% of calculated flow rate.

5. CONCLUSIONS

The methodology for hydraulic (friction) losses calculation in measurement pipe segment used to determine the flow rate by means of Gibson method was studied in the paper. The calculation procedures typical for this method, including the International Standard IEC 41, assume the friction losses to be proportional to the square of discharge. This approach leads to the error generation that results from instantaneous flow direction change while calculating the friction losses. The cases for which such change occurs while applying pressure-time method, are generally taking place during hydraulic machinery tests carried out in pump mode of operation. Introduction of calculation procedure modification appropriate to these conditions were proposed. Performed laboratory and field tests confirm the significance of introduced improvement, particularly in case when the Gibson method is used to investigate hydraulic machinery under pumping regime.

The lack of direct relationship between the end-up of flow shut-off device operation and time moment of flow cut-off makes it difficult to explicitly determine the end of integration range (upper integration limit) of differential pressure records obtained from flow rate measurement using Gibson method. The procedure of determination of the integration range end that is included in IEC 41 standard does not ensure the zero value of integral of free pressure oscillations related to the flow cut-off. Therefore, two ways that enable to eliminate the influence of these fluctuations on the calculated flow rate were presented. The methods, apart from eliminating the mathematical inaccuracy included in IEC 41 standard, have an effect on reducing the error of discharge calculation by means of pressure-time method.

REFERENCES

1. IEC 41: 1991, International Standard: Field acceptance tests to determine the hydraulic performance of hydraulic turbines, storage pumps and pump-turbines, European equivalent: EN 60041: 1999.

2. Gibson N. R.: The Gibson method and apparatus for measuring the flow of water in closed conduits, ASME Power Division, 1923, pp. 343-392.

3. Gibson N.R.: Experience in the use of the Gibson method of water measurement for efficiency tests of hydraulic turbines, ASME Journal of Basic Engineering, 1959, pp. 455-487.

4. Troskolański A.: Hydrometry, Pergamon Press Ltd., 1960.

5. Adamkowski A., Janicki W., Urquiza B. G., Kubiak J., Basurto M.: Water turbine tests using the classical pressure-time method with measuring instrumentation installed inside a penstock, Inter. Confer. HYDRO2007, Granada, Spain, 15-17 Oct. 2007.

6. Adamkowski A., Kubiak, J., Sierra Fernando, Z., Urquiza, G., Janicki, W., Fernández, J. M., An Innovative Approach to Applying the Pressure-Time Method to Measure Flow, HRW (Hydraulic Review Worldwide), Vol. 16, No. 6, December 2008, pp. 40-49.

7. Adamkowski A., Janicki W.: Influence of some components of pressure-time method instrumentation on flow rate measurement results, Inter. Confer. HYDRO2007, Granada, Spain, 15-17 Oct. 2007.

8. Adamkowski A., Kwapisz L.: Determination of the integration limits in the GIBSON method for measuring the flow rate in closed conduits. International Conference HYDROFORUM’2000, Czorsztyn, Poland, 18-20.10.2000, (in Polish) pp. 287-298.

9. Adamkowski A., Krzemianowski Z., Janicki W.: Improved Discharge Measurement Using the Pressure-Time Method in a Hydropower Plant Curved Penstock, J. Engng Gas Turbines and Power ASME, 131 (2009), pp. 053003-1 - 053003-6.

10. Cengel Y. A., Cimbala J. M.: Fluid Mechanics. Fundamentals and Applications, McGraw-Hill International Edition, New York, 2006.

11. Bortoni EC, Souza Z, Santos AHM, On the leakage flow measurement in Gibson method applied for hydro power plants, WaterPower XIV, 2005.

12. Adamkowski A., Janicki W.: A method measurement of leakage through closed water turbine wicket gate. Inter. Confer. HYDRO2009, Lion, France, 26-28 Oct. 2009

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