Multipoint Thermodynamic Measurements – A Statistical Approach to Uncertainty Levels

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1 ABSTRACT

The paper presents two thermodynamic efficiency measurements performed using several thermometers and velocimeters for detailed investigations of the energy at draft tube outlet.

The energy distributions are discussed, leading to a proposal for calculating the uncertainty levels. The uncertainty is further presented as a function of the number of probe locations at the outlet, computed by a statistical approach for a 95 % confidence level.

At one of the power plants, the energy at the outlet was also measured using a conventional collector device, revealing some discrepancies compared to the full investigation measurements. These discrepancies are discussed in the paper.

2 INTRODUCTION

The thermodynamic method is by far the preferred method for efficiency tests in the high-head Norwegian hydropower business. The low-head limitations for the method (IEC 41 [1]: 100 meters) still applies to some 240 of the 550 power stations in Norway, not counting the ones below 1 MW. A substantial part of these 240 plants are in the region where the thermodynamic method is desirable for practical reasons. No method offers the same accuracy within the same cost/time horizon. To the authors' knowledge, the method has been practised down to 50 meter head.

This rises the question of uncertainties associated with the temperature measurements. The 'normal' uncertainty for thermometers in use, are ± 1 mK, corresponding to ± 0.42 m head. Though not accounted for in [1], the physical uncertainties associated with the energy distribution both in the inlet and outlet *will intuitively* be more significant at decreasing head. The recommended systematic uncertainties due to the absence of exploration of the energy distribution amounts to ± 0.2 % for the inlet and ± 0.6% for the outlet of the turbine, regardless of the plant's head. This lack of physicality and general problems related to the energy distribution in draft tube outlet has been addressed at several occasions, namely by Alming, 1972 [2], Alming and Vinnogg, 1984 [3], Schedelberger [4], Brekke, 1992 [5], Brekke and Dahlhaug, 1996 [6], and 1998 [7], Dahlhaug et al., 1998 [8], Karlicek, 1998 [9] and 1999 [10]. The last publication by Karlicek proposes *tables* replacing the fixed values, for determining the mentioned systematic uncertainties in inlet and outlet, as a function of net head.

Although the criticism against this weakness in IEC 41 is becoming increasingly more profound, an improved method for attaining a representative specific energy in the outlet is the only means to improve the test *results*, and not only the *reports*.

In 1995, Dahlhaug performed measurements on turbine no. 2 at Bratsberg power plant in Trondheim, Norway, utilising 30 temperature/velocity probe positions in the draft tube outlet. After replacement of both turbines at Bratsberg, similar measurements have been performed, allowing numerous data to be analysed. One of these tests are discussed and analysed in this work, in addition to a test at Kaldestad power plant in 1998.

3 THE MEASUREMENTS

3.1 The Power Plants

The data for the two power plant measured is given in Table 1.

Table 1

Power Plant	Date of Test	Net Head	Power	Ann. Prod.
Kaldestad	Febr. 1999	75 m	23.1 MW	80 GWh
Bratsberg	March 1999	130 m	67.0 MW	180 GWh

Kaldestad power plant is situated near Norway's second largest town, Bergen, while Bratsberg in the surroundings of Trondheim, Norway's third largest town. Kaldestad has one turbine, Bratsberg two.

3.2 The Instrument Setup

The instrument set-up was basically the same for the two measurements, and the instruments were set up as shown in Figure 1.

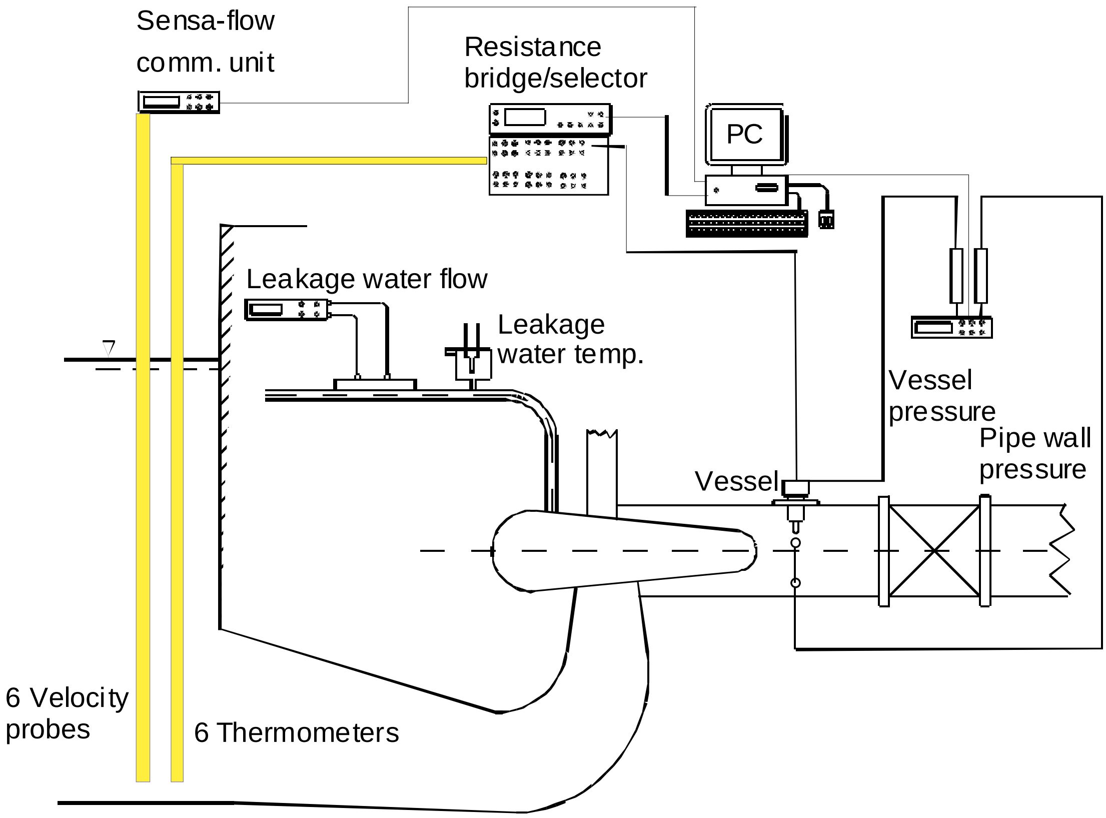

Figure 1. Instrument setup for the multipoint measurements.

The temperatures were measured using an ΑΣΛ F700 thermometer bridge with PT100 thermometers, and the velocities were measured using Sensa RMX electromagnetic velocity probes. The temperatures and the velocities were logged in the data acquisition system.

3.3 Measuring the Energy Distribution

The energy distribution at the outlet was measured using six thermometers and six velocity meters, mounted horizontally on a movable frame. For each operational point the frame was traversed in five heights, thereby covering a grid of 30 points distributed evenly over the outlet area.

At Bratsberg power plant, the energy was also measured using a sampling pipe mounted on the frame, collecting water and feeding it to a thermometer.

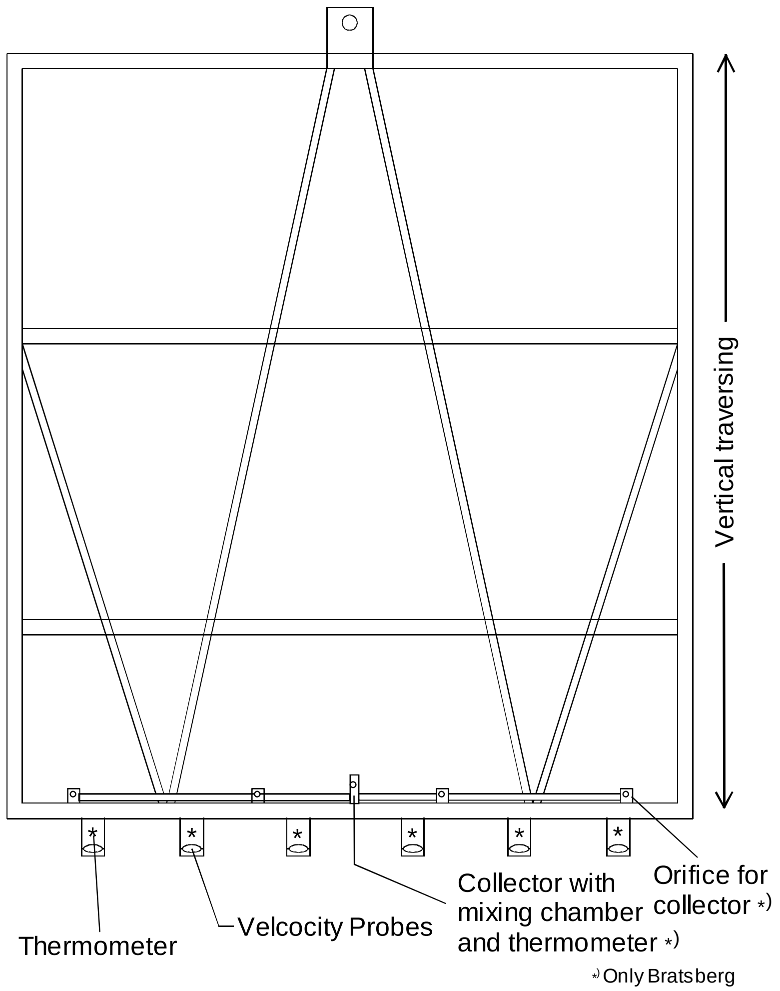

Figure 2. Frame for investigation of energy distribution.

3.4 Calculation of Specific Mechanical Energy

The equation for the specific mechanical energy will normally be given by its pressure, temperature, velocity and geodetic height terms

$$E_m = \bar{a} \cdot (p_{11} - p_{21}) + \overline{C}_p \cdot (\Theta_{11} - \Theta_{21}) + \frac{V_{11}^2 - V_{21}^2}{2} + \bar{g} \cdot (z_{11} - z_{21}) \qquad \text{[J/kg]}$$

If the specific mechanical energy at the outlet is measured in more than one point and weighted with regard to the measured velocity, the equation becomes more complicated. Each subarea *i* from 1 to n must weighted with respect to the measured flow.

$$E_m = \bar{a} \cdot \left(p_{11} - \frac{\sum_{i=1}^{n} A_{2-i} V_{2-i} \cdot p_{2-i}}{\sum_{i=1}^{n} A_{2-i} V_{2-i}} \right) + \overline{C}_p \cdot \frac{\sum_{i=1}^{n} A_{2-i} V_{2-i} \cdot (\Theta_{11} - \Theta_{2-i})}{\sum_{i=1}^{n} A_{2-i} V_{2-i}}$$

$$+ \frac{V_{11}^2}{2} - \frac{\sum_{i=1}^{n} A_{2-i} V_{2-i} \cdot \frac{V_{2-i}^2}{2}}{\sum_{i=1}^{n} A_{2-i} V_{2-i}} + \bar{g} \cdot \left(z_{1-1} - \frac{\sum_{i=1}^{n} A_{2-i} V_{2-i} \cdot z_{2-i}}{\sum_{i=1}^{n} A_{2-i} V_{2-i}} \right) \quad \text{[J/kg]}$$

3.5 The Measurement Conditions

3.5.1 Bratsberg Power Plant

The measuring conditions were very good, and the inlet temperature was stable within 44 mK through the entire measuring period – about 16 hours. No sudden temperature fluctuations were observed.

3.5.2 Kaldestad Power Plant

The measuring conditions were good. During the measuring period there were observed some instabilities in the temperature at the outlet, and these were removed before averaging the results. These instabilities were probably due to the drainage pump occasionally starting up.

It was impossible to lower the frame with water flowing through the turbine – the turbine had to be shut down between every measuring point. It was also impossible to rise the frame from position I to position II while the unit was running. this position was then only measured for the first measuring point. Then thermometer T_{2-2} was malfunctioning during the first test run. Finally the velocity probes malfunctioned after two test runs. However the temperature distributions on the remaining measuring points were very uniform, and hence – the influence of the weighting factor is minimal.

4 THE RESULTS

4.1 Multipoint Measurements

The relative efficiency is shown in Figure 3 and 4. The difference between the weighted and unweighted efficiencies is not significant.

The standard deviation of the distribution of the energy at the outlet is also shown (1 sigma level).

For Bratsberg power plant (H = 130 m) the energy distribution is quite narrow - about ± 0.2 % over the entire range. For Kaldestad the distribution is somewhat broader. Common for both measurements is the distinct rise in the standard deviation at best-efficiency point – 0.4% for Bratsberg and 1 % for Kaldestad.

4.2 Sampling the Energy Distribution Using a Collector

Figure 3 also shows the efficiency measured using the collector. The efficiency is 0.2 – 0.3 % lower than the multipoint measurement, but well within the +- 1 mK limit.

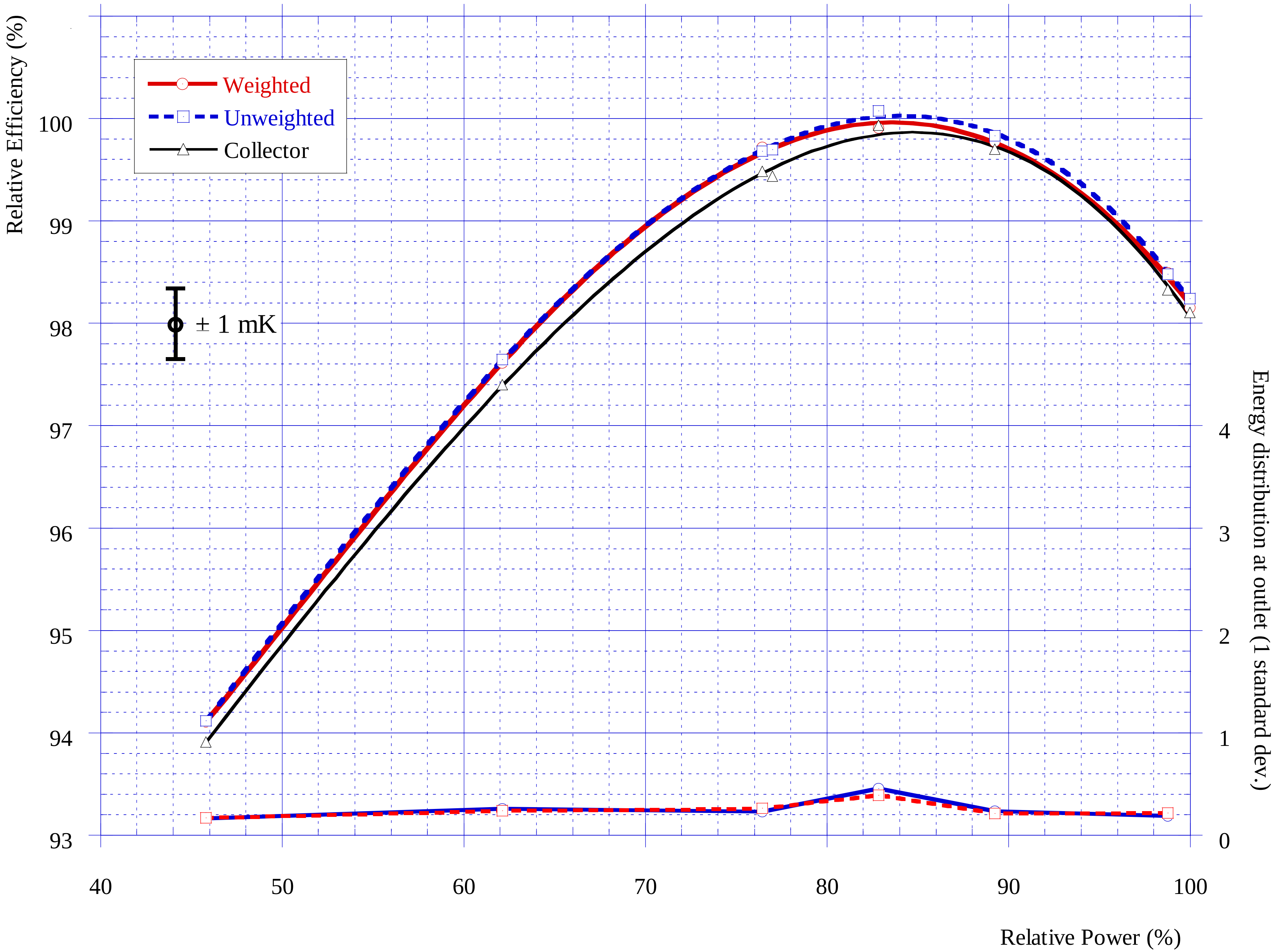

Figure 3. Bratsberg Power Plant

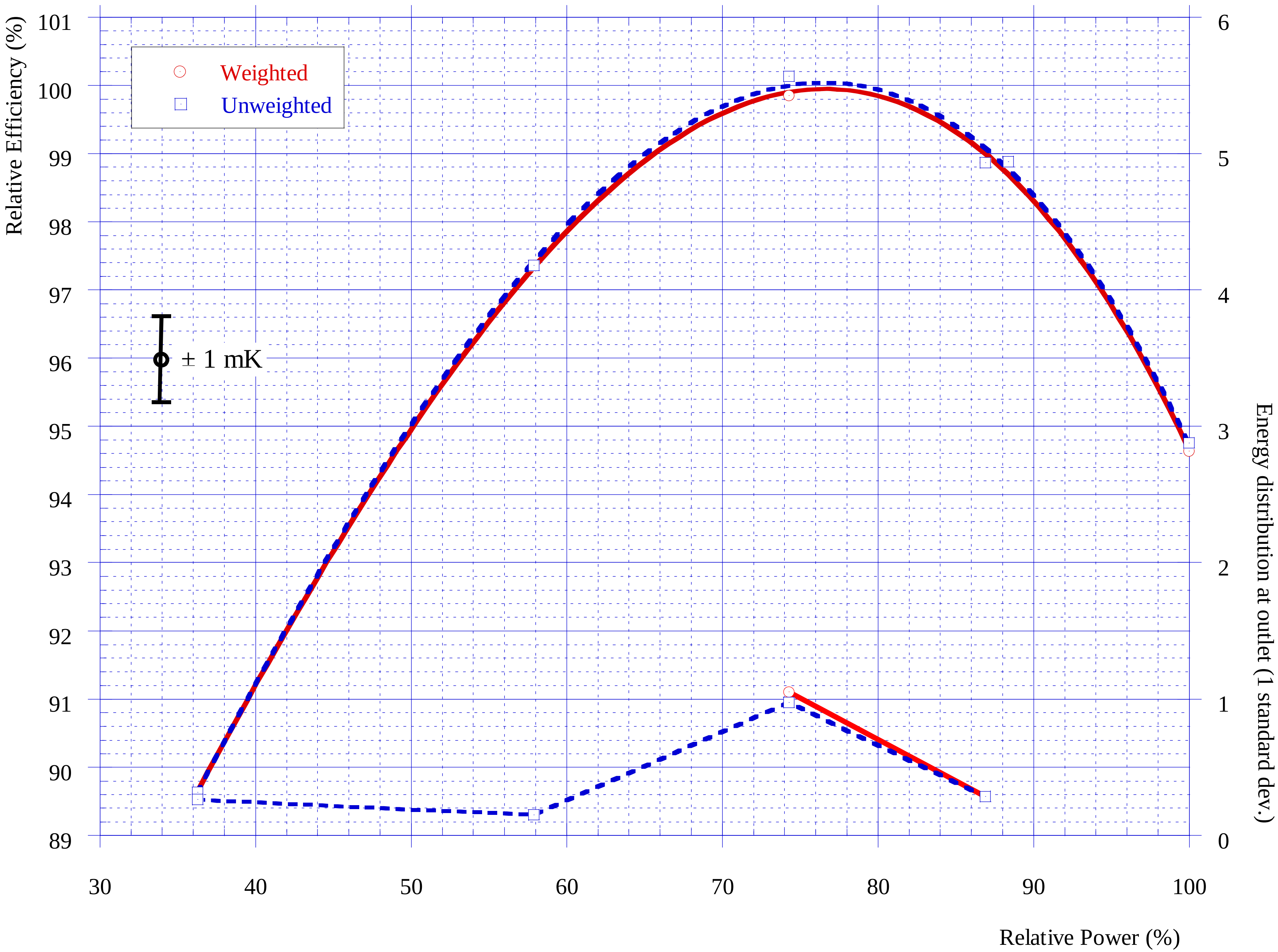

Figure 4. Kaldestad Power Plant

5 PROPOSAL FOR ESTIMATING THE UNCERTAINTIES AT THE OUTLET FOR MULTIPOINT MEASUREMENTS

It is reasonable to assume that the uncertainties due to the absence of exploration of energy distribution at the outlet is a function of

- The number of measuring points (at the outlet)
- The variation of the measured energy

In the following we have a suggestion on how to compute these uncertainties

$$e_{E_{20}} = \pm \frac{\sqrt{\sum_{i=1}^{n}\left(\mathrm{Max}\left(E_{m_{r_i}} - E_{m_{r_{i\pm a}}}\right)_a\right)^2 + \frac{1}{4}\sum_{i=1}^{n}\left(\mathrm{Max}\left(E_{m_{s_j}} - E_{m_{s_{j\pm a}}}\right)_a\right)^2}}{n+m}$$

where

index *r*	-	denotes subarea located at an edge of the cross section
index *s*	-	denotes subarea enclosed by 8 neighbouring areas
index *a*	-	stating that only neighbouring areas are taken into consideration
n	-	no. of subareas at the edge of cross section
m	-	no. of subareas enclosed by 8 neighbouring areas

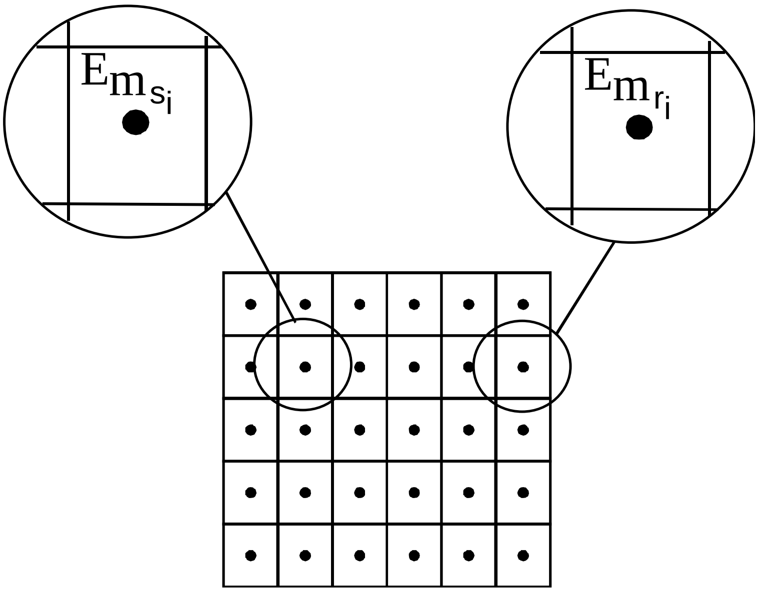

Figure 5. Cross-sectional grid in draft tube outlet.

6 THE UNCERTAINTY IN THE OUTLET - MEASURING 'N' POINTS

Using the data from Bratsberg Power Plant, we have studied the effects of the uncertainties as a function of the number of point measured at the outlet.

The following assumptions are used:

All other uncertainties are dealt with in other terms, i.e. the uncertainties due to temperature difference, temperature gradients etc. In this study only the "measured value" as indicated by Figure 6 is taken into consideration.

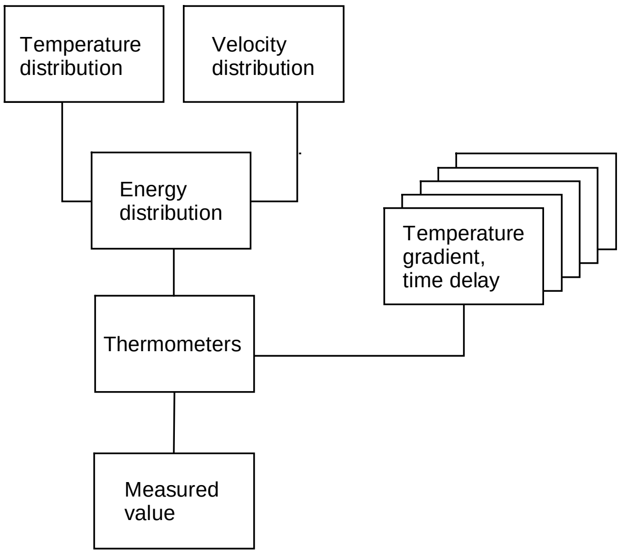

Figure 6. The origin of the measured value.

6.1 Statistical approach

Based on the measurement data from Bratsberg Power Plant, which are considered to be mutually consistent and repetitive, a statistical approach to the following question is achieved:

What is the error of reducing the number of probe positions?

The question needs further explanation: We assume that we have the answer to the 'correct' energy distribution sampled at 30 locations in the cross section – the maximum confidence level. By reducing the number of positions to 29 – what is the level of uncertainty introduced? And to 28? And all the way down to 1 probe?

Sampling at 'n' positions, we also assume that the chief of test has no knowledge of the actual energy distribution, hence placing his probes at arbitrary locations.

The average of the 'n' specific energy levels are used for calculation of the hydraulic efficiency.

For each number of 'n' probe locations, all combinations of 'n' points in the 5 by 6 point grid are selected, and their respective average calculated. These values now represent the 100% confidence level.

In order to perform this operation programmatically in MatLab, a high number of random combinations are used instead of systematically covering all possible combinations. This is a necessity, as the number of possible combinations vary from 1 (30 locations) to about 155 million (15 locations). For this calculation, 50,000 random combinations are employed. At a 166 MHz Pentium PC, the operation took 4 hours.

Example for n = 3: 50,000 combinations of $\overline{\Delta\eta} = \frac{\Delta\eta_i + \Delta\eta_j + \Delta\eta_k}{3}$ are calculated. These values are then sorted in ascending order. For a 95 % confidence level, the upper 2.5 % and lower 2.5 % are rejected, leaving the values covering the whole 95 % confidence level. The max and min value of the remaining values are then the upper and lower border of the confidence level, respectively.

6.2 Results from calculation

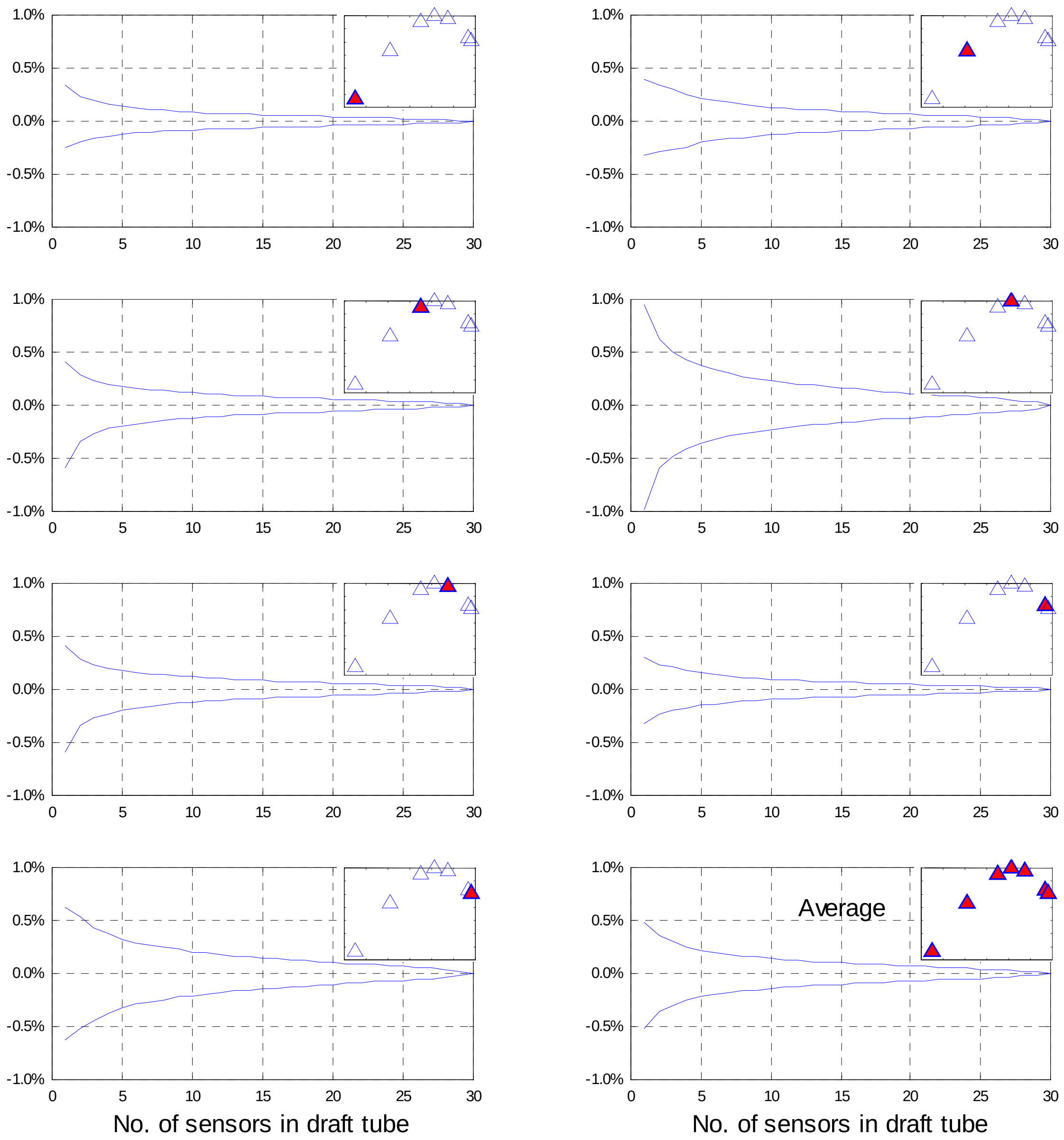

Figure 7. Results from calculating the 95 % confidence levels for 'n' probe locations, 'n' covering the range from 1 to 30 locations in the 5 by 6 point grid of the cross section. Each plot is from a single operational point of the turbine, indicated by the relative P-η curve.

A comparison of all of the operational point reveals, quite surprisingly, that the highest error level occurs at best efficiency point of the turbine! The variation of specific energy is largest for the point where the turbine outlet offers optimal conditions for the flow. The explanation to this puzzle is probably that the straight outlet from the turbine prevents mixing of the temperature zone. In a wider sense, this point is where we see remains of the true energy distribution from the turbine. At all other operational points, we have rotation in the draft tube and therefore higher diffusion of specific energy.

At full load we also see a tendency of the same phenomenon; a rise in error level.

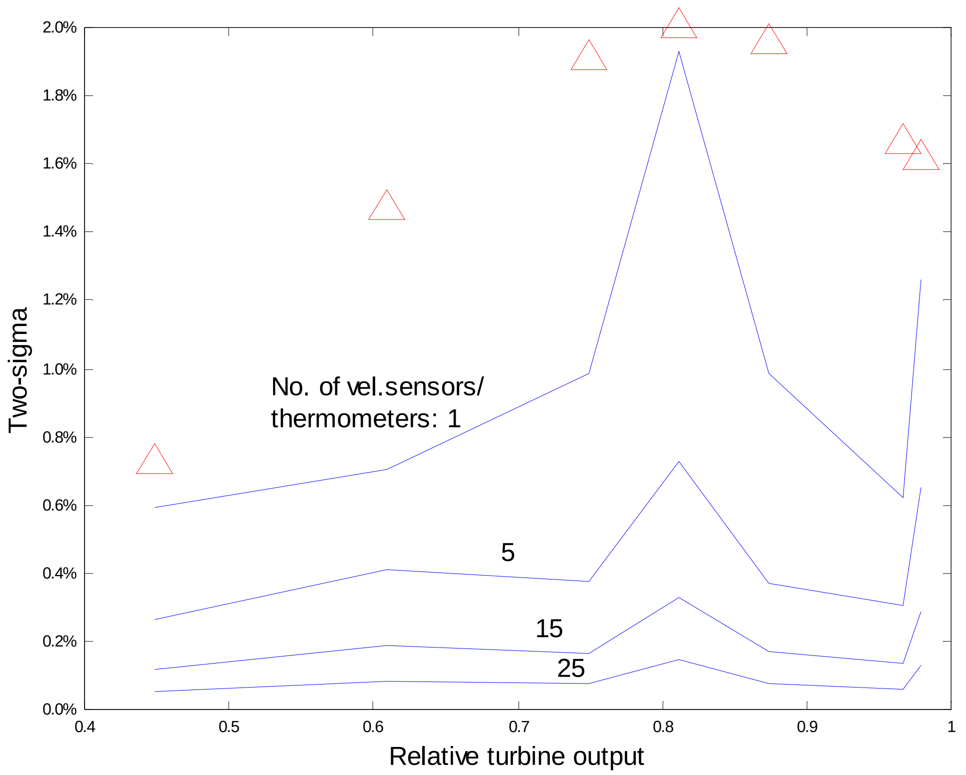

Figure 8. Computed error levels for 1, 5, 15 and 25 probe positions in the 5 by 6 point grid of the cross section, as a function of the turbine output. The highest error level occurs at best-efficiency point of operation. Efficiency curve is out of scale.

The computed error levels is somewhat compatible to the $2\sigma_f$ uncertainty level, but not completely. This statistical approach assumes – quite rightly - that each measured probe location is not independent of the other. Obviously a subarea will have a specific energy that has a physical relation to the neighbouring subareas. Strictly speaking, this conditions excludes calculation of standard deviation and variance from classical statistics. The applied method is chosen for its mutually comparable results.

Figure 6 clearly shows the distinct rise in error levels at best-efficiency points, and also shows how the error level is effectively flattened out due to increasing the number of probe locations. According to the original assumption, at n = 30 locations the curve will be flat at $2\sigma_f = 0.0$ %.

7 CONCLUSIVE REMARKS

The presented data show that, for these measurements:

- The difference between weighted and unweighted efficiency with respect to the water velocity, is not significant
- The difference between measuring with a conventional collector traversed in 5 levels, and six separate thermometers traversed in the same levels, is around 0.2 %, but well within the ± 1 mK uncertainty level.
- The variation of specific energy is by far largest at best-efficiency point, probably due to lack of effective diffusion in the draft tube bend.
- The uncertainty level associated with the energy distribution in the draft tube outlet is effectively reduced by increasing the number of probe locations.

However, it is still a matter of discussion whether a multipoint measurement greatly reduces the overall uncertainties. It is proven that one can reduce the error level associated with variation of the specific energy, but at what cost? One must certainly realise that by applying six thermometers and six velocity probes instead of one thermometer in the outlet, new sources of error are introduced. The chief of test will have to take extreme care to calibrate the thermometer offset in situ, taking into consideration the self-heating of for instance the thermometers in the middle of a bundle being calibrated in a thermos with pouring water from the penstock.

This is a field that will hopefully be more explored in the future.

8 ACKNOWLEDGEMENT

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