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A Kalman filter based approach for measuring sound speed, axial and transversal flow component in ATT measurements

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Abstract

The ATT method for measuring the flow in hydropower applications is an accurate and well established method. The method uses normally a high number of transit time measurements in up- and downstream direction, from which the axial and transversal flow components are determined. In order to obtain a sufficient accuracy, the obtained transit times and transit time differences have to be filtered and averaged. Typical values of a pair of up- and downstream measurement for one path is 50 to 100 pairs per second. In the case that a transverse component of the flow is present, crossed paths are used in one layer, reducing therefore the rate of determining the axial and transverse flow by a factor of two. The conventional determination of the axial velocity of one path relies on the assumption that the speed of sound does not change during the measurement of the up- and downstream measurement process. If this assumption is violated, the conventional method treats this change as a measurement error for the transit times. In the following a Kalman filter based approach is chosen, which allows to distinguish between system noise and measurement noise. The system noise corresponds to the variations of the speed of sound, the axial and the transverse flow component of the flow, while the measurement noise reflects the inaccuracies introduced by determining the transit and transit time differences. The chosen system model is the so called random walk model for the three states speed of sound, the axial and the transverse flow component of the flow and four output quantities for the up- and downstream transit times for two crossed paths. The Kalman filter approach allows to weigh the variation of each state separately and to relate the magnitude of each of the noise sources to the measurement noise. Different situations of estimating the states in a noisy environment are examined with the help of simulation: stepwise change in speed of sound, axial and transverse flow. The influence of the choice of the weighting of the noise sources are investigated, showing the trade-off of noise rejection and signal tracking capabilities of the different filter parameterizations.

INTRODUCTION

The widely used ATT method for discharge measurement determines the flow Q by a weighted sum over a number N of averaged axial path velocities $\bar{v}_{ax}$, which can be obtained from the measured transit times of the acoustic pulses along the paths.

$$Q = k\frac{D}{2}\sum_{i=1}^{N} W_i\, b(z_i)\, \bar{v}_{ax}(z_i) \qquad (1)$$

The positions (heights) z_i (or d_i) and weights W_i of the paths are determined by the integration method used, $b(z_i)$ is the width of the conduit at position z_i, D the diameter of the conduit and k a possible geometrical correction factor. Fig. 1 illustrates the velocity components which play a role. The three dimensional velocity $v(s)$ at a position s along the path A can be split in a vertical component which does not contribute to the path velocity and in two horizontal velocities $v_{ax}(s)$ and $v_{tr}(s)$ which build the layer velocity $v_{layer}(s)$. If the layer velocity is projected on the path, the transverse component leads to an erroneous velocity contribution $v_c(s)$. In order to eliminate this contribution a second crossed path B is added at the same height. Assuming the same averaged transversal component on each of the two crossed paths, the influence of the transversal component to the averaged path velocities can be eliminated.

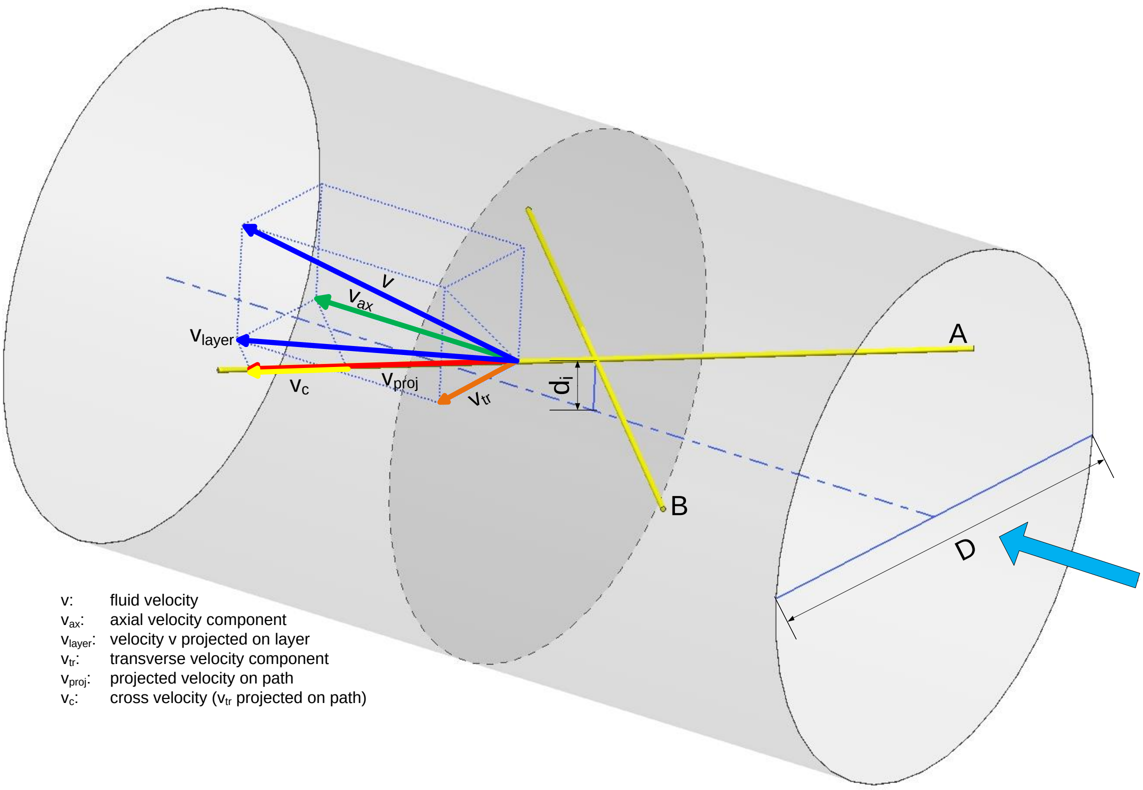

Fig. 1: velocity components and acoustic paths

Equation (2) and (3) give the relation between projected path velocity and transit time t :

$$t = \frac{L}{c \pm \bar{v}_{proj}} \qquad (2)$$

$$\bar{v}_{proj} = \frac{1}{L}\int_0^L v_{proj}(s)\,ds \qquad (3)$$

The speed of sound c is actually also an average speed along a path. If a crossed two path arrangement is chosen, four different transit times (two in up- and downstream direction) can be determined:

$$t_{d1} = \frac{L}{c + \bar{v}_{\mathrm{ax}} \cos\varphi + \bar{v}_{\mathrm{tr}} \sin\varphi}$$

$$t_{u1} = \frac{L}{c - \bar{v}_{\mathrm{ax}} \cos\varphi - \bar{v}_{\mathrm{tr}} \sin\varphi}$$

$$t_{d2} = \frac{L}{c + \bar{v}_{\mathrm{ax}} \cos\varphi - \bar{v}_{\mathrm{tr}} \sin\varphi}$$

$$t_{u2} = \frac{L}{c - \bar{v}_{\mathrm{ax}} \cos\varphi + \bar{v}_{\mathrm{tr}} \sin\varphi} \qquad \text{(4a-4d)}$$

This quadruple of transit times is repeated up to 50 to 100 times per second, depending on the ping rate of the system. Assuming constant speed of sound and averaged velocities during the measuring process of the four transit times, the well-known formulae for the averaged axial velocity and the speed of sound are found:

$$\bar{v}_{ax,1} = \frac{L}{2\cos\varphi}\left(\frac{1}{t_{d1}} - \frac{1}{t_{u1}}\right) - \bar{v}_{tr} \tan\varphi$$

$$\bar{v}_{ax,2} = \frac{L}{2\cos\varphi}\left(\frac{1}{t_{d2}} - \frac{1}{t_{u2}}\right) + \bar{v}_{tr} \tan\varphi \qquad \text{(5a, b)}$$

$$\bar{v}_{ax} = \frac{\bar{v}_{ax,1} + \bar{v}_{ax,2}}{2}$$

$$c = \frac{L}{4}\left[\left(\frac{1}{t_{d1}} + \frac{1}{t_{u1}}\right) + \left(\frac{1}{t_{d2}} + \frac{1}{t_{u2}}\right)\right] \qquad \text{(6a, b)}$$

Commercial measurement techniques apply the above formulae for the evaluation of the flow and the speed of sound. If consecutive measurements are logged without being filtered, a noisy behaviour of the instantaneous velocities can be observed, which can amount to a magnitude of up to +/- 20% of the long term average. Therefore averaging of the transit time and differences of the transit time must be applied in order to stabilize the flow measurement. Typically time constants of 1 sec or more are used. It is an open question from where these variations in time measurement do occur. Is it due to underlying physical processes of turbulence or is it due to the acquisition of the signal and the determination of the times from the recorded signals?

1. TIME-VARYING MODEL FOR VELOCITIES

The determination of the velocities as given by equations (4) assumes a noise free environment and constant velocity components during the measurement process. In many ultrasonic devices the measurement process is done sequentially and not at the same time instant k. A possible time varying measurement process could be formulated as follows:

$$t_{d1}(k) = \frac{L}{c_1(k) + \bar{v}_{\mathrm{ax},1}(\mathrm{k}) \cos\varphi + \bar{v}_{\mathrm{tr},1}(\mathrm{k}) \sin\varphi} + n_1(k)$$

$$t_{u1}(k)=\frac{L}{c_1(k+1)-\bar{v}_{\mathrm{ax},1}(\mathrm{k}+1)\cos\varphi-\bar{v}_{\mathrm{tr},1}(\mathrm{k}+1)\sin\varphi}+n_2(k)$$

$$t_{d2}(k)=\frac{L}{c_2(k)+\bar{v}_{\mathrm{ax},2}(\mathrm{k})\cos\varphi-\bar{v}_{\mathrm{tr},2}(\mathrm{k})\sin\varphi}+n_3(k)$$

$$t_{u2}(k)=\frac{L}{c_2(k+1)-\bar{v}_{\mathrm{ax},2}(\mathrm{k}+1)\cos\varphi+\bar{v}_{\mathrm{tr},2}(\mathrm{k}+1)\sin\varphi}+n_4(k) \qquad \text{(7a-7d)}$$

This model takes into account that the two acoustic paths are averaging different local path velocities. There are six time dependent averaged velocities involved:

$$c_1(k),\bar{v}_{\mathrm{ax},1}(\mathrm{k}),\bar{v}_{\mathrm{tr},1}(\mathrm{k}),c_2(k),\bar{v}_{\mathrm{ax},2}(\mathrm{k}),\bar{v}_{\mathrm{tr},2}(\mathrm{k}) \qquad (8)$$

and for independent noise sources $n_1(k),n_2(\mathrm{k}),n_3(\mathrm{k}),n_4(k)$. With the abbreviations

$$\underline{x}(k)=\left[c_1(k),\bar{v}_{\mathrm{ax},1}(\mathrm{k}),\bar{v}_{\mathrm{tr},1}(\mathrm{k}),c_2(k),\bar{v}_{\mathrm{ax},2}(\mathrm{k}),\bar{v}_{\mathrm{tr},2}(\mathrm{k})\right]^T \text{ =system state}$$

$$\underline{y}(k)=\left[t_{d1}(k),t_{\mathrm{u}1}(\mathrm{k}),t_{\mathrm{d},2}(\mathrm{k}),t_{u2}(k)\right]^T \text{ =system output}$$

$$\underline{n}(k)=\left[n_1(k),n_2(\mathrm{k}),n_3(\mathrm{k}),n_4(k)\right]^T \text{ =measurement noise} \qquad \text{(9a-9c)}$$

The measurement process can be put in a compact form:

$$\begin{aligned}\underline{x}(k+1)&=\underline{f}(\underline{x}(k))\\ \underline{y}(k)&=\underline{g}(\underline{x}(k+1),\underline{x}(k))+\underline{n}(k)=\underline{g}(\underline{f}(\underline{x}(k)),\underline{x}(k))+\underline{n}(k)\end{aligned} \qquad \text{(10a,b)}$$

The nonlinear vector function $\underline{g}$ is given by the expressions (7a-7d), while the unknown vector function $\underline{f}$ describes the time evolvement of the averaged velocity components.

The problem with this model is the definition of $\underline{f}$. This definition must contain the effect of local turbulences on the vector components and is of a random nature. At the moment no such model was yet applied but if suitable random models are available, it certainly would be interesting to use these in this context.

2. RECURSIVE ESTMATION OF VELOCITIES VIA KALMAN FILTER

Lanzensdörfer [1] proposed a simplified model for which a least square solution can be found easily. It is assumed that

$$\underline{x}=\begin{bmatrix} c_1(k)=c_2(k)=c=const\\ \bar{v}_{\mathrm{ax},1}(\mathrm{k})=\bar{v}_{\mathrm{ax},2}(\mathrm{k})=\bar{v}_{\mathrm{ax}}=const\\ \bar{v}_{\mathrm{tr},1}(\mathrm{k})=\bar{v}_{\mathrm{tr},2}(\mathrm{k})=\bar{v}_{\mathrm{tr}}=\mathrm{const}\end{bmatrix} \qquad (11)$$

and additionally that the inverse of the transit times are defined as outputs.

$$
\begin{aligned}
y_1(k) &= \frac{1}{t_{d1}(k)} = \frac{1}{L}\left[c + \bar{v}_{\text{ax}}\cos\varphi + \bar{v}_{\text{tr}}\sin\varphi\right] + \varepsilon_1(k) \\
y_2(k) &= \frac{1}{t_{u1}(k)} = \frac{1}{L}\left[c - \bar{v}_{\text{ax}}\cos\varphi - \bar{v}_{\text{tr}}\sin\varphi\right] + \varepsilon_2(k) \\
y_3(k) &= \frac{1}{t_{d2}(k)} = \frac{1}{L}\left[c + \bar{v}_{\text{ax}}\cos\varphi - \bar{v}_{\text{tr}}\sin\varphi\right] + \varepsilon_3(k) \\
y_4(k) &= \frac{1}{t_{u2}(k)} = \frac{1}{L}\left[c - \bar{v}_{\text{ax}}\cos\varphi + \bar{v}_{\text{tr}}\sin\varphi\right] + \varepsilon_4(k)
\end{aligned}
\tag{12}
$$

The new measurement noise sources $\underline{\varepsilon}(k)$ are rescaled quantities from the original noise sources $\underline{n}(k)$ under the assumption that the noise contribution is small. Equations (12) can be put into matrix form

$$\underline{z}(k) = H\,\underline{x} + \underline{\varepsilon}(k)$$

$$
H = \begin{bmatrix} h_1^T \\ h_2^T \\ h_3^T \\ h_4^T \end{bmatrix} = \begin{bmatrix} 1/L & \cos\varphi/L & \sin\varphi/L \\ 1/L & -\cos\varphi/L & -\sin\varphi/L \\ 1/L & \cos\varphi/L & -\sin\varphi/L \\ 1/L & -\cos\varphi/L & \sin\varphi/L \end{bmatrix}
\tag{13}
$$

and a linear problem least square problem for the unknown $\underline{x}$ can be formulated (Lanzensdörfer [1]). For N consecutive measurements the minimization of the performance index

$$
J = \sum_{k=1}^{N}\sum_{k=1}^{4}\varepsilon_i(k)^2 = \sum_{k=1}^{N}\left(\underline{z}(k) - H\underline{x}\right)^T \bullet \left(\underline{z}(k) - H\underline{x}\right) = \sum_{k=1}^{N}\sum_{i=1}^{4}\left(z_i(k) - h_i^T\,\underline{x}\right)^2 = \min
\tag{14}
$$

leads to the normal equations. The solution for $\underline{x}$ can be found by direct inversion of a block of measurements or it can also be solved by an iterative procedure via a recursive least square method. By introducing a forgetting factor λ it is possible to weigh new measurements heavier than older ones. This weighting prevents the recursive methods from getting insensitive to changes in the $\underline{x}$ (velocities) values. In flow measurement applications changes in these values are a fact. So either the block length or the forgetting factor must be chosen adequately in order to obtain satisfying results.

Here another approach is followed. A dynamic model of 3rd order and constant coefficients is assumed for the time evolvement of the velocities, see Fig. 1.

The states of this model are driven by the system noise $\underline{w}(k)$.

$$
\begin{aligned}
\underline{x}(k+1) &= A\underline{x}(k) + B\underline{w}(k) \\
\underline{z}(k) &= H\,\underline{x}(k) + \underline{\varepsilon}(k)
\end{aligned}
\tag{15}
$$

The measurement equation for $\underline{z}(k)$ the system output $\underline{y}(k) = H\,\underline{x}(k)$ corrupted by the measurement noise $\underline{\varepsilon}(k)$. The dynamic model used is a random walk model (Gruber, Tödtli [2]) for the unknown but time varying velocities:

$$
\begin{aligned}
c(k+1) &= c(k) + w_1(k) \\
\bar{v}_{\text{ax}}(k+1) &= \bar{v}_{\text{ax}}(k) + w_2(k) \\
\bar{v}_{\text{tr}}(k+1) &= \bar{v}_{\text{tr}}(k) + w_3(k)
\end{aligned}
\tag{16}
$$

This can be written in matrix form (equation (15)) with

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{17}$$

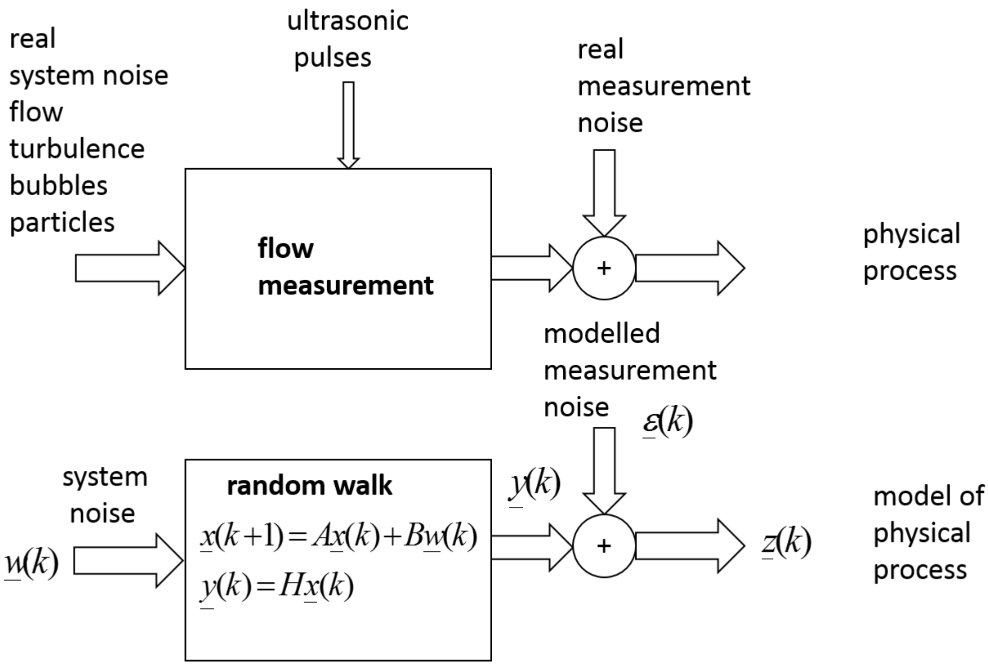

Fig.2: Random walk model for the acoustic measurement process

All noise sources are assumed to be Gaussian, independent and uncorrelated:

$$E\{\underline{w}(k)\} = E\{\underline{\varepsilon}(k)\} = 0 \qquad E\{\underline{x}(0)\} = \underline{x}_0$$

$$\text{cov}\{\underline{w}(j), \underline{w}(k)\} = Q = \begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{bmatrix} \qquad \text{cov}\{\underline{\varepsilon}(j), \underline{\varepsilon}(k)\} = R = \begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{bmatrix} \tag{18}$$

$$\text{cov}\{\underline{w}(j), \underline{\varepsilon}(k)\} = \text{cov}\{\underline{x}(0), \varepsilon(k)\} = \text{cov}\{\underline{w}(k), \underline{x}(0)\} = 0 \qquad \text{var}\{\underline{x}(0)\} = P'_{\underline{x}_0}(0)$$

The random walk model for a state K has the statistical properties, that the mean is constant and the variance is growing linear in time:

$$E\{\underline{z}(k)\} = \underline{x}_0 \qquad \text{var}\{\underline{z}(k)\} = P'_{\underline{x}_0}(0) + kQ + R \tag{19}$$

With the formulation from equation (13), (15), (17), and (18) a standard Kalman filter can be applied to the dynamic system (Sage, Melsa [3]. The Kalman filter is a filter, that tries to estimate the unknown state $\underline{x}(k)$ of the system by minimizing the error covariance between true state and an estimate $\hat{\underline{x}}(k)$ of the state

$$P(k) = \text{var}\{\underline{x}(k) - \hat{\underline{x}}(k)\} \tag{20}$$

The filter consists of two parts: a first part with a one step ahead prediction $\underline{\hat{x}}^*(k)$ of the state estimate $\underline{\hat{x}}(k)$ and a one step ahead prediction $P^*(k)$ of the error covariance matrix $P(k)$. In a second part the predicted value is updated and corrected with the weighted (with the Kalman gain) difference between the new measurement $\underline{z}(k+1)$ and the output prediction without measurement noise

$$H\underline{\hat{x}}^*(k) = HA\underline{\hat{x}}(k) .$$

If the model is time invariant, the steady state Kalman gain matrix K can be computed off-line:

$$\begin{aligned}
&P(0) = P'_{\underline{x}_0} \qquad k = 0,1,\ldots\ldots \\
&P^*(k+1) = AP(k)A^T + Q \\
&K(k+1) = P^*(k+1)H^T(HP^*(k+1)H^T + R)^{-1} \\
&P(k+1) = P^*(k+1) - K(k+1)A^T + Q
\end{aligned} \tag{21}$$

If the fixed point of the difference equations is reached, K can be obtained. K is in this case only a function of A, H, Q and R. K can then be used for the update equation of the state estimate corresponding to the velocities:

$$\begin{aligned}
&\underline{\hat{x}}(0) = \underline{x}_0 \qquad k = 0,1,\ldots\ldots \\
&\underline{\hat{x}}(k+1) = (I - KH)A\underline{\hat{x}}(k) + K\underline{z}(k+1)
\end{aligned} \tag{22}$$

Fig. 3 shows the block diagram of the filter with the real measurements $\underline{z}(k)$. Equation (22) allows estimate the speed of sound of water $c(k)$, the axial and the transversal component $\bar{v}_{ax}(k)$ and $\bar{v}_{tr}(k)$ of the flow velocity.

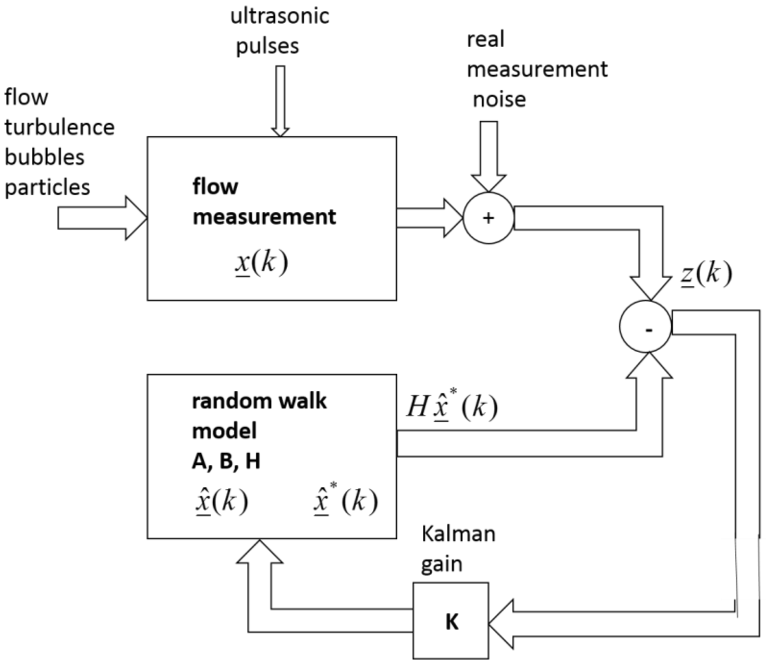

Fig. 3: Steady state Kalman filter implementation

3. FILTER PARAMETRIZATION STUDY

The benefit of the Kalman filter compared to other recursive estimation methods is that it is possible to influence the time behavior of the filter in two ways:

1) The ratio of the covariance matrices of the system noise Q over the measurement noise R. If Q/R is much larger than 1, the measurement noise is much less weighted than the system noise, that means the filter tries to follow the measured output in a fast way. Each variation in the measurement is considered to stem from a change in $\underline{x}(k)$. If Q/R is much lower than 1, the system noise is much less weighted than the measurement noise, that means in this case the filter acts as a low pass filter. With this ratio it is possible to tune the filter such that a predefined settling time for step change in the velocities can be achieved. The higher the ratio Q/R is, the more confidence in the measured value is assumed.
2) As the system noise consist of 3 noise sources, one for each velocity, it is possible to weigh them individually. That makes sense in this application because changes in speed of sound, axial and transversal components are not the same. This possibility enhances the flexibility of the filter.

We consider a situation with typical values as given in Fig. 4a-4d. In order to make the time behavior of the filter visible, step changes for the speed of sound at k=1000, the axial velocity at k=2000 and the transversal velocity at k=3000 are applied sequentially and the whole run is simulated during 4000 time steps.

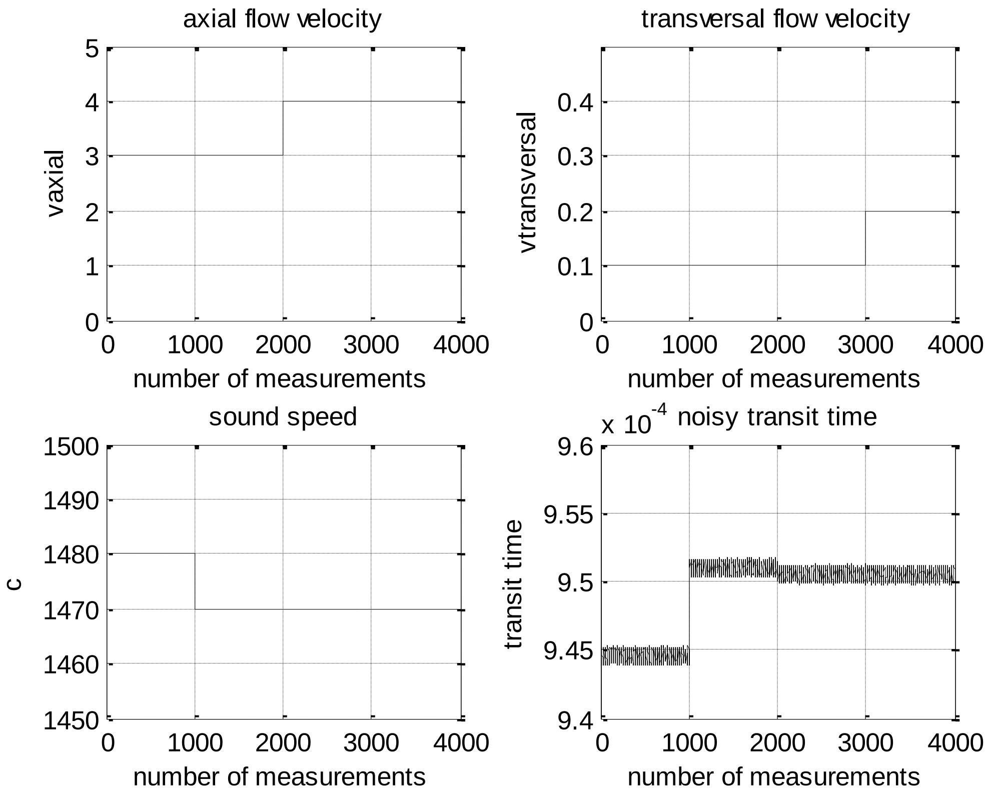

Fig. 4: simulated process, a: axial flow velocity, b: transversal velocity, c: speed of sound, d: measured transit time (td1)

Fig. 5a -5d displays the ideal and noisy velocity components and the noisy transit time measurements due to velocity noise and measurement noise. The scaling of the horizontal axes is such that a time period without step change in the velocities is shown. The scaling of the vertical axes is for each graph different.

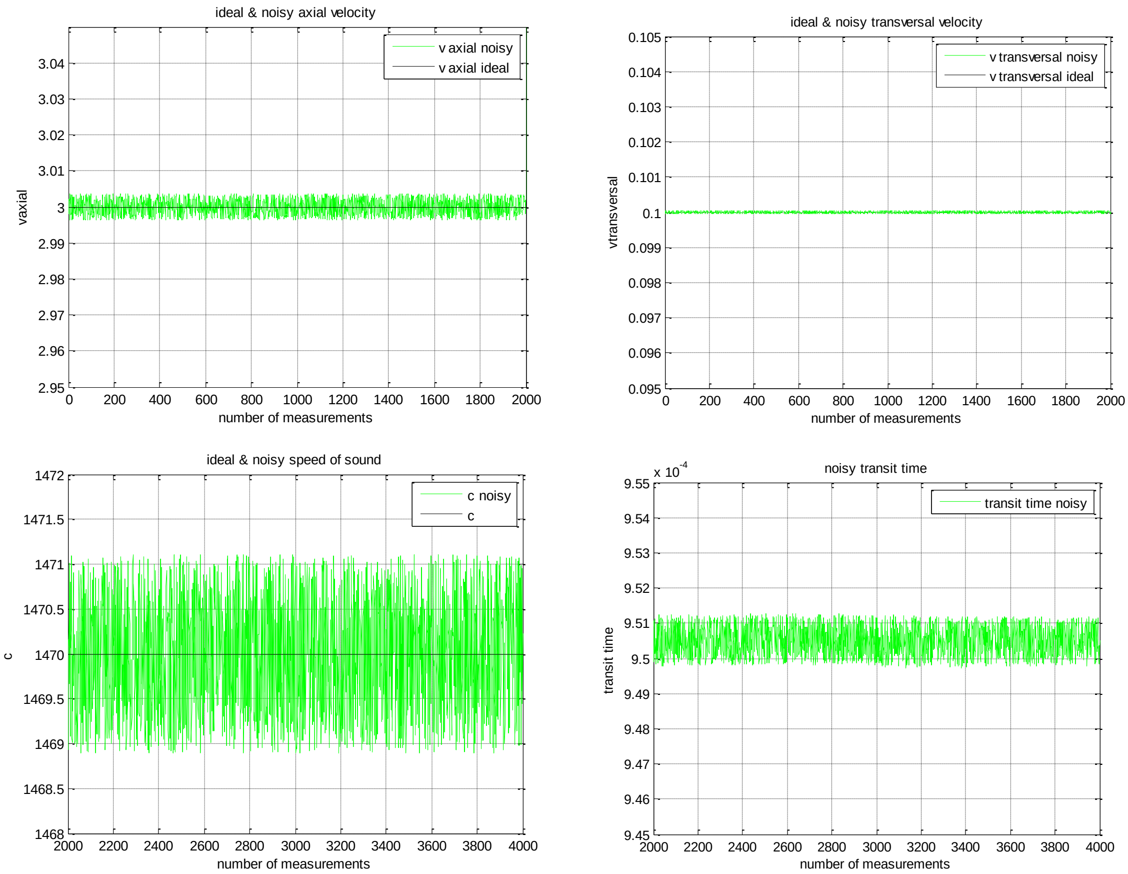

Fig. 5: examples of noisy velocity components and transit time measurement

Example 1: A first example of a Kalman filter treats all noise sources as equal, that means all noise covariance matrices are chosen as the unity matrix.

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The steady state Kalman filter gain matrix is then given by

$$K = \begin{bmatrix} 0.2573 & 0.2573 & 0.2573 & 0.2573 \\ 0.3090 & 0.3090 & 0.3090 & 0.3090 \\ 0.3062 & 0.3062 & 0.3062 & 0.3062 \end{bmatrix}$$

Fig. 6a shows the determination of the axial velocity if equations (5a, b) are used. No additional low pass filtering was applied. Fig. 6b shows the tracking capability and noise rejection of the chosen Kalman filter for the estimated axial path velocity $v_{axial\_estimated}$ and a low pass filtered (moving average of length 25) signal $v_{axial\_filtered}$ of the measured axial velocity together with the ideal axial velocity. Fig. 6c shows the same for the transversal component. Both figures indicate that the Kalman filter is not well tuned to the simulation conditions, while the low pass filter behaves much better. In order to compare the filters the following performance criteria have been used:

$$
\begin{aligned}
J_{Kalman} &= \sum_{k=1}^{N} \left| v_{axial\_ideal}(k) - v_{axial\_estimated}(k) \right| \\
J_{measured} &= \sum_{k=1}^{N} \left| v_{axial\_ideal}(k) - v_{axial\_measured}(k) \right| \\
J_{filtered} &= \sum_{k=1}^{N} \left| v_{axial\_ideal}(k) - v_{axial\_filtered}(k) \right|
\end{aligned} \tag{23}
$$

The values of these indices are tabulated in Table 1. Fig. 6d shows the performance of the Kalman filter for the estimation of the speed of sound.

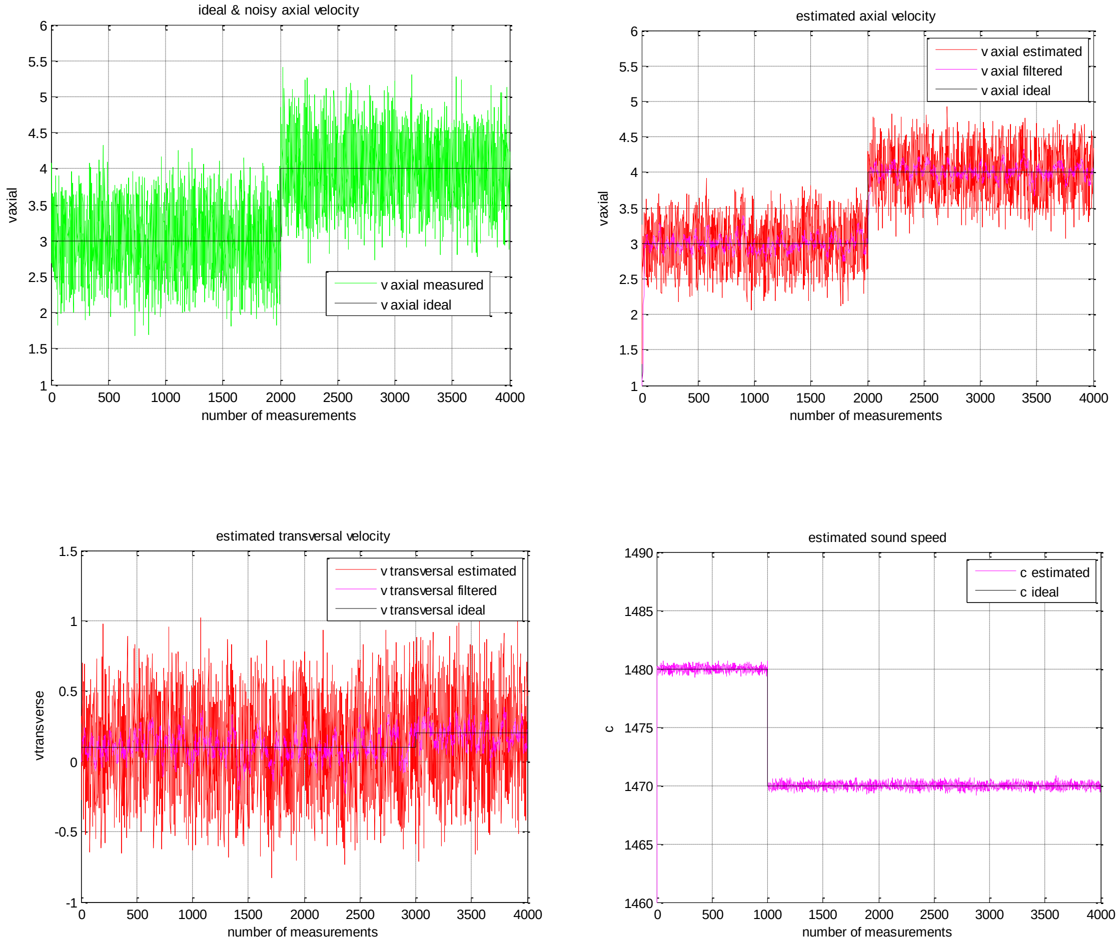

Fig 6:a) measured axial velocity, b) Kalman filter estimate and filtered measured axial velocity, c) Kalman filter estimate and filterede measured transversal velocity, d) Kalman filter estimate for sound speed.

Example 2: In this example the covariance matrices are adapted as follows: The diagonal elements of the covariance matrix are weighted individually with diminishing magnitude of the velocity components. The values 1000, 10 and 1 are chosen arbitrarily (the same holds for R), the ratio between the elements however is chosen according to the magnitudes. Also the diagonal elements of R are all equal and at least ten times larger than the values for Q. Therefore a low pass filter effect can be expected.

$$Q = \begin{bmatrix} 1000 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} 10000 & 0 & 0 & 0 \\ 0 & 10000 & 0 & 0 \\ 0 & 0 & 10000 & 0 \\ 0 & 0 & 0 & 10000 \end{bmatrix}$$

The steady state Kalman filter gain matrix is now changed to

$$K = \begin{bmatrix} 0.1264 & 0.1264 & 0.1264 & 0.1264 \\ 0.0156 & -0.0156 & 0.0156 & -0.0156 \\ 0.0050 & -0.0156 & -0.0156 & 0.0156 \end{bmatrix}$$

Figure 7 displays the same quantities as in example 1. It is clearly visible that the Kalman filter performs much better than before and even outperforms the moving averaged filtered estimate (see Table 1). That means that with an appropriate Kalman filter parametrization good results can be obtained. Keep in mind that the above choice of the filter was not optimized explicitly to the noise sources but were chosen by inspection.

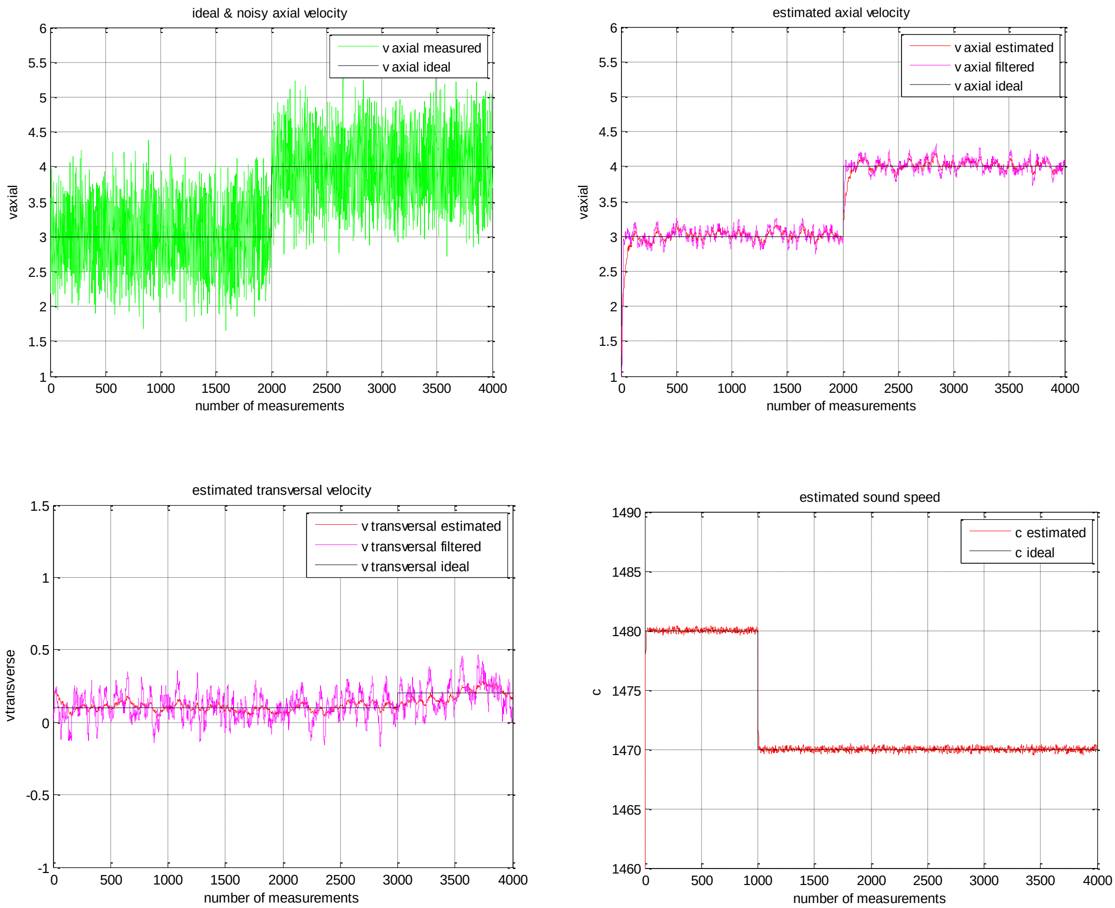

Fig 7:a) measured axial velocity, b) Kalman filter estimate and filtered measured axial velocity, c) Kalman filter estimate and filtered measured transversal velocity, d) Kalman filter estimate for sound speed.

Performance index	Example 1	Example 2
Unfiltered $J_{measured}$	1386	1420
moving average length 25 $J_{filtered}$	304.6	336.0
Kalman filter J_{Kalman}	933.5	251.4

Table 1: Performance indices for the different filtering techniques for the two examples

4. OUTLOOK AND CONCLUSION

By tuning a Kalman filter for a random walk model of the physical measurement process properly it has been shown that a recursive estmation of the time varying velocity components $\bar{v}_{ax}(k), \bar{v}_{trans}(k), c(k)$ are possible without the use of the delta time information found via the correlation of up- and downstream acoustic pulses. Due to this fact, the following problems could be addressed:

- Behavior of the filter for slowly varying situation (sinusoidal, ramp-like)
- Optimization of the filter for measured noise levels
- Comparison of the filter performance for different methods to determine the transit times to a sufficiently high accuracy
- Evaluate the filter with real measurements
- Can the Kalman filter solution be extended to a situation where also the delta time between up- and downstream pulses can be included?
- Modelling the measurement noise dependent on the method of transit time determination
- Modelling of the system noise by using some turbulence model.

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