UNCERTAINTY AND PERFORMANCE OF A LOW HEAD THERMODYNAMIC MEASUREMENT

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ABSTRACT

Hol II power plant has a nominal head of 46.5 meters where a thermodynamic efficiency measurement has been performed with a thorough analysis of the outlet energies. In addition to this an uncertainty analysis has been performed. This has pointed out the importance of stable temperature conditions, and the importance of reducing the systematic uncertainty in temperature read-off. It has also resulted in a discussion, when using absolute temperature measurements, if it is the uncertainty for the temperature difference or the uncertainty for the temperature measurements itself that should be used in the overall uncertainty calculation.

The thermodynamic efficiency measurement method is according to IEC 41 [2] not valid for hydro power plants with a head lower than 100 meters. One of the largest uncertainties in the thermodynamic method is the measurement of outlet energy. The international standard IEC 41 states that 0.6 % uncertainty should be added just for the outlet energy. By introducing several control volumes at the outlet, where both water velocity and temperature are measured, this uncertainty can be reduced. It also gives the opportunity to use so called weighted average in the calculations. Using weighted average contra non-weighted average seems to give a more correct efficiency curve. Studying the energy distribution and flow pattern at the outlet of a Francis turbine is also interesting for other research matters.

At Hol II the inlet diameter is so large that two thermo elements were needed according to the IEC 41. Difference in the recorded temperatures from the two thermo elements has caused problems that have been discussed. The reason for the difference may be of hydraulic matter, but it can also be a systematic difference that should have been found during zero point calibration. The reason for the difference could also have been detected if the upper and the lower temperature probe on the inlet had been shifted during the test, but unfortunately this was not done. The uncertainty for inlet energy should be set to 0.2 % according to the IEC 60041, and this is also used in these measurements. It could be though that a butterfly valve that was positioned just upstream the inlet measurement point, caused so much disturbance in the flow that this uncertainty should have been increased slightly, but the thermo elements measured values are within each others systematic and random uncertainties.

The paper concludes that to perform thermodynamic measurements on low head hydro power plants, it is important to have stable temperature conditions, and control with the outlet energy. But just as important it is to get a good calibration of the thermo elements in use and a good documentation of the calibration. Making a good zero-point calibration jig, where one could be sure that the temperature on all thermo elements is as good as equal can be a solution for future tests. Performing a thermodynamic measurement under good conditions and with good and thoroughly calibrated equipment makes this method competitive to other methods.

INTRODUCTION

For turbines with head above 100 meters water column, the thermodynamic method is very well suited. The method has an accuracy of better than 1 % depending on the power station and measuring conditions. It is also a fast method to perform and it demands few stops in the production, which is a huge advantage from an economic point of view. For power plants with a head less than 100 meters the thermodynamic method is not valid according to IEC 41. However, if the conditions are favourable, the measurements are quite possible to perform

One of the greatest uncertainties in the thermodynamic method is the measurement of outlet energy. The international standard IEC 41 states that 0.6 % uncertainty should be added just for the outlet energy. By introducing several control volumes at the outlet, where both speed and temperature are measured, we believe that this uncertainty can be reduced so much that the method will have a sufficient accuracy also for lower heads. It is also of general interest to study the energy distribution and flow pattern at the outlet of a Francis turbine.

Hol II power plant in Hallingdal, Norway, has a nominal head of 46.5 meters and a power output of 30 MW. The best efficiency point is at 23 MW. Its waterway design together with its limited magazine capacity basically ruled out the use of ultrasonic equipment and the pressure-time method, and the owners experience with current meters is not good. Therefore a thermodynamic measurement is chosen despite that IEC 41 claims that it is not preferable. To improve the conditions the measurement was performed in the winter while the reservoirs are covered with ice in order to get stable inlet temperatures. Note the difference between test point and control volume in the text below. When test point is used in the text, this means a test that is running on a constant power output. The result of a test point is made up from several control volumes at the outlet.

METHODOLOGY

The thermodynamic method results from the application of the principle of conservation of energy or the first law of thermodynamics [4]. In other words, any loss (volumetric losses excluded) will result in a small temperature increase in the water. If the process is shown in an enthalpy-entropy diagram shown in Figure , a perfect process without loss would go from point 1-1 in the diagram to the point s as the pressure is reduced through the turbine. The real process however goes to the point 2-1.

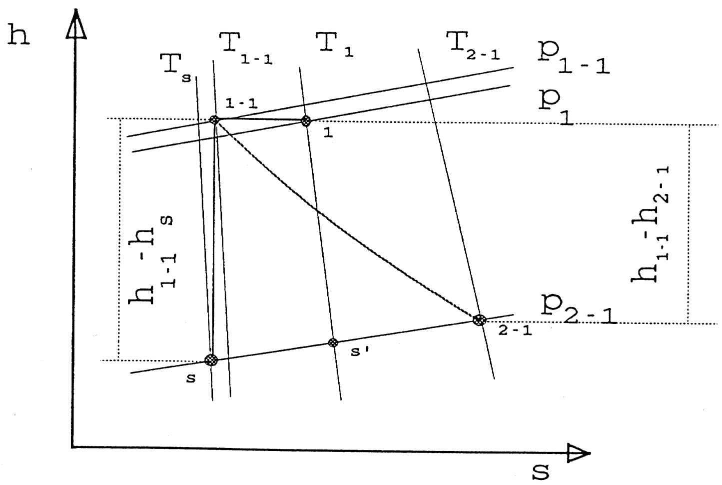

Figure 1: Enthalpy- Entropy diagram

The efficiency of the process is given by the relation:

$$\eta = \frac{h_{1-1} - h_{2-1}}{h_{1-1} - h_s} \quad [-] \qquad (1)$$

The equations can by using relations given in thermodynamics be derived to the final equation

$$\eta = \frac{E_{mechanical}}{E_{hydraulic}} = \frac{\overline{a} \cdot (p_{1-1} - p_{2-1}) + \overline{c_p} \cdot (T_{1-1} - T_{2-1}) + g \cdot (z_{1-1} - z_{2-1}) + \frac{1}{2} \cdot \left(c_{1-1}^2 - {c_{2-1}}^2 \right)}{\frac{1}{\rho} \cdot (p_1 - p_2) + g \cdot (z_1 - z_2) + \frac{1}{2} \cdot \left({c_1}^2 - {c_2}^2 \right)} \tag{9}$$

Which has parameters that are possible to measure.

INSTRUMENT AND DATA ACQUISITION SETUP

The same method and a similar setup have been used in earlier occasions. Two of these are described in the work by Hulaas et. al in the proceedings from the IGHEM conference in Kempten, Germany 2000 [5].

At Hol II power station the diameter at the inlet of the turbine is so large that according to the IEC 41 two measurement points has to be used. Therefore one temperature sensor was installed at the top and one at the bottom of the inlet cross section. These sensors were mounted in probes that also were connected to pressure transmitters. The velocity through the probes was found by filling a container with known volume and using a stop watch. Inlet pressure was recorded using a pressure transducer connected to the inlet ring main. At the outlet altogether four temperature sensors were mounted in frames that could be traversed. All temperature sensors were of the type Seabird SBE38 absolute temperature sensors. The turbine outlet was split by a pillar, therefore two frames were needed. Five different heights for measurement were predestined, giving 20 control volumes. Next to the temperature sensors at the outlet, electromagnetic velocity meters were mounted. The velocity found by these was used to weight the energy in each control volume relative to each other. The outlet pressure was found manually using a laser distance meter to find the water level. For calculation of mechanical energy, the power stations own power output read-off, and the generator efficiency curve were used. The data acquisition system was made by National Instruments and the software was LabView. An outline of the instrument and data acquisition setup is shown in Figure 2.

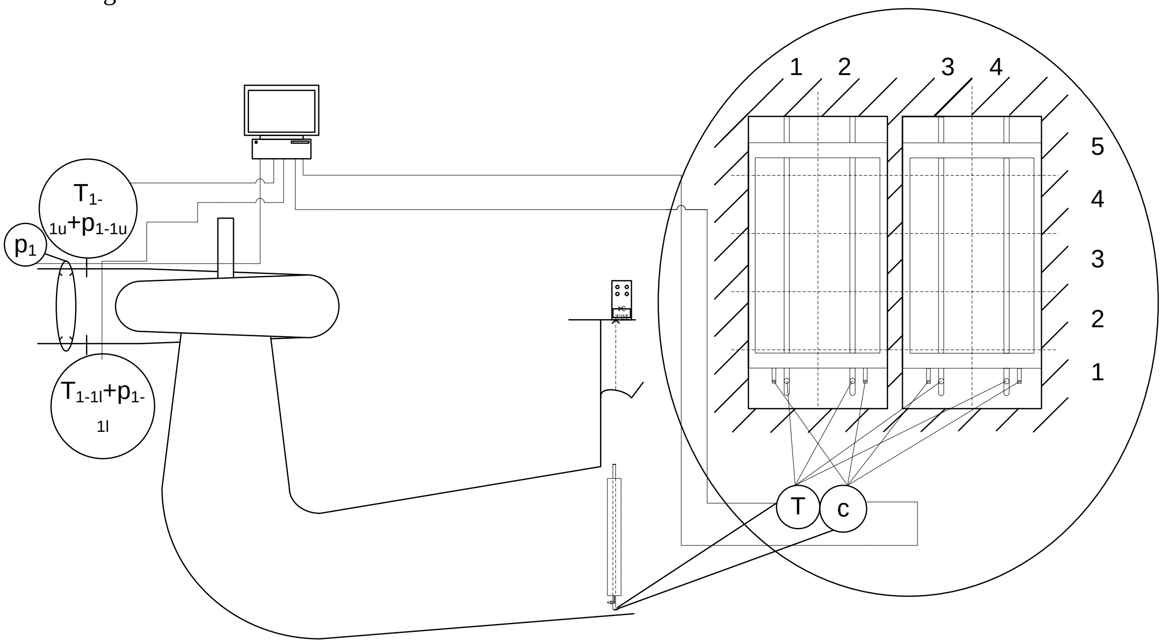

Figure 2: Instrument and data aquisition setup

For the calculation of efficiency, each of the control volume's contribution is relative to the water velocity in the control volume divided by the accumulated water velocities for the whole test point. Since they also have to be calculated with respect to two temperature elements at the inlet, it gives that altogether 40 part efficiencies has to be summed.

$$\eta = \sum_{i}^{n} \frac{\eta_i \cdot c_{2-1i}}{\sum c_{2-1}} \qquad [-] \tag{10}$$

By using this method, control volumes with small water velocity, will affect the total efficiency less.

THE CASE OF ENERGY DISTRIBUTION ON THE OUTLET

The IEC 41 [2] claims "*the uncertainty due to faulty exploration of energy distribution (for the low pressure side) is estimated to be 0.6 % of mechanical energy*." The main aim for this test was to see if the uncertainty of energy distribution for the low pressure side could be reduced by a more thorough measurement of the energy distribution. Figure shows the measured flow together with the deviations in mean efficiency for each control volume. The diagrams for part load at (15 MW), best point (23 MW), and full load (27 MW) illustrate the development of flow pattern as power increase. The diagram for 24 MW is shown because this had the most extreme deviation in efficiency between control volumes. It is chosen to show the graphs in a bar diagram in stead of contour diagram since the grid for the sensors actually is quite coarse with 1.5 meters between the sensors horizontally, and 1.1 meters vertically.

In the diagrams the numbers 1 and 2 on the x-abscissa is for the control volumes in the left draft tube, and 3 and 4 in the right. For the y-abscissa, 1 is on the lowest level of the traversable measurement frame, and 5 the uppermost.

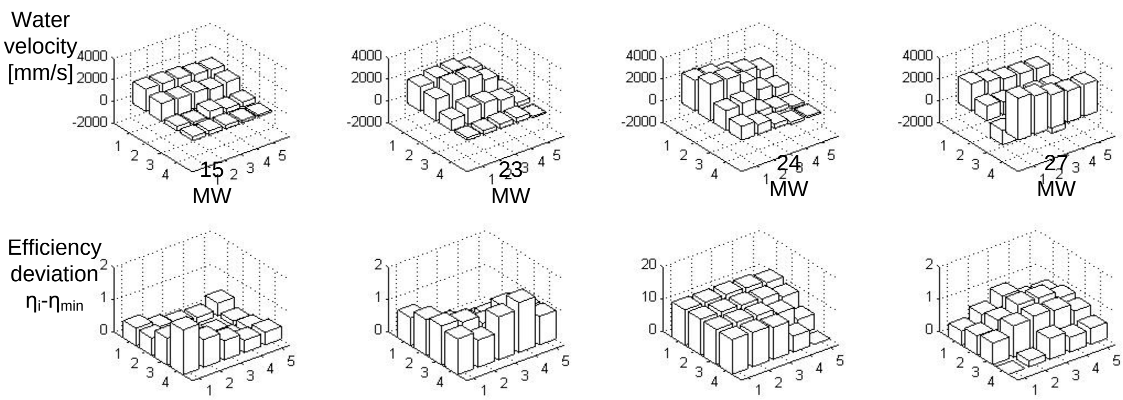

Figure 3: Flow and efficiency distribution in oulet control volumes

As can be seen from the diagrams there is quite a lot of difference between the water velocities in the different control volumes. The higher the efficiency output is, the more chaotic the flow distribution becomes. Some of the control volumes also experience backflow. This can be expected since the flow increases and the draft tube swirl give a non uniform flow. But even on the best efficiency point, where the swirl should be of less significance, there is backflow in a control volume. This might imply that the draft tube bend affects the flow.

To illustrate the usefulness of weighted average special attention can be put on the point at 23.71(≈24) MW. Looking at the efficiencies in each control volume at the outlet, one control volume has a deviation of 10 percent, and two of the neighboring control volumes have almost 5 percent deviation from the "normal". These control volumes are situated in the upper right corner of the outlet cross section. The water velocities for these control volumes are small and one of the control volumes have a small backflow. The best guess as to what happens in this region is that there is a kind of standing whirlpool in this upper right corner. If the efficiency had been calculated by using the mean of all control volumes, this point would get an efficiency that is 1.44 percent lower than the one when weighted average is used. This is the only point where you get such an extreme deviation between a weighted average, and a normal average. Under normal circumstances this point would probably be rejected due to the large deviations in the control volumes, but as mentioned, it gets a reasonable result when using weighted average. For the other points the deviation between weighted average and normal average became less than 0.4 %. Table 1 shows the calculated uncertainty for the outlet energy for each test point and the difference between a weighted average calculation and a normal average calculation. Similar tests were done by Hulaas and Dahlhaug et. al. [3][5], but contrary to this case their tests showed no significant difference between weighted average and normal average. But since all turbines have their own unique design, it is almost impossible to know in advance how the flow pattern on the outlet is. Using weighted average is therefore a security for calculating the outlet energy correctly. Especially in low head measurement where even small temperatures can give large effects on the efficiency calculation.

An interesting test to do when having backflow or a standing swirl would be to replace the multiple control volume measurement with a measurement with a regular collecting frame. This would detect if a regular frame collects the water in a way that resembles the correct outlet energy.

Using the formula of random uncertainty with 20 points and a student t distribution, the uncertainty of energy distribution on the outlet becomes 1.33 percent. It is not assumed though that this gave much error in the calculation of efficiency since the method of weighted average was used.

Table 1: Uncertainty in outlet energy distribution and difference between weighted and normal average

Power [MW]	9.77	12.98	14.85	16.69	18.36	20.62	21.84	22.78	22.85	23.71	25.12	27.15	29.45
Uncertainty in outlet energy distribution [%]	0.22	0.15	0.15	0.15	0.16	0.29	0.28	0.19	0.18	1.33	0.69	0.14	0.44
Difference in efficiency using weighted average and normal average [%]	0.01	0.02	-0.10	-0.16	-0.17	-0.27	-0.21	-0.11	-0.17	1.44	0.39	-0.24	-0.15

THE UNCERTAINTY ANALYSIS

For the uncertainty analysis the Root-Square-Sum method gives that the relative uncertainty of the efficiency is

$$f_\eta = \sqrt{f_{E_m}^2 + f_{E_h}^2} \quad [-] \tag{11}$$

Where the uncertainties for mechanical and hydraulic energies are

$$e_{E_{mtotal}} = \sqrt{e_{E_{mp}}^2 + e_{E_{mc}}^2 + e_{E_{mz}}^2 + e_{E_{mT}}^2} \text{ [J/kg] and } e_{E_{htotal}} = \sqrt{e_{E_{hp}}^2 + e_{E_{hc}}^2 + e_{E_{hz}}^2} \text{ [J/kg]} \tag{12}$$

Each of these parameters are then again broken down to each parts uncertainty down to the uncertainty of calibration/systematic uncertainty and random uncertainty. Then they are added together again using the root sum square method. For parameters that have the same dimension the absolute uncertainties are added, while for the parameters with different dimensions, the relative uncertainties have to be added. As an example the uncertainty of the thermal energy is given.

$$E_{mT} = \overline{c_p} \cdot (T_{1-1} - T_{2-1}) \text{ [J/kg]} \tag{13}$$

$$f_{E_{mT}} = \sqrt{f_{Cp}^2 + \frac{(e_{T_{1-1}}^2 + e_{T_{2-1}}^2)}{(T_{1-1} - T_{2-1})^2}} \quad [-] \tag{14}$$

$$e_{Cp} = \sqrt{\left(\left.\frac{\partial Cp}{\partial T}\right|_{T=\frac{T_{1-1}+T_{2-1}}{2}}\right)^2 \cdot \left(e_{T_{1-1}}^2 + e_{T_{2-1}}^2\right) + \left(\left.\frac{\partial Cp}{\partial p}\right|_{p=\frac{p_{1-1}+p_{2-1}}{2}}\right)^2 \cdot \left(e_{p_{1-1}}^2 + e_{p_{2-1}}^2\right) + e_{Cp_{table}}^2} \text{ [J/kg·K]} \tag{15}$$

$$e_T = \sqrt{e^2_{T_{systematic}} + e^2_{T_{random}}} \quad [^oC] \tag{16}$$

For uncertainty of calibration of pressure transducers, the method for finding uncertainty of slope in regression analysis was used [1]. This reduces the systematic uncertainty of the pressure probes. The random uncertainty of the test data was found using student t-distribution as suggested by IEC 41 [2], defining the random uncertainty to be

$$e_{E_{m_{random}}} = \frac{t_{0,95} \cdot \sigma}{\sqrt{n}} \quad [J/kg] \tag{17}$$

Knowing that the student t factor ($t_{0,95}$) is reduced as the number of recorded points, n, is increased, it is evident that more measurement points will decrease uncertainty as long as the standard deviation for the measured points is not too large. This method was also used for finding the uncertainty of the outlet energy described earlier. For the outlet energy distribution the number of recorded points is 20, which is the same as the number of control volumes. It was also used for uncertainty in inlet and outlet energy distribution in what is chosen to be called the averaged control volume uncertainty. However now with n equal to 40 because of 2 measurement probes on the inlet and 20 control volumes on the outlet.

Random and systematic uncertainty was added using the root-square-sum method. The systematic uncertainty of the thermo elements was set to 1 mK according to the manufacturers specifications. Values for uncertainties used in the calculation (random uncertainties excluded as they are individual for each instrument in each test) are given in Table 2.

Table 2: Systematic uncertainties for parameters

Parameter	**Symbol**	**Value**
Systematic uncertainy for temperature elements	$e_{Tsystematic}$	1 mK
Systematic uncertainty for pressure probes	$f_{psystematic}$	0,045 %
Systematic uncertainty for p_{amb}	$f_{pambsystematic}$	0,02 %
Uncertainty for calculation of specific heat capacity	f_{Cp}	0,5 %
Uncertainty for calculation of isothermal factor	f_a	0,2 %
Uncertainty of given geodetic heights	e_z	0,02 m
Uncertainty of measured water levels	e_{Dz}	0,02 m
Uncertainty of inlet radius measurement	e_{r1}	0,01 m
Uncertainty of outlet area	f_{A2}	0,4 %
Uncertainty for measurement of velocity in outlet control volumes	f_{c2-1}	10 %
Uncertainty for measurement of velocity in temperature probes	f_{c1-1}	0,65 %
Uncertainty for generator output read-off	e_P	0,1 MW

Table 3 and Figure 16 show the efficiency curve with uncertainty.

Table 3: Efficiencies and uncertainties

Power	9.77	12.98	14.85	16.69	18.36	20.62	21.84	22.78	22.85	23.71	25.12	27.15	29.45
Calculated efficiency [%]	81.41	86.67	88.78	90.91	92.58	93.92	94.38	94.54	94.62	94.47	94.04	92.06	88.14
Calculated uncertainty [%]	1.81	1.73	1.63	1.66	1.50	1.81	1.75	1.44	1.54	1.77	1.60	2.70	1.98

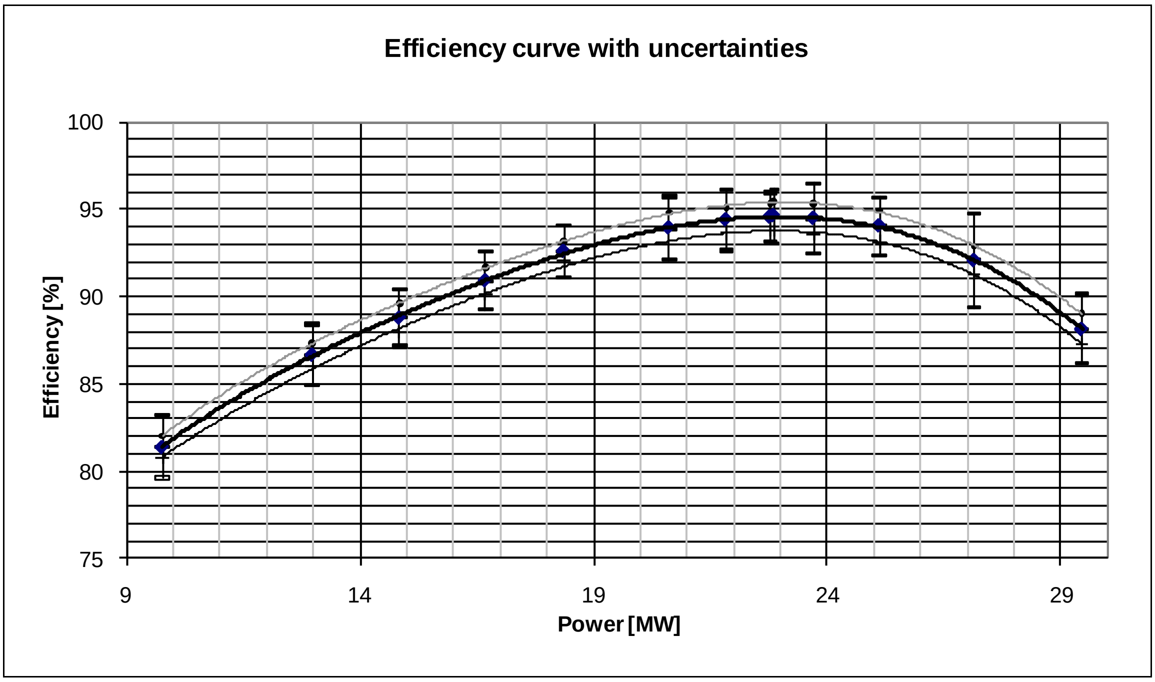

Figure 16: Efficiency curve with uncertainties

For this case since the head is low, the largest contribution to the uncertainty of the calculated efficiency is due to the systematic uncertainty of the temperature measurement of 1 mK. This one alone counts for 1.37 percent uncertainty on the best efficiency point. By better calibration, the systematic uncertainty might be considerably reduced. This will be discussed further down in this article.

THE CASE OF THE 27.15 MW POINT, AND RANDOM UNCERTAINTIES

A test point that should get some extra attention is the point at 27.15 MW which has an uncertainty of 2.7 percent. This is dominated by the random uncertainty of the temperature recording, as the temperature changes during the test point were quite large. (The test was made on a sunny day, and the point was taken in the evening when the sun settled behind the mountains and shadows were cast on the power plant inlet). This point out the importance of stable temperature conditions during a test like this. But even tough the random uncertainties of the measurements become large; the random uncertainty for the temperature difference between inlet and outlet is small. The changes in recorded temperature at the inlet are followed by an equal change in the temperatures at the outlet. This means that if a temperature difference measurement bridge had been used, this uncertainty would not have been an issue. The random uncertainties for the temperature recordings for the different control volumes were varying from 0.68 mK to 2.7 mK, while the random uncertainties for the temperature difference between inlet and outlet control volumes were only varying from 0.15mK to 0.25 mK. If the random uncertainties for the absolute measurements are replaced by the random uncertainty for the temperature difference, the uncertainty for this point would be only 1.5 %.

THE CASE OF DIFFERENT TEMPERATURE MEASUREMENTS AT INLET

In the efficiency diagram the efficiency of the upper and the lower temperature on the inlet is plotted as thin grey lines. The upper temperature probe has ca. 1.4 percent better efficiency than the lower one. I.e. the *recorded* water temperature is a little higher on this point. With the setup used during measurement it is impossible to say whether this is due to different temperature layers in the flow, or if it is a systematic difference in reading of the instrument. It is also discussed if the turbine inlet valve, or the geometry of the pressure shaft just in front of the inlet could cause a loss and thereby a heating in the upper part of the flow. However the recorded temperature difference seems to be quite constant and it is therefore most probable that the difference is due to a systematic difference in the temperature read off. If the temperature difference had been flow induced, some kind of dependency upon flow velocity should be expected. This once again brings forth the issue of a better zero point calibration for the temperature sensors which will be discussed later. Another possible action to detect if the difference was due to systematic difference or flow related, would be to shift the upper and the lower temperature sensor while the turbine runs at stationary conditions.

The large deviation between upper and lower temperature measurement also question how to calculate the uncertainty. The deviation between them is so large that it almost uses most of the calculated uncertainty band. The uncertainties shown are calculated using the earlier described average control volume method. But since the measurement consists of two measurements on the inlet the uncertainty could be calculated adding uncertainties for the upper and lower measurement using the root-square-sum method. This would increase the uncertainty quite a lot. The uncertainty can also be said to stretch from the lowest uncertainty band for the lower temperature probe, to the upper uncertainty band for the upper temperature probe. This would increase the uncertainty even more. For the best point the first method gives an uncertainty from 93.11 to 95.98, the second method an uncertainty from 92.53 to 96.55, and the third method an uncertainty from 92.34 to 96.72.

DISCUSSION OF DIFFERENTIAL TEMPERATURE UNCERTAINTY VS ABSOLUTE TEMPERATURE UNCERTAINTY

The important parameter in a thermodynamic measurement is the temperature difference, and not the exact temperature in the measurement point. Therefore from an uncertainty calculation point of view, it should be better to use a temperature difference measurement bridge, rather than absolute temperature measurements that was used in this case. However the experience with the temperature sensors that were used in this measurement (SBE 38), is that they are very stable. The temperature read off may have a systematic error, but this error seems constant and also the slope also seems very constant. This experience is based upon calibration history, repeatability of reference points, and that the random uncertainty of the temperature difference between the sensors during tests. (The zero point temperature read off over time may change a little bit, but according to calibration history less than 0.7 mK over a 4 year period for the worst case in the six SBE 38’s that we possess). The question becomes then whether the systematic uncertainty given by the manufacturer can be rejected, and replaced by a random uncertainty found in a zero point calibration. The calibration system and setup would have to be of such a quality that the temperature on all the compared temperature sensors is the same, and the recording of temperature should happen simultaneously in the time domain for all sensors.

Said that the temperature difference between compared sensors is the same for all temperatures put into the calibration system, the systematic error given by the manufacturer could be replaced by the random uncertainty from the calibration. Or even, to make the demand stricter, use the standard deviation from the calibration as uncertainty. As an example, standard deviation from the Hol II measurement test points can be used. The point with the largest standard deviation on temperature difference had a standard deviation of approximately 0.7 mK. This standard deviation also account for the waters travel time from inlet to outlet, varying flow, e.g. caused by draft tube swirl, displacement in time of temperature recording etc. But 0.7 mK is smaller than 1.37 mK which is the uncertainty if the manufacturers specifications and the combination of two absolute measurements are to be used. However, the blunder that was made for this test was that the zero point calibration system was not good enough to find a certain displacement between the different sensors, or to find out if they all have the same and a constant slope. However, to use this other kind of calibration, the system in use has to be very stable according to experience, such as we have with the SBE 38 sensors.

The calibration issue if it is the differential temperature or the absolute temperature that is the correct parameter to use is a very important source for solving many of the challenges regarding the high uncertainties in this measurement.

FINAL CONSIDERATIONS

This method can if the temperature variation is small and the number of control volumes is large enough reduce the uncertainty of the outlet energy distribution. But it can also increase the uncertainty in case the flow and energy distribution is very non uniform. Anyway when one has the data of the flow patterns and energies available, one is more capable of evaluating the significance of this. The analysis has also shown that the uncertainty for a low head thermodynamic measurement is mainly dependent on the systematic uncertainty in temperature measurement (1 mK for this case). A system for reducing the temperature measurement uncertainty should be given more attention, and is the most important measure for making the thermodynamic method available for low head measurements. A good and officially approved zero calibration jig and zero calibration method is essential in so matters, and a discussion as to what should be the procedures for such a calibration is needed.

The analysis has also shown the importance of stable temperature conditions. As soon as the water temperature started varying during a measurement the uncertainty of the measurement point would increase considerably. However, varying water temperature is not necessarily the same as varying differential temperatures, so this matter will only appear in cases where absolute temperature sensors are used, contrary to a differential temperature measuring bridge.

The two temperature probes at the inlet have a difference in temperature measurement. This can be caused by disturbance, swirl etc. in the flow at the inlet, but since the temperature difference seems to be constant, the most likely reason is systematic difference in the temperature read off in the two temperature probes. A test where the temperature probes at the inlet were shifted should have been performed. This would have cancelled out a lot of the uncertainty due to deviation between the upper and lower temperature probes. This big difference can also cause discussion of the validity of the measurement. Also in this case a good calibration jig and method could be a solution to the problem.

It is believed that if all suggested improvements are performed, and the conditions at the power plant are good, the uncertainty can be decreased so much that this method can be competitive to other methods.

The authors hope to use the same method on other power plants in the future, in order to get enough statistics to support the conclusion. It is also a goal to build a suitable calibration jig that removes possible errors and uncertainties in a zero point calibration.

NOMENCLATURE

Symbols

E	Energy	J/kg
p	Pressure	kPa
c	Water velocity	m/s
T	Water temperature	^{o}C
c_p	Specific heat capacity of water	J/kg·K
a	Isothermal factor of water	m^3/kg
z	Geodetic height	m a.s
h	Efficiency	
$t_{0,95}$	Student t factor for 95% confidence interval	
e	Absolute uncertainty	
f	Relative uncertainty	
h	Enthalphy	J
s	Entropy	J/K
n	number of samples	
s	population mean standard deviation	
ρ	Water density	kg/m^3

Subscripts

m	Mechanical
h	Hydraulic
table	Value derived from table
1	High pressure section
2	Low pressure section
1-1	High pressure temperature measurement section/point
2-1	Low pressure temperature measurement section/point
u	Upper measurement position inlet
l	Lower measurement position inlet
i	Sample number
mp	Pressure part of mechanical energy
mc	Velocity part of mechanical energy
mz	Potential energy/geodetic height part of mechanical energy
mT	Temperature part of mechanical energy
hp	Pressure part of hydraulic energy
hc	Velocity part of hydraulic energy
hz	Potential energy/geodetic height part of hydraulic energy

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