# Feasibility study of the cascading mass flow rate evaluation as a new variant of Gibson's method

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**Abstract**

We present a feasibility study of Gibson's method for low and medium head plants using not a single and complete closure of the wicket gates but a stepwise closing procedure. One has to wait until flow stabilizes after each step to initiate another step of abrupt closing operation. Therefore, n steps before complete closure gives n measuring points instead of a single measuring point in applying the common procedure. That decreases the overall time consumption of the test session and reduces the rejected loads impacting the stability of the electrical grid and the mechanical components. In this paper, we show the theoretical basis and the corresponding uncertainty estimation for this variant of Gibson's method. Instationary simulations and results will be presented at the conference.

# 1 Introduction

The application of the Pressure-time method aka Gibson's method on units in hydropower plants is of great interest. It is rather used on turbines than on pumps (e.g., [1]). The minor efforts related to time consumption of installation makes it an attractive alternative compared to other primary flow measurement methods especially on low and medium head units. The successful outcome of a testrun is closely related to the deceleration rate of the water inside the closed piping. The steeper the temporal decrease of flow $\partial v / \partial t$ – the steepness of this value is comparable to that one during an emergency shutdown – the better the signal-noise-ratio (SNR) of the pressure signal(s) and the higher the reliability. The so-induced water hammer gives understandably rise to limitations with respect to allowable mechanical stresses. However, operating companies often refuse the application of the Pressure-time method because of the negative impact of the fast load-rejection onto the grid stability. The quality of power grid networks with low capacity suffer hard under such circumstances and are able to collapse if the rejected load cannot be compensated. In these cases one has to choose another measuring method, which is most likely more costly, since alternative variants of the Pressure-time method are neither indicated nor recommended by the leading test codes [2, 3, 4].
We discuss the significance of a new variant of Gibson's method in this publication. This variant differs from the original procedure in applying several small deceleration steps of the water instead of a single and complete deceleration by complete closure of the flow regulation device (e.g., wicket gates). One can test the unit's behavior at $n$ operating points before minimum opening of the closure device instead of a single operating point if using the common procedure. The grid network should suffer therewith only from small amount of abrupt load-changes. It can react within an short time frame to compensate the power needs by changing the loads on other energy generating units. Therefore, we should be able to apply this variant of Pressure-time method even on a unit which is part of a small isolated network.
Since we could not yet test the new procedure in practice, we have to rely on simulation data obtained from one-dimensional instationary flow calculations by the software *SIMSEN* to check the feasibility and the limitations. These results will be presented at the conference. Although this paper primarily deals with flow through turbine units in both modes, i.e., using deceleration and acceleration procedure, the workflow for testing pumps is similar but is not treated in here.

## 2 Theoretical basis

We do not give a detailed mathematical and physical background of the general Pressure-time method since it has already been discussed excellently elsewhere [4, 5].
Consider a turbine operating at high load and under steady head conditions. We start our data recording. The Gibson differential pressure $p_{\mathrm{G}}(t)$ fluctuates randomly around the running line pressure $p_{\mathrm{RL}}$. Let us add the index 1 to the last-mentioned symbol, i.e., $p_{\mathrm{RL},1}$, to indicate that this pressure value belongs to the evaluation procedure of the initial mass flow $(\rho Q)_1$. After a fast but partly deceleration of the water in the piping the pressure conditions stabilize again and we end up with a Gibson pressure denoted by the static line pressure $p_{\mathrm{SL}}$. We put the index 1 into this symbol and call it $p_{\mathrm{SL},1}$. Between the transient stabilization of $p_{\mathrm{RL},1}$ and $p_{\mathrm{SL},1}$ the mass flow changed from $(\rho Q)_1$ to $(\rho Q)_2$, where $(\rho Q)_2 < (\rho Q)_1$. At this stage $p_{\mathrm{G}}(t)$ fluctuates in stochastic manner around $p_{\mathrm{RL},2}$. Consequently, we equate this running line pressure with the static line pressure of the previous step $p_{\mathrm{RL},2} = p_{\mathrm{SL},1}$ or, generally speaking,

$$p_{\mathrm{RL},i+1} = p_{\mathrm{SL},i} \quad \forall i \in \{\mathbb{N} | i < n\} \, . \tag{1}$$

We repeat the procedure of deceleration a total of $n$ times until leakage flow $\rho q$ (or zero flow) is reached. See reference [6] for leakage flow determination. After $p_{\mathrm{G}}(t)$ stabilizes significantly at $p_{\mathrm{SL},n}$ we can stop the data recording. The analysis starts now at the very end of the collected data. Mass flow rate $(\rho Q)_n$ is calculated by

$$(\rho Q)_n = \frac{1}{F} \int\limits_{t_n}^{t_n+\Delta t_n} \left[ p_{\mathrm{G}}(t) - p_{\mathrm{R}}(t) \right] dt + \rho q \tag{2}$$

with

$$\frac{p_{\mathrm{R}}(t) - p_{\mathrm{SL},n}}{p_{\mathrm{RL},n} - p_{\mathrm{SL},n}} = \frac{[(\rho Q)(t)]^2 - (\rho q)^2}{(\rho Q)_n^2 - (\rho q)^2} \, . \tag{3}$$

The parameters $t_n$ and $\Delta t_n$ denote an appropriate choice of the integration starting time and the time length [5], respectively. The behavior of the recovery line pressure $p_{\mathrm{R}}(t)$ between the time interval $[t_n, t_n + \Delta t_n]$ needs iterative computation with the previous two equations. Thus, we are able to determine the loss coefficient $\zeta$ at mean Reynolds number

$$\overline{\mathrm{Re}}_n \propto \frac{1}{\mu \Delta t_n} \int\limits_{t_n}^{t_n+\Delta t_n} (\rho Q)(t) \; dt \tag{4}$$

by reordering

$$\frac{p_{\mathrm{RL},n} - p_{\mathrm{SL},n}}{(\rho Q)_n^2 - (\rho q)^2} = \frac{1}{2\rho} \cdot \left[ \frac{\alpha_2}{A_2^2} \left(1 + \zeta(\overline{\mathrm{Re}}_n)\right) - \frac{\alpha_1}{A_1^2} \right] . \tag{5}$$

The parameters $A_1$ and $A_2$ label the cross-section of the Gibson measurement section 1 (upstream) and 2 (downstream). Since we do not have any information of the available flow profile at both measurement sections, the correction factors $\alpha_1$ and $\alpha_2$ are set to unity.
Hopping back in time of the recorded data we compute the relevant parameters analogously by

$$(\rho Q)_i = \frac{1}{F} \int_{t_i}^{t_i+\Delta t_i} \left[p_{\mathrm{G}}(t) - p_{\mathrm{R}}(t)\right] dt + (\rho Q)_{i+1} , \tag{6}$$

$$\frac{p_{\mathrm{R}}(t) - p_{\mathrm{SL},i}}{p_{\mathrm{RL},i} - p_{\mathrm{SL},i}} = \frac{[(\rho Q)(t)]^2 - (\rho Q)_{i+1}^2}{(\rho Q)_i^2 - (\rho Q)_{i+1}^2} , \tag{7}$$

$$\overline{\mathrm{Re}}_i \propto \frac{1}{\mu \Delta t_i} \int_{t_i}^{t_i+\Delta t_i} (\rho Q)(t)\, dt \tag{8}$$

and

$$\zeta(\overline{\mathrm{Re}}_i) = \left(\frac{A_2}{A_1}\right)^2 - 1 + 2\rho A_2^2 \cdot \frac{p_{\mathrm{RL},i} - p_{\mathrm{SL},i}}{(\rho Q)_i^2 - (\rho Q)_{i+1}^2} . \tag{9}$$

We do not require the calculation of the loss coefficient dependency on the Reynolds number to determine the contributing mass flow steps. But it reveals the minor value changes with respect to the frictional behavior of fluid and pipe wall between both Gibson measurement sections. Hence, this variant elaborates more sensitively on changes in friction than the common procedure does, which uses only one loss coefficient for the complete closing procedure. By increasing the number of mass flow steps $n$ one gains an even higher resolution of $\zeta(\mathrm{Re})$. Anyhow, the integer value of $n$ is limited by the SNR of the pressure measurement devices in use.

# 3 Uncertainty estimation

Detailed uncertainty analyses have already been published for the commonly used procedure [7]. Here, we give a rough estimate of the expectable measurement uncertainty, stated by the expanded uncertainty $U[(\rho Q)_i] = ku[(\rho Q)_i]$

[8], in using the cascadian technique.

Consider $n$ steps equidistant in massflow starting from $(\rho Q)_1$ down to zero leakage. That is,

$$(\rho Q)_i = (n+1-i)/n \cdot (\rho Q)_1 \tag{10}$$

$$(\rho Q)_{n+1} = \rho q = 0 \text{ kg/s} . \tag{11}$$

We assume that the relative standard uncertainty of the measurement conditions[1] $u_\mathrm{M}/(\rho Q)$ and of the integration area $u(B)/(\rho Q)$ remain unchanged for the individual steps. The combined standard uncertainty of the final cascadian step before zero leakage thus yields

$$u_n = \frac{u_\mathrm{c}[(\rho Q)_n]}{(\rho Q)_n} = \frac{\sqrt{u^2(B) + u^2(F) + u_\mathrm{M}^2}}{(\rho Q)_n} \tag{12}$$

including the standard uncertainty of the pipe factor $u(F)$. We bear in mind that the massflow equation for the previous steps consists of the integrational term $B_i/F$ and of the subsequent massflow $(\rho Q)_{i+1}$. Therefore, with the aid of equation (10) we obtain the relative standard uncertainty of massflow $i$ recursively by

$$u_i = \frac{\sqrt{u_n^2 + (n-i)^2 \cdot u_{i+1}^2}}{n+1-i} \quad \forall i \in \{\mathbb{N} | i \leq n\} . \tag{13}$$

Expanding $u_{i+1}$ until $u_n$ finally yields

$$\frac{u_i}{u_n} = \frac{1}{\sqrt{n+1-i}} \tag{14}$$

$$\frac{u_\mathrm{c}[(\rho Q)_i]}{u_\mathrm{c}[(\rho Q)_n]} = \sqrt{n+1-i} \tag{15}$$

for all $i \in \{\mathbb{N} | i \leq n\}$, respectively. Equation (15) indicates that the relative standard uncertainty increases with every step-down massflow compared to the preceding step; the absolute standard uncertainty decreases vice versa (view eq. 15). The ratio of the corresponding expanded uncertainties coincides with the right-hand side of latter equation consequently.

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[1] We use this uncertainty proportion to account for any deviation from ideal measurement conditions [2] (e.g., $u_\mathrm{M} = (0.25 \ldots 1.00\%) \cdot (\rho Q)$).

**Example** We want to apply $n = 4$ equidistant massflow steps from $(\rho Q)_1$ to zero flow on the basis of (10). Assuming a relative expanded uncertainty of $U[(\rho Q)_4]/(\rho Q)_4 = 2.00\%$ $(k = 2)$ for the final massflow before zero flow, we can expect expanded uncertainties for the other massflow values according to table 1.

Table 1: Expanded uncertainty $(k = 2)$ of massflow values using equation (15)

| $i$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Eq. (15) | $\sqrt{4}$ | $\sqrt{3}$ | $\sqrt{2}$ | $\sqrt{1}$ |
| $\frac{U[(\rho Q)_i]}{(\rho Q)_i}$ | 1.00% | 1.15% | 1.41% | 2.00% |

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Although higher massflows really tend to lower relative uncertainty values, the estimation of $u(B)/(\rho Q)$ requires an individual numerical treatment for each step.

# Nomenclature

| Symbol | Description | Unit |
|---|---|---|
| $A$ | Cross-section | (m$^2$) |
| $B$ | Integration area | (Pa·s) |
| $F$ | Pipe factor | (1/m) |
| $k$ | Coverage factor (e.g., $k = 2$ for confidence level 95%) | (-) |
| $\rho q$ | Leakage mass flowrate | (kg/s) |
| $\rho Q$ | Mass flowrate | (kg/s) |
| $n$ | Number of massflow steps | (-) |
| $p_{\mathrm{G}}$ | Gibson (differential) pressure | (Pa) |
| $p_{\mathrm{RL}}$ | Running line pressure | (Pa) |
| $p_{\mathrm{SL}}$ | Static line pressure | (Pa) |
| $q$ | Leakage volume flowrate | (m$^3$/s) |
| $Q$ | Volume flowrate | (m$^3$/s) |
| Re | Reynolds number | (-) |
| $t$ | Time | (s) |
| $u$ | Standard uncertainty | (a.u.) |
| $U$ | Expanded uncertainty ($= ku_{\mathrm{c}}$) | (a.u.) |
| $u_{\mathrm{c}}$ | Combined standard uncertainty | (a.u.) |
| $u_{\mathrm{M}}$ | Standard deviation accounting for the measurement conditions ($= (0.25 \ldots 1.00\%) \cdot (\rho Q)$) | (kg/s) |
| $\alpha$ | Velocity profile coefficient ($\approx 1$) | (-) |
| $\zeta$ | Loss coefficient | (-) |
| $\mu$ | Dynamic viscosity of water | (Pa·s) |
| $\rho$ | Water density | (kg/m$^3$) |

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