Theoretical considerations of tracer dilution methods for flow evaluation (IGHEM 2012)

J. Lanzersdorfer[1]
ANDRITZ Hydro GmbH, Ostring 17, 4031 Linz, Austria

Abstract

Tracer methods are chemical techniques that exploit the dilution ratio of injected tracers in the water to determine the stationary flow of a water-bearing system. The scope of this paper is to represent theoretical considerations on the evaluation of volume flow and mass flow. It does not matter which kind of tracer (salt, dye, radioactive tracers) is used which only affects the practical determination of the dilution factor. The calculation scheme of the flow remains the same for all types of tracers. In accordance with the fundamental law of mass conservation within the Newtonian physics the mass of tracer m_T injected at any upstream measuring section (1) into the system equals the mass of tracer which is measured anywhere downstream (2):

$$m_{\mathrm{T},1} = \int_{\mathcal{I}} \dot{m}_\mathrm{T}(t)\, dt = \int_{\mathcal{I}} \dot{m}_\mathrm{T}(t+\tau)\, dt = m_{\mathrm{T},2} \tag{1}$$

This equation assumes no leakage or drainage of water–tracer mixture between those two sections. The following equation can be extracted showing the determination of mass flow, and discharge respectively, in its generalized form

$$\dot{M} = \frac{\int_{\mathcal{I}} \dot{m}(t)\, dt}{\int_{\mathcal{I}} D(t+\tau)\, dt} \qquad Q = \frac{\int_{\mathcal{I}} q(t)\, dt}{\int_{\mathcal{I}} D^\star(t+\tau)\, dt} \tag{2}$$

where $\dot{m}(t)$ and $q(t)$ denote the injection rates with respect to mass and volume of the initial solution with constant tracer concentration C_1. $D(t)$ ($D^\star(t)$) is called the dilution factor, representing the ratio of the final tracer concentration measured at time t and the initially injected tracer concentration. With equation (2), the special evaluation technique using constant rate injection may be derived from. The parameter τ represents the mean-time difference or delay which takes for an infinitesimal small tracer particle to get detected at the downstream measuring section after injection into the system to be tested. The delay time may be represented by summing up three terms

$$\tau = \tau_1 + \tau_2 + \tau_3 \tag{3}$$

which are related to the time between pre-dilution and injection, the average transit time between both sections, and the time period between extraction and measurement of the dilution factor. It can be shown that equation (2) may only lead to correct results if each time delay term can be kept constantly during the measuring time. The impact onto the measured temporal behavior of the dilution factor is presented by means of instationary sample extraction using some simplifications. The author introduces a transformation scheme which reveals the necessity of monitoring such auxiliary flows like pre-dilution flow and extraction flow.

[1]Contact: johannes.lanzersdorfer@andritz.com

Introduction

Absolute efficiency testing on hydro plants requires the determination of several parameters. The most critical one represents the volume flow Q, respectively the mass flow $\dot{M} = \rho \cdot Q$. Since its absolute value can only be measured with higher uncertainty compared to other parameters such as static head, density or acceleration due to gravitation it directly affects the uncertainty of the efficiency value.
The tracer dilution method represents a high-quality technique to evaluate the volume flow. Although this method provides low uncertainty ($|f(Q)| < 1.0\%$) and costs are low, it is only rarely applied on hydraulic plants. Tracer dilution techniques are applicable in open channels [1] and closed conduits [2]. The basic principle is simple. The degree of dilution of a previously injected aqueous solution into the water-bearing system gives information about the discharge in this conduit. A sufficient degree of intermixture of tracer and water is a prerequisite for successful flow measurements. The investigation in the last years mainly focused on this issue resulting in a few rules of thumb to estimate the required hydraulic path length of intermixture [3]. However, prognoses of the intermixture process are not satisfying so far. Detailed descriptions on the practical application of those techniques can be found elsewhere, e.g. in the standards ISO 9555 and ISO 2975.

Three procedures are currently in use. The first one is based on the mean transit time between two measuring sections. It is cost-intensive since it requires the twofold instrumentation for the dilution measurements. And the knowledge of the geometric dimensions between both measuring sections is necessary. Nowadays, this variant is of minor interest and it is hardly applied. The other ones (constant rate injection, sudden injection) obey to the dilution principle which implies that there is no need to know any geometric dimension of the conduit. Both important standard test codes for efficiency measurements on hydraulic machines, IEC 60041–1991 [4] and ASME PTC-18–2011[5], prefer the application of the constant-rate injection technique notwithstanding it represents a special case of the general technique of sudden injection. In cases where no distinct *plateau* of the dilution factor can be determined only the integration method may give plausible values.

The subsequent pages deal with the theoretical background of flow evaluation using tracer dilution methods. In addition to this, a time transformation scheme is introduced to correct any impact of instationary extraction flow on the measured transient behavior of the dilution factor when using measuring devices outside the water-system. This procedure may be applied on instationary pre-diluting flows too.

Theory

Principle of the method

Consider a non-circulating hydraulic system without any deadwater regions having the mass flow $\dot{M} = \rho Q$. The law of mass conservation implies that an injected mass of pure tracer m_T equals the mass of pure tracer anywhere downstream the water-bearing system after a certain period of time. Assuming

total intermixture of the injected tracer with the water leads to the subsequent derivation

$$\int \dot{m_{\mathrm{T}}}(t)\, dt \quad = \quad \int \dot{m_{\mathrm{T}}}(t+\tau)\, dt \tag{4}$$

τ denotes the average delay time between tracer injection and mass detection downstream. $\dot{m_{\mathrm{T}}}(t)$ may be expanded by

$$\begin{aligned} \dot{m_{\mathrm{T}}}(t) &= \lim_{\Delta t \to 0} \frac{m_{\mathrm{T}}(t+\Delta t) - m_{\mathrm{T}}(t)}{\Delta t} \\ &= \lim_{\Delta t \to 0} \frac{m_{\mathrm{T}}(t+\Delta t) - m_{\mathrm{T}}(t)}{m(t+\Delta t) - m(t)} \cdot \frac{m(t+\Delta t) - m(t)}{\Delta t} \\ &= C_1(t) \cdot \dot{m}(t) \end{aligned} \tag{5}$$

where $C_1(t)$ denotes the concentration of the injected tracer solution (mass of tracer per total mass). $\dot{m}(t)$ represents the mass flow of injection of that solution. The expansion of $\dot{m_{\mathrm{T}}}(t+\tau)$ provides

$$\begin{aligned} \dot{m_{\mathrm{T}}}(t+\tau) &= \lim_{\Delta t \to 0} \frac{m_{\mathrm{T}}(t+\tau+\Delta t) - m_{\mathrm{T}}(t+\tau)}{\Delta t} \\ &= \lim_{\Delta t \to 0} \frac{m_{\mathrm{T}}(t+\tau+\Delta t) - m_{\mathrm{T}}(t+\tau)}{(M(t+\tau+\Delta t) - M(t+\tau)) + (m(t+\tau+\Delta t) - m(t+\tau))} \\ &\quad \cdot \frac{(M(t+\tau+\Delta t) - M(t+\tau)) + (m(t+\tau+\Delta t) - m(t+\tau))}{\Delta t} \\ &= C_2(t+\tau) \cdot \left(\dot{M}(t+\tau) + \dot{m}(t+\tau) \right) \\ &\cong C_2(t+\tau) \cdot \dot{M}(t+\tau) \\ &\cong C_2(t+\tau) \cdot \dot{M} \end{aligned} \tag{6}$$

The simplification done in the penultimate line of the previous formula is generally allowed, since the ratio $\dot{M}(t)/\dot{m}(t) > 10^5$. Together with the constancy of the total flow $\dot{M}(t+\tau) = \dot{M}(t) = \dot{M}$ (last line of equation 6) these equations lead to the final representation

$$\int C_1(t) \cdot \dot{m}(t)\, dt \quad = \quad \int C_2(t+\tau) \cdot \dot{M}\, dt \tag{7}$$

Possessing the knowledge of the initial concentration $C_1(t) = C_1$ yields when integration within the time interval of interest $\mathcal{I}$ is applied

$$\dot{M} = \frac{\int\limits_{\mathcal{I}} \dot{m}(t)\, dt}{\int\limits_{\mathcal{I}} \frac{C_2(t+\tau)}{C_1}\, dt} = \frac{\int\limits_{\mathcal{I}} \dot{m}(t)\, dt}{\int\limits_{\mathcal{I}} D(t+\tau)\, dt} \tag{8}$$

$D(t+\tau)$ is called the dilution factor and, here, it is defined as the ratio of the measured final concentration at time $t+\tau$ and the initial tracer concentration. Using another definition for the concentration parameter (mass of tracer per total volume) leads to a similar derivation. It finally reveals an expression for

the discharge

$$Q = \frac{\int\limits_{\mathcal{I}} q(t)\,dt}{\int\limits_{\mathcal{I}} \frac{C_2^\star(t+\tau)}{C_1^\star}\,dt} = \frac{\int\limits_{\mathcal{I}} q(t)\,dt}{\int\limits_{\mathcal{I}} D^\star(t+\tau)\,dt} \tag{9}$$

$q(t)$ is the injection discharge of the initial solution. The average time delay τ may be repesented as a summation of three terms

$$\tau = \tau_1 + \tau_2 + \tau_3 \tag{10}$$

where the first one takes into account the time consumption of the injection of the initial tracer solution into the water bearing system by means of any pre-dilution process. τ_2 is related to the averaged temporal consumption of an infinitesimal drop of water travelling from the point of injection to the downstream measuring section. The last one takes into account when doing dilution measurements outside the water-bearing system using continuous sample extraction.

Variants of the method

Sudden injection method (integration method) This case is the general one using the generalized formulae (8) and (9). A certain amount of initial solution is injected upstream. After complete intermixture the integration of the measured dilution factor within the time interval $\mathcal{I}$ yields to the mass flow (discharge). This method requires a smaller mass of tracer and less time than the constant rate injection method.

Constant-rate injection method This method represents a special case of the general method of sudden injection. Establishing the injection of initial solution with constant rate over a longer time period (several minutes) causes more or less a stabilisation of the final concentration around a certain value at the downstream measuring section. The dilution factor forms a plateau within the time interval $[t_1, t_2]$ which can be used to calculate the mass flow (discharge)

$$\dot{M} = \frac{\overline{\dot{m}}\big|_{t_1-\tau}^{t_2-\tau}}{\overline{D}\big|_{t_1}^{t_2}} \qquad Q = \frac{\overline{q}\big|_{t_1-\tau}^{t_2-\tau}}{\overline{D^\star}\big|_{t_1}^{t_2}} \tag{11}$$

The averaging of the required parameter values should be done for the same time length taking into account the delay time τ. In principle, the consumption of initial solution is higher than with sudden injection due to longer injection periods.

Transit time method This variant is based on the determination of the average transit time between two measuring sections. Therefore it requires the geometric dimensions of the conduit/channel between those sections to calculate the flow. It does not exploit the advantages of the dilution principle described above and, hence, it is not treated in this publication.

Time transformation when using dilution measurements outsite the water-system

Equations (8) and (9) may only be used in a correct sense if each of the terms of delay time in (10) can be kept constantly during the recordings of any operating point. Because any instationarity of the contributing flows (pre-dilution flow, main flow, sample extraction flow) impacts negatively on the measurements of the temporal behavior of the dilution factor. The main flow may be considered to be constant at any distinct operating point after pressure stabilization, i.e. τ_2 is constant and there is no negative influence expected with respect to the discharge calculations. The constancy of τ_1 and τ_3 may be fulfilled in most cases too. But each flow, pre-dilution flow and extraction flow, should be monitored. The following derivation of a simplified model may reveal the impact of an instationary extraction flow onto the measurements of the final concentration[2].

Considering an infinitesimally small water sample[3] extracted at time t with flow rate $q(t)$. It is assumed that it does not mix with any other neighboring infinitesimal water sample and there is no change in the temporal order of extracted samples. Furthermore, compressibility effects are neglected. It takes the time $\tau_3(t)$ that the concentration $C_{\text{in}}(t)$ of this *drop* is getting measured by the relevant device after extraction. That is, the measured concentration outsite the system $C_{\text{out}}(t+\tau_3(t))$ corresponds to the actual value $D_{\text{in}}(t)$ inside.

As a consequence of this, the temporal transformation $\mathcal{T}$ has to be found yielding

$$\mathcal{T}\{C_{\text{out}}(t+\tau_3(t))\} = C_{\text{in}}(t) \tag{12}$$

The time shift $\tau_3(t)$ depends on the average discharge $\overline{q(\tau_3(t))}$ and the hydraulic volume V between extraction point and device entrance, i.e. the inner volume of the connecting pipes.

$$\tau_3(t) = \frac{V}{\overline{q(\tau_3(t))}} \tag{13}$$

The average discharge may be calculated by

$$\overline{q(\tau_3(t))} = \frac{1}{\tau_3(t)} \cdot \int_t^{t+\tau_3(t)} q(t)\, dt \tag{14}$$

Combining (13) and (14) leads to an implicit equation to determine the delay time $\tau_3(t)$

$$V = \int_t^{t+\tau_3(t)} q(t)\, dt \tag{15}$$

and, respectively,

[2]The dilution factor is proportional to the final concentration of a water–tracer intermixture. Hence, the presented scheme remains the same when using terms of dilution factor.

[3]volume dV, density $\rho(t)$ and concentration $C_{\text{in}}(t)$ of the intermixture tracer–water

$$\bar{\rho}V = \int_{t}^{t+\tau_3(t)} \dot{m}(t)\, dt \tag{16}$$

taking into account the mass flow $\dot{m}(t)$ and the average density $\bar{\rho}$ inside the volume.

The necessity of the transformation (12) is subsequently be presented in a few cases which may occur at typical measurements. A Gaussian shaped curve is used as a reference curve for $C_{\mathrm{in}}(t)$. The area under this graph is normalized to the value 1. This time signal is then retransformed by means of (12) to those measurement conditions outside the conduit ($C_{\mathrm{out}}(t + \tau_3(t))$). It takes into account the time behavior of the discharge of extraction $q(t)$. The area under the graph of $C_{\mathrm{out}}(t)$ is determined successively and compared with the reference value.

Constant extraction The condition present for a constant value of $q(t) = a$ is depicted in figure 1. The time delay is independent of time and may derived by (15). It yields

$$\tau_3(t) = \tau_3 = \frac{V}{a} \tag{17}$$

The shape of $C_{\mathrm{out}}(t)$ is identical with the measurements inside shifted by τ_3. Integrations of both graphs coincide perfectly. Hence, a transformation is not required to apply the constant-rate injection technique and the sudden injection technique according to the relevant standard test codes.

Linear increasing/decreasing extraction The transient discharge is obeying here to a linear function of type $q(t) = a + b \cdot t$. The time delay is no longer independent of time and yields therefore

$$\tau_3(t) = -\left(t + \frac{a}{b}\right) + \sqrt{\left(t + \frac{a}{b}\right)^2 + \frac{2V}{b}} \tag{18}$$

The signal $C_{\mathrm{out}}(t)$ is broadened compared with $C_{\mathrm{in}}(t)$ in case of a negative slope ($b < 0$), and it is narrowed when the slope is positive ($b > 0$). Hence, the integrals do not match.

Oscillating Extraction It is assumed that the discharge follows an oscillating function of type

$$q(t) = a + b \cdot \sin(c \cdot t)$$

The time delay can be determined by solving the implicit equation

$$V = a\tau_3(t) + \frac{b}{c} \cdot [\cos(c \cdot t) - \cos\left(c \cdot (t + \tau_3(t))\right)] \tag{19}$$

The signal $C_{\mathrm{out}}(t)$ is squeezed and stretched locally due to the periodicity of the oscillations. The negative impact on the measurements increases with higher amplitude and lower frequency. Of course, the integrals do not coincide.

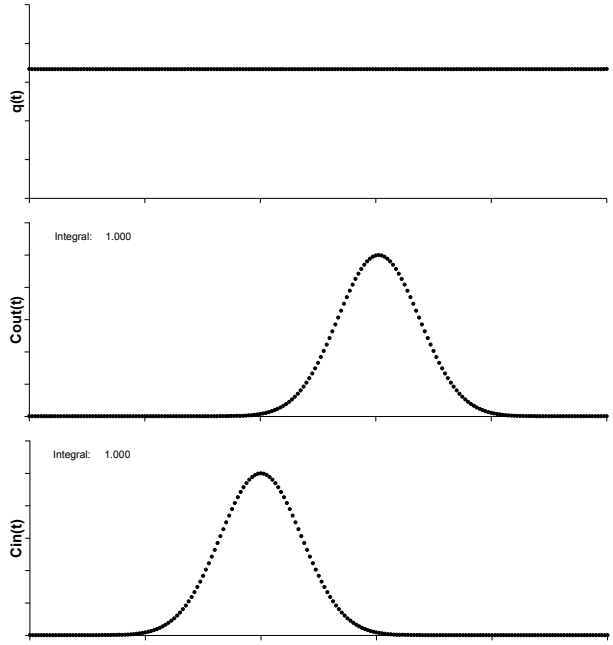

Figure 1: Constant extraction discharge $q(t) = a$

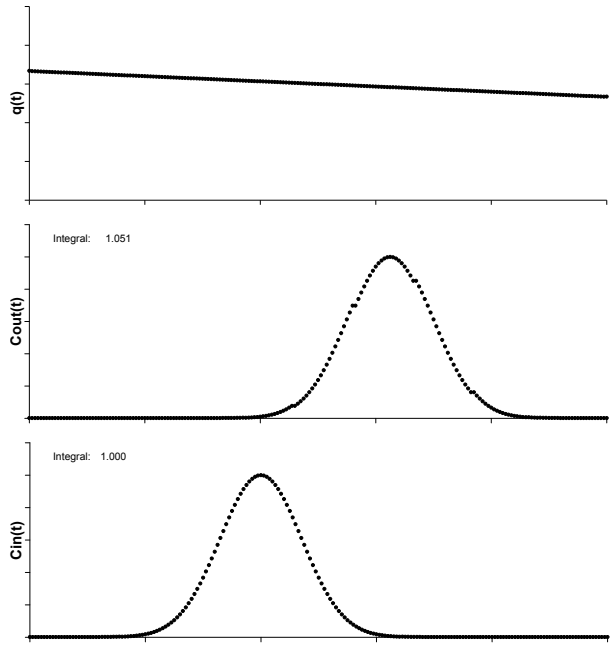

Figure 2: Extraction discharge $q(t) = a + b \cdot t$

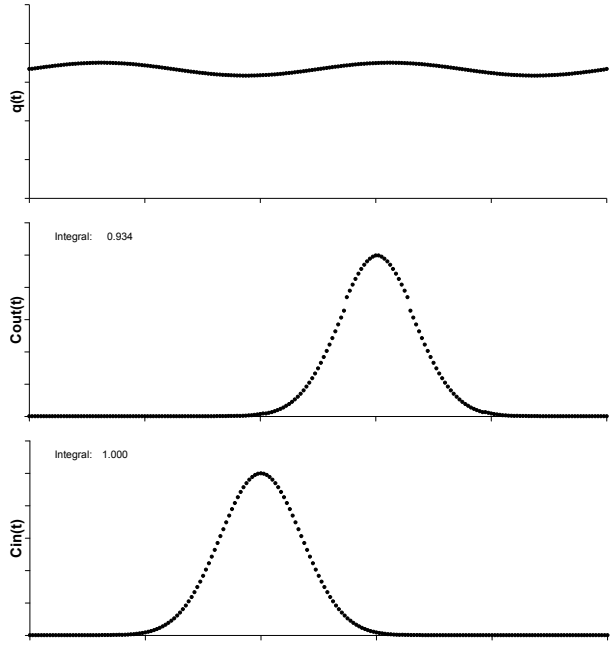

Figure 3: Extraction discharge $q(t) = a + b \cdot \sin(c \cdot t)$

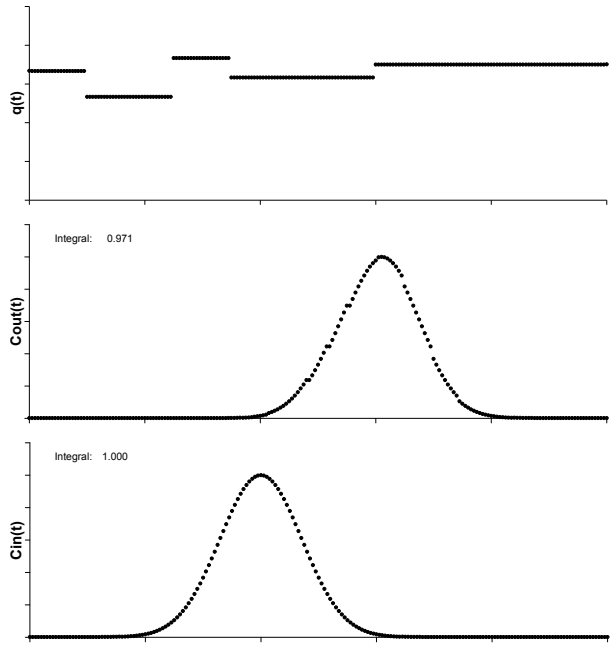

Figure 4: Discrete extraction discharge $q(t) = a_i$ for $t \in [t_i, t_{i+1}]$

Discrete Extraction Finally consider the following time behavior of the discharge of extraction

$$q(t) = \sum_{i=1}^{n} a_i \cdot [\Theta(t - t_i) - \Theta(t - t_{i+1})]$$

whereas $\Theta(t-t_0)$ denotes the Heaviside function. The time delay is only constant over a certain period of time. The evaluation of the time delay has to be done by the implicit formula

$$V = a_i \cdot (t_i - t) + \sum_{j=i+1}^{n-1} a_j \cdot (t_{j+1} - t_j) + a_n \cdot (\tau_3(t) - t_n) \tag{20}$$

The shape of the signal $C_{\text{out}}(t)$ is reproduced only correctly within fragments. This affects the shape of $C_{\text{out}}(t)$ and therefore also the integration value.

Conclusions

The definition of the concentration parameter (mass of tracer per total mass, mass of tracer per total volume) impacts directly onto the calculation of the corresponding flow parameter ($\dot{M}$, Q), and it may affect successively the measurement uncertainty. That is, searching for the mass flow $\dot{M}$ with low uncertainty prefers the definition of tracer mass per total mass. Furthermore, it has an influence on the choice of flowmeter for monitoring the injection rate of initial solution. Using the mass per total mass definition prefers such high accuracy devices like Coriolis mass flowmeters ($|f(\dot{m})| \cong 0.25\%$) whereas special manufactured, piston-driven pumps may lead to lower measurement uncertainties using the definition of tracer mass per total volume.
A general formula for determining the discharge Q or the mass flow $\dot{M}$ could be deduced what represents the general version of the sudden injection procedure. The author prefers – in contrast to the relevant standard test codes – this evaluation technique, since the flow calculation is considered to be more trustworthy. The special case of the constant rate injection procedure could be deduced from the general formulation. Unfortunately, there exists no clear instructions in the test codes in choosing the time window at the plateau to obtain an average value of the dilution factor. Hence, the author recommends to use similar lengths of time when averaging the injection rate and the diluton factor. Everything else gives rise to arbitrariness.
A simplified time transformation scheme is introduced revealing the possibility to correct transient measurements of the dilution factor in presence of flow instabilities of the extraction flow and/or pre-dilution flow.

Vitae

Johannes Lanzersdorfer graduated in Technical Physics from the Technical University of Graz in 2008. He is working at ANDRITZ Hydro in Linz as a measurement engineer since 2009. His field of activity includes field measurements and the development of measuring techniques for the hydraulic laboratory and for field testing.

References

[1] ISO 9555, editor. *Measurement of liquid flow in open channels – Tracer dilution methods for the measurement of steady flow.* ISO, 1 edition, 1994.

[2] ISO 2975, editor. *Measurement of water flow in closed conduits – Tracer methods.* ISO, 1974.

[3] Sung-keun Park Sun-ki Lee, Sun-chul Chung. Chemical Tracer Method for Measurement of Flow Rates in Closed Conduits. *Journal of Nuclear Science and Technology*, 44(11):1467–1473, 2007.

[4] IEC 60041, editor. *Field acceptance tests to determine the hydraulic performance of hydraulic turbines, storage pumps and pump-turbines.* IEC, 3 edition, 1991.

[5] ASME PTC-18, editor. *Hydraulic Turbines and Pump-Turbines - Performance Test Codes.* ASME, 2011.