A SAMPLE OF UNCERTAINTY ANALYSIS ON FIELD PERFORMANCE TEST

By Albert F. Mikhail, P.Eng., Ontario Power Generation

IGHEM 2002 TORONTO

ABSTRACT

The field performance test results should have an uncertainty analysis to provide a band of acceptable data values. The uncertainty will consist of the systematic and random components. All the test result values will be acceptable within the uncertainty band. Unrealistic presentation of uncertainties will impact on confidence of the test results

There are subjective and objective measures of the uncertainties. The requirement of an objective measures of uncertainties are very important to establish a true value of the test data accuracy.
A subjective reasoning of uncertainties may describe the measured data as “accurate”, “right on”, “astonishing close”, “highly accurate”.
The objective uncertainty evaluation can be described as the basic concept of elemental uncertainty analysis which consist of two parts, the random uncertainties (Precision), and the systematic uncertainties (bias).

The concept of analyzing both uncertainties combined together (random and systematic) to a single number to describe measurement uncertainties which combine the effect of precision and bias.

This is an attempt to apply the IEC test code uncertainties definitions in a practical way to evaluate the field test uncertainties.

This paper will present the measurement elements of a field performance test. The Flow was measured by the Pressure Time Method using the Gibson Instrument. The head, output and wicket gate opening were measured by high accuracy instrumentation.

1. MEASUREMENT OF FLOW (GIBSON PRESSURE-TIME METHOD)

The following uncertainty analysis is prepared according to the test codes IEC - 41, and ASME PTC 18-1992

The uncertainties, both systematic and random, associated to the measurement of flow are dependent on the measurement of many parameters as described below. Table 1 presents a summary of the uncertainty values associated with the measurement flow. While, the detailed analysis of flow uncertainty is presented in Table 2.

In the uncertainty analysis the following parameters are used

	Description	Units
A_{GIBSON}	net area of the Gibson diagram	in^2
e_r	Random error	
e_s	Systematic error	
e	Total Uncertainty = $(e_r^2 + e_s^2)^{1/2}$	
F	pipe factor	ft^{-1}
g	gravitational acceleration	m/sec^2
K	Gibson Instrument Constant	
M	Specific gravity of mercury	lb/ft^3
P_{DIA}	Penstock Diameter	metres
P_{LG}	Distance Between Gibson Piezometer Taps	metres
Q	turbine flow	m^3/s
$Q_{LEAKAGE}$	Leakage from the wicket gates	ft^3/s
R	Ratio of cross sectional area of glass tube to area of the riser of Gibson Instrument	
S	Time constant of Gibson Diagram	inches

2. Calibration of the Gibson Instrument (K)

The Gibson Instrument was calibrated before the tests. The calibrations were carried out at our facilities to determine the Gibson Instrument Constant. The uncertainty of the Gibson instrument consists of the following elements.

3. Determination of the Acceleration due to Gravity (g)

The acceleration of gravity is calculated from the standard acceleration due to gravity and adjusted to the proper latitude and altitude. The systematic uncertainty is based on IEC code recommendations.

4. Determination of the Specific Gravity of Mercury (M)

The mercury specific gravity was measured by an independent laboratory and found to be 13.5757 lb/ft^3 at 25° C. The specific gravity was adjusted to 13.5749 lb/ft^3 at 20°C.

5. Ratio of Areas - Two Legs of the Gibson Apparatus Manometer Tubes (R)

The Gibson Apparatus is fabricated with very high precision. The Gibson instrument was calibrated immediately before the test using a micrometer and a precise pressure source.

6. Pipe Factor (F)

7. Penstock Diameter Measurement ($P_{Dia.}$)

The penstock diameter was measured using a surveying technique with checking the upper and lower piezometer taps using a custom made micrometer. The measurement was carried out by Remote Access Technology. Analysis and computations were undertaken by Capital Projects Group Inc. (CPG Inc.)

8. Measurement of distance - Upper and Lower Gibson Piezometer Taps (P_{LG})

Measurement of distance between the upper and lower piezometer taps uncertainties are from the measuring Tape length uncertainty (100 ft), Thermal Expansion & Contraction, Measuring procedure

9. Leakage Past Turbine after Gate Closure ($Q_{LEAKAGE}$)

The uncertainty arising from determining the leakage through the wicket gates after the closure of the gates. The uncertainty is a function of the gate clearance area calculations and the coefficient of flow.

10. Setting up the Lens of the Gibson Apparatus

The setting up of the lens of the apparatus to produce a 1 to 1 ratio of object and image. The Gibson instrument is built with a very high level of precision. The lens was calibrated, following uncertainties were determined during this calibration process:

11. Time Measurement by the Suspension Rod of the Pendulum (S)

The uncertainty is determined by measuring the time lines on the diagram.

12. Determining the Gibson Diagram Area (A_{GIBSON})

The uncertainty evaluation depends on the digitizer accuracy, print correction factor and integration techniques and the operation of digitizing. The uncertainties associated with each of these factors are described below.

13. Digitizer Accuracy

Accuracy was specified by the digitizer manufacturer.

14. Print Correction

The print correction factor is determined by comparing four corners on the film to the same four points on the print.

15. Integration Techniques

Determining the area under the pressure wave diagram, the starting and diagram ending points, the harmonic effect, and the method of fitting a graph to the digitized data.

16. Digitizing Technique

The uncertainty from the actual digitizing, the number of digitized points, the selection of the location of the points, etc.

17. "Nebenbewegungen" or Conduit Friction Tolerance

Accessory motion of the water column in the penstock as per ASME Paper 58-A-78.

18. Friction in the Mercury Manometer

Mercury column friction in the manometer, as per ASME paper 58-A-78

19. Inertia of the Mercury

Mercury column inertia in the mercury manometer, as per ASME paper 58-A-78.

20. Elasticity, Water Pulsation, and Reflected Waves in the Penstock

As per ASME paper 58-A-78.

21. MEASUREMENT OF HEAD

The uncertainties, both systematic and random, associated to the measurement of head depend on the measurement of elevation and pressure as described in the following sections. The uncertainties associated with head measurements, are summarized in Tables 1, while, Tables 3 and Table 4 present a detailed computation of the uncertainty.

22. MEASUREMENT OF GENERATOR OUTPUT

The uncertainties, both systematic and random, associated to the measurement of generator output depend on the measurement of generator output, generator losses and the phase shift as summarized in Tables 1. Theses uncertainties are included in the analysis of turbine and overall unit results, Tables 5 and 6.

23. UNCERTAINTY OF TURBINE AND OVERALL UNIT RESULTS

The uncertainties associated with the test results are dependent on all parameters measured. For turbine results these include flowrate, net head, turbine power output, and wicket gate opening. Tables 5 provide a detail of the analysis of the uncertainty attributed to the turbine results. The overall unit results are dependent of the measurement of flow, gross head, generator output and wicket gate opening. Table 6 provides a detail of the analysis of the uncertainty attributed to the overall unit test results.

TABLE 1 **SUMMARY OF UNCERTAINTY ANALYSIS VALUES**

Ontario Power Generation Inc.
Sir Adam Beck G.S. II
Unit #26

DATA INPUT

Normalized Gross Head	88.00	metres		
Normalized Net Head	87.80	metres		
Q=Average Flow near Best Efficiency	102.61	m^3/sec		
Full Servometer Stroke	332.02	mm		
Turbine Output at Best Efficiency	84.55	MW		
Unit Output at Best Efficiency	80.54	MW		
Gravity	9.808531354	m/sec^2	32.1804	ft/sec^2
Mercury Specific Gravity	13583		13.5778	
Distance Between Upper and Lower Taps	21.84996595	m	71.6862	ft
Leakage Flow	0.785583007	m^3/sec	27.7467	cfs
Total Leakage Area	0.019045196	m^2	0.205	$feet^2$
Wicket Gate Circumference	15.42066462	m	607.113	in

		Systematic Uncertainty			Random Uncertainty		
FLOWRATE							
1.1.1	Acceleration due to Gravity	± 0.0500	% of flow	e			
1.1.2	Specific Gravity of Mercury	± 0.1000	% of flow	e			
1.1.3	Ratio of Areas - Two Legs of the Gibson Apparatus Manometer Tu	± 0.1000	% of flow	e	± 0.013380	%	e
1.2.1	Penstock Diameter Measurement	± 0.10000	feet	d	± 0.164400	%	e
1.2.2	Length between Upper & Lower Gibson Taps	± 1.4893	feet	d	± 0.081390	%	e
1.3	Leakage Past Turbine after Gate Closure	± 0.2500	inches	e			
1.4	Lens of the Gibson Apparatus	± 0.0500	%	?			
1.5	Time Measurement by the Suspension Rod of the Pendulum	± 0.0500	%	d	± 0.080000	%	d
1.6.1	Digitizer Accuracy	± 0.1000	%	d			
1.6.2	Printing Correction	± 0.0500	%	e			
1.6.3	Integration Techniques	± 0.0800	%	e			
1.6.4	Digitizing	± 0.5000	%	e	± 0.750000	%	e
1.7	Nebenbewegungen" or Conduit Friction Tolerance	± 0.1500	%	e			
1.8	Friction in the Mercury Manometer	± 0.1250	%	e			
1.9	Inertia of the Mercury	± 0.0600	%	e			
1.1	Elasticity, Water Pulsation, & Reflected Waves in the Penstock	± 0.0500	%	e			
HEAD							
2.1	Station Headwater (% of IR psi)	± 0.100	% of 10	d	± 10.0	mm	d
2.2	Station Tailwater (% of IR in psi)	± 0.100	% of 10	d	± 10.0	mm	d
2.3	Vertical Control Survey (mm)	± 1.0	mm	d	± 3.0	mm	d
2.4	Net Head Measurement (% of IR in psi)	± 0.050	% of 100	d	± 25.0	mm	d
2.5	Unit Tailwater (% if IR in psi)	± 0.100	% of 5	d	± 10.0	mm	d
OUTPUT							
3.1	Generator Output	± 0.050	% of measurement	d	± 2.0	kW	d
3.2	Generator Losses Values	± 0.100	% of measurement	d	± 2.0	kW	d
3.3	Phase Shift	± 0.100	% of measurement	d	± 1.0	kW	d
3.4	Transformer Inaccuracies	± 0.400	% of measurement	d	± 2.0	kW	d
WICKET GATE OPENING							
	Wicket Gate Opening (% of IR)	± 0.050	% of 1000	d	± 0.50	mm	d

e - estimated
d - determined

kW - kilowatt
mm - millimeter

IR - instrument range

TABLE 2 **SUMMARY OF UNCERTAINTY ANALYSIS VALUES**

Ontario Power Generation Inc.
Sir Adam Beck G.S. II
Unit #26

Gravity	32.18043	ft/sec2
Mercury Specific Gravity	13.57783	
Average Discharge (Q)	102.6	cfs
Distance Between Upper and Lower Taps	71.68624	ft
Leakage Flow	27.74669	cfs
Total Leakage Area	0.205001	0.21
Wicket Gate Circumference	607.1128	in

FLOW UNCERTAINTY ANALYSIS

References	DESCRIPTION	SYSTEMATIC UNCERTAINTY				RANDOM UNCERTANTY			
		Uncertainty		es % of Q	es^2	Uncertainty		er % of Q	er^2
1.1.1	Acceleration due to Gravity	0.05000 %		0.05000	0.002500				
1.1.2	Specific Gravity of Mercury	0.10000 %	0.10000	0.09745	0.009497				
1.1.3	Ratio of Areas - Two Legs of the Gibson Apparatus Manometer Tubes	0.10000 %	0.10000 m/sec	0.09745	0.009497	0.013380 %	0.01338 %	0.01338	0.000179
1.2.1	Penstock Diameter Measurement	0.10000 ft	m/sec			0.164400 %	0.16440 %	0.16440	0.027027
1.2.2	Length between Upper & Lower Gibson Taps	1.48930 ft	1.84792 m³/sec	1.80084	3.243037	0.081390 %	0.08139 %	0.08139	0.006624
1.3	Leakage Past Turbine after Gate Closure	0.25000 %	0.00494 m/sec	0.00482	0.000023				
1.4	Lens of the Gibson Apparatus	0.05000 %	0.05000 m/sec	0.04873	0.002374				
1.5	Time Measurement by the Suspension Rod of the Pendulum	0.05000 %	0.05000 m/sec	0.04873	0.002374	0.080000 %	0.08000 %	0.08000	0.006400
1.6.1	Digitizer Accuracy	0.10000 %	0.10000 m/sec	0.09745	0.009497				
1.6.2	Printing Correction	0.05000 %	0.05000 m³/sec	0.04873	0.002374				
1.6.3	Integration Techniques	0.08000 %	0.08000 m²	0.07796	0.006078				
1.6.4	Digitizing	0.50000 %	0.50000 m³/sec	0.48726	0.237423	0.750000 %	0.75000 %	0.75000	0.562500
1.7	Nebenbewegungen" or Conduit Friction Tolerance	0.15000 %	0.15000 m³/sec	0.14618	0.021368				
1.8	Friction in the Mercury Manometer	0.12500 %	0.12500 m³/sec	0.12182	0.014839				
1.9	Inertia of the Mercury	0.06000 %	0.06000 m/sec	0.05847	0.003419				
1.1	Elasticity, Water Pulsation, & Reflected Waves in the Penstock	0.05000 %	0.05000						
		Sums of the Squares			3.56430				0.60273
		Square Root of the Sum of the Squares		(a)	1.88794			(b)	0.77636

Random Uncertainty at 95% Confidence = (c) 1.23518

(c) = (Student's t x (b)) /sqrt(n)
Student's t = 3.182
n = average number of replicates runs = 4

sqrt ($(a)^2 + (c)^2$)
Overall Uncertainty at 95 % Confidence 2.25610

95 % CONFIDENCE **2.26**

TABLE 3 **SUMMARY OF UNCERTAINTY ANALYSIS VALUES**

Ontario Power Generation Inc.
Sir Adam Beck G.S. II
Unit #26

Net Head instrument range 150 psi
Unit Tailwater instrument range 5 psi

NET HEAD UNCERTAINTY ANALYSIS

Net Head 88.00 metres

References	DESCRIPTION	SYSTEMATIC UNCERTAINTY				RANDOM UNCERTAINTY		
		Uncertainty		es	es^2	Uncertainty	er	er^2
		Instrument Uncertainty	Actual Uncertainty	% of Net Head			% of Net Head	
2.4	NET HEAD WATER LEVEL MEASUREMENT	0.05 %	0.05277 m	0.05996	0.0036	100 mm	0.11364	0.0129
2.5	UNIT TAILWATER LEVEL MEASUREMENT	0.10 %	0.00352 m	0.00400	0.0000	10 mm	0.01136	0.0001
2.3	VERTICAL CONTROL MEASUREMENT	1 mm	0.00100 m	0.00114	0.0000	3 mm	0.00341	0.0000

Sums of the Squares: 0.00361 | 0.01305

Square Root of the Sum of the Squares: (a) 0.06011 | (b) 0.11425

Random Uncertainty at 95% Confidence = (c) 0.18178

(c) = (Student's t x (b)) /sqrt(n)

Student's t = 3.182

n = average number of replicates runs = 4

$\mathrm{sqrt}((a)^2 + (c)^2)$

Overall Uncertainty at 95 % Confidence 0.19146

95 % CONFIDENCE **0.19**

TABLE 4 **SUMMARY OF UNCERTAINTY ANALYSIS VALUES**

Ontario Power Generation Inc.
Sir Adam Beck G.S. II
Unit #26

Sation Headwater instrument range 10 psi
Station Tailwater instrument range 5 psi

GROSS HEAD UNCERTAINTY ANALYSIS

Gross Head 87.8 metres

References	DESCRIPTION	SYSTEMATIC UNCERTAINTY				RANDOM UNCERTANTY		
		Uncertainty		es	es^2	Uncertainty	er	er^2
		Instrument Uncertainty	Actual Uncertainty	% of Gross Head			% of Gross Head	
II-2.1	STATION HEAD WATER LEVEL MEASUREMENT	0.100 %	0.00704 m	0.00801	0.0001	10 mm	0.01139	0.0001
II-2.2	STATION TAILWATER LEVEL MEASUREMENT	0.100 %	0.00352 m	0.00401	0.0000	10 mm	0.01139	0.0001
II-2.3	VERTICAL CONTROL MEASUREMENT	1 mm	0.00100 m	0.00114	0.0000	3 mm	0.00342	0.0000

Sums of the Squares: 0.00008 | 0.00027

Square Root of the Sum of the Squares: (a) 0.00903 | (b) 0.01647

Random Uncertainty at 95% Confidence = (c) 0.02620

(c) = (Student's t x (b)) /sqrt(n)

Student's t = 3.182

n = average number of replicates runs = 4

sqrt ($(a)^2 + (c)^2$)
Overall Uncertainty at 95 % Confidence 0.02771

95 % CONFIDENCE **0.03**

TABLE 5 SUMMARY OF UNCERTAINTY ANALYSIS VALUES

Ontario Power Generation Inc.
Sir Adam Beck G.S. II
Unit #26

Net Head	88.00	metres
Power Output	84.55	MW
Average Discharge (Q)	102.6	m^3/sec
Full Servomoter Stroke	332.0	mm
Servomotor Stroke instrument range	1000	mm

TURBINE EFFICIENCY UNCERTAINTY ANALYSIS

References	DESCRIPTION	SYSTEMATIC UNCERTAINTY			RANDOM UNCERTAINTY		
		Uncertainty	es (%)	es^2	Uncertainty	er (%)	er^2
TABLE 2	DISCHARGE MEASUREMENTS (GIBSON)		1.88794	3.56430		0.77636	0.60273
TABLE 3	NET HEAD MEASUREMENT		0.06011	0.00361		0.18178	0.03304
II-3.1	GENERATOR OUTPUT MEASUREMENTS	0.05 %	0.05000	0.00250	2.0 kW	0.00237	0.00001
II-3.2	GENERATOR LOSSES VALUES	0.10 %	0.10000	0.01000	2.0 kW	0.00237	0.00001
II-3.3	PHASE SHIFT	0.10 %	0.10000	0.01000	1.0 kW	0.00118	0.00000
II-3.4	TRANSFORMER INACCURACIES	0.40 %	0.40000	0.16000	2.0 kW	0.00237	0.00001
II-4	SERVOMOTOR STROKE	0.05 %	0.15059	0.02268	0.5 mm	0.15059	0.02268

Sums of the Squares: 3.77309 | 0.65847

Square Root of the Sum of the Squares: (a) 1.94245 | (b) 0.81146

Random Uncertainty at 95% Confidence = (c) 1.29104

(c) = (Student's t x (b)) /sqrt(n)
Student's t = 3.182
n = average number of replicates runs = 4

sqrt ($(a)^2 + (c)^2$)
Overall Uncertainty at 95 % Confidence 2.33235

95 % CONFIDENCE **2.33**

TABLE 6 **SUMMARY OF UNCERTAINTY ANALYSIS VALUES**

Ontario Power Generation Inc.
Sir Adam Beck G.S. II
Unit #26

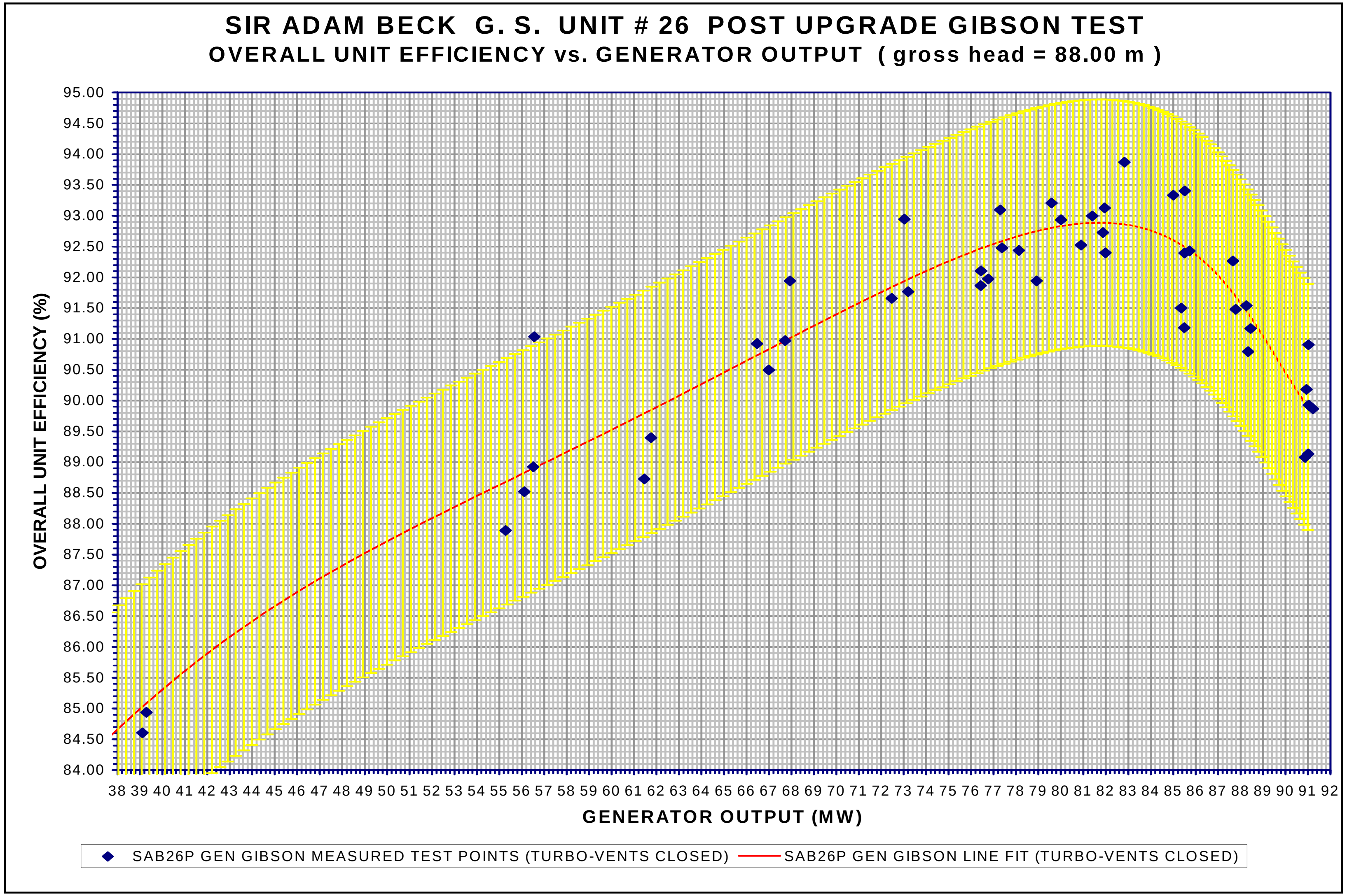
Gross Head	87.80	metres
Unit Output	80.54	MW
Average Discharge (Q)	102.6	m^3/sec
Full Servomoter Stroke	332.0	mm
Servomotor Stroke instrument range	1000	mm

OVERALL UNIT EFFICIENCY UNCERTAINTY ANALYSIS

References	DESCRIPTION	SYSTEMATIC UNCERTAINTY			RANDOM UNCERTAINTY		
		Uncertainty	es (%)	es^2	Uncertainty	er (%)	er^2
TABLE 2	DISCHARGE MEASUREMENT (GIBSON)		1.88794	3.56430		0.77636	0.60273
TABLE 4	GROSS HEAD MEASUREMENT		0.00903	0.00008		0.01647	0.00027
II-3.1	GENERATOR OUTPUT MEASUREMENTS	0.05 %	0.05000	0.00250	2.0 kW	0.00248	0.00001
II-3.3	PHASE SHIFT	0.10 %	0.10000	0.01000	1.0 kW	0.00124	0.00000
II-3.4	TRANSFORMER INACCURACIES	0.40 %	0.40000	0.16000	2.0 kW	0.00248	0.00001
II-4	SERVOMOTOR STROKE	0.05 %	0.15059	0.02268	0.5 mm	0.15059	0.02268

Sums of the Squares	3.75956	0.62569
Square Root of the Sum of the Squares	(a) 1.93896	(b) 0.79101
Random Uncertainty at 95% Confidence = (c)		1.25849
Overall Uncertainty at 95 % Confidence, sqrt ($(a)^2 + (c)^2$)		2.31157
95 % CONFIDENCE		**2.31**

(c) = (Student's t x (b)) /sqrt(n)
Student's t = 3.182
n = average number of replicates runs = 4


SIR ADAM BECK G. S. UNIT # 26 POST UPGRADE GIBSON TEST
OVERALL UNIT EFFICIENCY vs. GENERATOR OUTPUT (gross head = 88.00 m)
OVERALL UNIT EFFICIENCY (%)
95.00
94.50
94.00
93.50
93.00
92.50
92.00
91.50
91.00
90.50
90.00
89.50
89.00
88.50
88.00
87.50
87.00
86.50
86.00
85.50
85.00
84.50
84.00
38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92
GENERATOR OUTPUT (MW)
SAB26P GEN GIBSON MEASURED TEST POINTS (TURBO-VENTS CLOSED)
SAB26P GEN GIBSON LINE FIT (TURBO-VENTS CLOSED)