Investigation of the pressure-time method using design of experiments

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Abstract: The present work presents a sensibility analysis by means of factorial design, a statistical method to design experiments (Cervantes and Engström, 2004), applied to a numerical analysis. The procedure presented by Adamkowski and Janicki (2013) for flow rate estimation based on differential pressure measurements, and developed further by Dunca et al (2016) is used. It considers both the liquid compressibility and the pipe walls deformability. The influence of the following parameters over the flow rate estimation error is determined: Reynolds number of the flow, *Re*, pressure wave speed, *a*, and the measuring length, *L* between the differential pressure measuring sections. Further, the factorial design analysis will be validated using laboratory data, obtained in the Waterpower Laboratory from the Norwegian University of Science and Technology, Norway.

Keywords: flow rate measurement, estimation error, factorial design

1. Introduction

High precision determination of hydraulic machines efficiency is a constant concern of researchers in the field. Despite considerable advances in measuring techniques, the flow measurement is a challenge even for the most experienced teams of specialists.

The pressure-time method is a simple method for flow rate estimation, recommended by IEC 60041 (1991) and ASME PTC 18 (2011). It consists in measuring the pressure variation between two hydrometric sections of a closed conduit during a machine shutdown, using the transformation of momentum into pressure. The value of the flow rate is then obtained by integrating the pressure variations during the induced transient regime.

But this method is subjected to limitations and considers some simplifications. Over time, eliminating or at least relaxing those restrictions has been tried. Many papers were presented enhancing the method performance by experimental and numerical means after its standardization. In the most recent works performed, the following can be mentioned: Jonsson et al. (2007, 2008) analysed the method pressure-time situations outside of standard measurement criteria; they developed a numerical model of the method for applying pressure-time method on low head machines. The method has been successfully used for measuring lengths shorter than those stipulated in the standard, obtaining a more accurate estimation with 0.4%. Adamkowski, 2012, and Adamkowski and Janicki, 2010, developed applications of the method pressure-time for special conditions of use in hydroelectric plants (curved penstockes with special instruments inside the pipes, with irregular cross sections between measuring sections etc.). The results were satisfactory.

In the present work, a development based on the method described by Adamkowski and Janicki (2013) using the water hammer equation is applied, considering an unsteady model for the friction factor instead of a constant one. It considers both the liquid compressibility and the pipe walls deformability. The influence of the following input parameters variation over the model precision for flow rate evaluation is estimated using design of experiments: Reynolds number of the flow, *Re*, pressure wave speed, *a*, and the measuring length, *L* between the differential pressure measuring sections. The results of the numerical analysis are analyzed using factorial design.

2. Method

The model used in the present paper is based on rewriting the water hammer classical equations in the form presented by Adamkowski and Janicki (2013). The hyperbolic equations are solved using the method of characteristics (MOC). The pressure head *H* and the flow velocity *V* are replaced with the pressure head difference, *dH*, between two cross-sections and the discharge, *Q*.

In this way the equations used to compute the pressure head and flow rate variation inside the pipe during the transient regime are Eq. 1 and Eq. 2, (fig. 1):

- along the positive characteristic C^+

$$dH_P - dH_M + \frac{a}{g \cdot A}\left(Q_P - Q_M\right) + \frac{f \cdot \Delta x}{2g \cdot D \cdot A^2} \cdot Q_P \cdot \left|Q_M\right| = 0 \qquad (1)$$

- along the negative characteristic C^-

$$dH_P - dH_N - \frac{a}{g \cdot A}\left(Q_P - Q_N\right) - \frac{f \cdot \Delta x}{2g \cdot D \cdot A^2} \cdot Q_P \cdot \left|Q_N\right| = 0 \qquad (2)$$

where: Δx is spatial discretization, g is the acceleration due to gravity, *f* is the friction factor, *D* is the pipe diameter and *a* is the pressure wave speed. The parameter *a* depends on the pipe walls Young modulus *E*, liquid density ρ, and bulk modulus ε, pipe wall thickness *e*, and diameter, *D*, according to the relation $a = \sqrt{\varepsilon/\rho}\Big/\sqrt{1+(\varepsilon D)/(eE)}$.

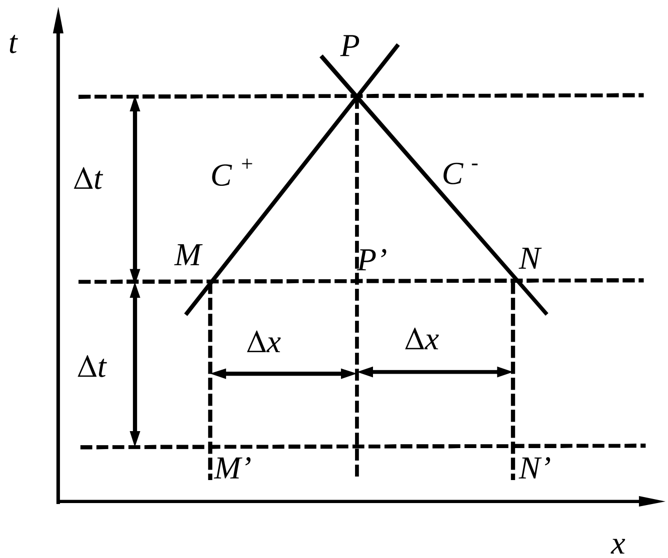

Fig. 1. Characteristics in the plane *xOt*.

The model developed by Adamkowski and Janicki (2013) obtained the discharge flowing through a pipe using the pressure head difference measured between two cross-sections, similar to the pressure-time method. The effects of liquid compressibility and pipe walls deformability are considered using the Eq. (1) and (2) via the speed of sound, *a*. The computational procedure implies defining certain moments in time, which characterize the water hammer transient phenomenon (Fig. 2):

- t_1 – beginning of the analysed time-history
- t_2 – end of the initial steady state
- t_3 – end of the transient state corresponding to the forced flow rate change
- t_4 – end of the analysed time-history.

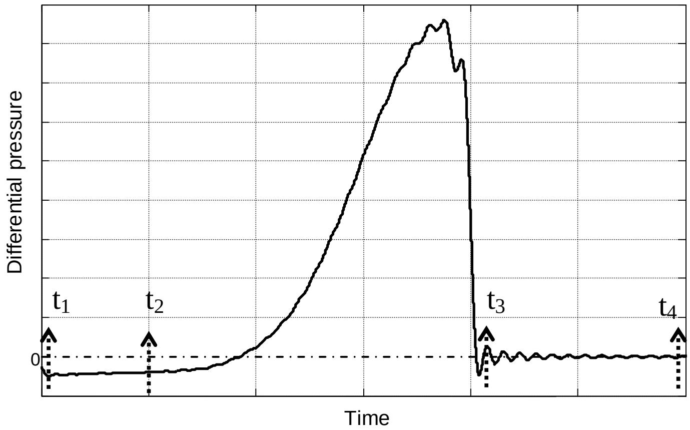

Fig. 2. Differential pressure variation and time definition.

In the method presented by Adamkovski, the friction factor *f* is considered constant. This hypothesis is acceptable for pipes with high roughness and a quasi-steady-transient phenomenon, i.e., slow transient. For the fast transient regimes, an unsteady friction factor should be used. Bergant *et al.* (2001) analysed some of the unsteady friction expressions obtaining the best results with the Brunone model. This model gave good results in other studies as Jonsson *et al.* (2012) and Dunca *et al.* (2013). In the present work, the model is implemented in the method proposed by Adamkowski to evaluate possible improvement in the error associated with the discharge estimation.

The Brunone model is described by Bergant *et al.* (2001). It consists in expressing the friction factor *f* as:

$$f = f_q + \frac{k \cdot D}{V \cdot |V|}\left(\frac{\partial V}{\partial t} - a \cdot \frac{\partial V}{\partial x}\right) \tag{3}$$

where f_q is the quasi-steady friction factor, k is the Brunone friction coefficient, $\partial V/\partial t$ is the instantaneous local acceleration and $\partial V/\partial x$ is the instantaneous convective acceleration. The coefficient k can be determined either by trial and error method or analytically using the Vardy's coefficient (Vardy's shear decay coefficient C^*), $k = \sqrt{C^*}/2$, empirically calibrated. Coefficient C^* is 0.00476 for laminar flows while for turbulent flows is computed using the equation:

$$C^* = \frac{7.41}{\mathrm{Re}^{\log\left(14.3/\mathrm{Re}^{0.05}\right)}} \tag{4}$$

The quasi-steady part of the friction factor, f_q, is computed using Darcy equation for laminar flow ($f_q = 64/\mathrm{Re}$) and the Haaland equation for turbulent flow:

$$\frac{1}{\sqrt{f_q}} = -1.8\log\left[\left(\frac{\Delta/D}{3.7}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right] \quad (5)$$

with Δ the roughness of the pipe wall.

In order to evaluate the flow rate with the proposed evaluation procedure, the following information is needed:

- pressure head difference, *dH*, measured between two cross-sections. An initial discharge value is imposed as initial guess for this algorithm.
- definition of the moments t_1, t_2, t_3 and t_4 based on the pressure head difference *dH*.
- geometrical characteristics of the pipe (*D* – diameter, *E* – pipe walls Young modulus, *L* – distance between the pressure head measuring sections), and the liquid properties (ρ – density, ε – bulk modulus).

As described by Adamkowski and Janicki, 2013, the method is iterative. First, an initial guess for the flow rate, *Q*, is made. The value for the friction factor, *f*, in the steady state regime is obtained from the measured pressure head difference in t_1-t_2 time interval.

Starting with these values for *Q*, and *f*, the MOC is applied, using the boundary conditions:

- at upstream end (first measuring section):

  d*H* (t) = 0, while *Q*(t) results from Eq (2) along C-

- at downstream end (second measuring section):

  d*H* (t) according with measured data, while *Q*(t) results from Eq (1) along C+

A new value of the steady state flow rate *Q*, is then derived as the average value of the discharge trace during the steady state $t_1 - t_2$ time-period.

The obtained flow rate value is compared with the previous one and if the difference between them is less than an imposed value the computation stops. If this condition is not accomplished, the computation resumes with the new *Q*.

In Dunca et al, 2016, the flow rate estimation errors using the developed method are presented, for numerical experiments and for laboratory experiments. The obtained values for flow rate error were lower than 0.1% in all studied cases.

Further in this paper the influence of three parameters variation considered in the flow rate evaluation over the accuracy of this method is analysed, using factorial design.

3. Factorial design

Factorial design represents a statistical method for predicting the influence of each individual parameter and the interaction of different factors on one or more quantities of the evaluated process (Box et al, 1978, Montgomery, 2013). This method was originally applied in clinical or military trials, but it was also applied in engineering scientific research with encouraging results (Cervantes and Engström, 2004).

In this method, the factors influencing a process are varied in a certain pattern leading to a set of experimental runs. The general factorial design considers multiple factors, with multiple levels and with multiple repetitions of every case. For each factor, a main effect will be determined, then, for each combination of two factors, the interaction effect is computed. The method can be applied to obtain the three-factors effects and finally the N-factors effect. In the present paper a 3^3 factorial design will be applied, considering three factors (Reynolds number of the flow, *Re*, pressure wave speed, *a*, and the measuring length, *L* between the differential pressure measuring sections) each having three levels.

In order to establish the correct regression model, a statistical analysis in made. As a result, the parameters having an important influence over the observed response, and the possible interaction between the parameters can be determined.

For example, consider the three-factor analysis of variance model:

$$y_{ijkl} = \mu + \tau_i + \beta_j + \gamma_k + (\tau\beta)_{ij} + (\tau\gamma)_{ik} + (\beta\gamma)_{jk} + (\tau\beta\gamma)_{ijk} + \varepsilon_{ijkl}, \qquad (6)$$

with $i = 1,2,...,a$, $j = 1,2,...,b$, $k = 1,2,...,c$, $l = 1,2,...,n$, where μ is the overall mean effect, τ_i is the effect of the i^th^ level of the row factor A, β_j is the effect of the j^th^ level of column factor B, γ_k is the effect of the k^th^ level of column factor C, $(\tau\beta)_{ij}$, $(\tau\gamma)_{ik}$, $(\beta\gamma)_{jk}$ are the effect of the interactions between τ_i, γ_k and β_j, and ε_{ijk} is a random error component.

The analysis of variance table is shown in Table 1. The F tests on main effects and interactions follow directly from the expected mean squares. According to Popa and Neagoe, 2008, the F_0 estimator is compared to the critical value from the Fisher-Snedecor distribution estimator, f_{cr}, having (a-1), (b-1)…. (a-1)(b-1)(c-1) degrees of freedom, considering a value for the confidence level interval, 1-α. If F_0, determined for each parameter and for the interactions, is higher than the critical value f_{cr}, the influence of that parameter is important over the response value and it will be considered in the fitted regression model equation.

Table 1. The analysis of variance for the three factor effects model

Source of variation	Sum of squares	Degrees of Freedom	Mean Square	F_0
A	$SS_A = \frac{1}{bcn}\sum_{i=1}^{a} y_{i...}^2 - \frac{y_{....}^2}{abcn}$	a-1	MS_A	MS_A / MS_E
B	$SS_B = \frac{1}{acn}\sum_{i=1}^{b} y_{.j..}^2 - \frac{y_{....}^2}{abcn}$	b-1	MS_B	MS_B / MS_E
C	$SS_C = \frac{1}{abn}\sum_{i=1}^{c} y_{..k.}^2 - \frac{y_{....}^2}{abcn}$	c-1	MS_C	MS_C / MS_E
AB	$SS_{AB} = \frac{1}{cn}\sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij..}^2 - \frac{y_{....}^2}{abcn} - SS_A - SS_B$	(a-1)(b-1)	MS_{AB}	MS_{AB} / MS_E
AC	$SS_{AC} = \frac{1}{bn}\sum_{i=1}^{a}\sum_{k=1}^{c} y_{i.k.}^2 - \frac{y_{....}^2}{abcn} - SS_A - SS_C$	(a-1)(c-1)	MS_{AC}	MS_{AC} / MS_E
BC	$SS_{BC} = \frac{1}{an}\sum_{j=1}^{b}\sum_{k=1}^{c} y_{.jk.}^2 - \frac{y_{....}^2}{abcn} - SS_B - SS_C$	(b-1)(c-1)	MS_{BC}	MS_{BC} / MS_E
ABC	$SS_{ABC} = \frac{1}{n}\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} y_{ijk.}^2 - \frac{y_{....}^2}{abcn} - SS_A ...$ $- SS_B - SS_C - SS_{AB} - SS_{AC} - SS_{BC} = ...$ $SS_{Subtotals} - SS_A - SS_B - SS_C - ...$ $SS_{AB} - SS_{AC} - SS_{BC}$	(a-1)(b-1)(c-1)	MS_{ABC}	MS_{ABC} / MS_E
Error	$SS_E = SS_T - SS_{Subtotals}$	abc(n-1)	MS_E	
Total	$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} y_{ijkl}^2 - \frac{y_{....}^2}{abcn}$	abcn-1		

4. Numerical analysis

In the present work, a sensibility analysis of the developed method is performed, considering only numerically generated data. Using the Matlab code, data needed for the developed method to be applied (the pressure head variation during the valve closure) was computed with MOC, in different conditions considered further in the factorial design analysis. A hypothetical configuration of the pipe was considered.

In order to have an argued sensibility analysis of the developed method, the variation ranges for the analysed parameters had to be carefully chosen.

For analysing the Reynolds number influence over the method accuracy, three Reynolds number values were chosen in the usual range corresponding to real on site flow. The values considered for the pressure wave speed were chosen considering that in the wave speed estimation there can be an uncertainty of 7% (determined comparing the computed and the measured pressure wave speed from the numerically simulated data, Karadzic et al, 2014). For the measuring length variation range, the standard lengths measuring uncertainty was considered: ±0.5 mm. Every simulated case was repeated three times, in order to determine also the uncertainty estimation. The analysed cases and the corresponding flow rate estimation error obtained with the developed method are presented in Table 2.

Table 2. Estimation error $\varepsilon_Q[\%] = \dfrac{Q - Q_{ref}}{Q_{ref}} \cdot 100$ data and the analysed cases

Reynolds number (A)	**Pressure wave speed (B)**								
	963			**900**			**837**		
	Measuring length (C)			**Measuring length (C)**			**Measuring length (C)**		
	9-0.0005	**9**	**9+0.0005**	**9-0.0005**	**9**	**9+0.0005**	**9-0.0005**	**9**	**9+0.0005**
25464791	-7.159	-7.159	-7.159	-0.039	-0.038	-0.038	8.269	8.269	8.269
	-7.114	-7.114	-7.113	0.010	0.010	0.011	8.321	8.321	8.322
	-7.069	-7.068	-7.068	0.059	0.059	0.059	8.373	8.374	8.374
16976527	-6.741	-6.741	-6.741	-0.022	-0.022	-0.022	7.717	7.717	7.717
	-6.710	-6.710	-6.710	0.011	0.011	0.011	7.753	7.753	7.753
	-6.679	-6.679	-6.679	0.044	0.044	0.044	7.788	7.788	7.789
12732395	-6.636	-6.636	-6.636	-0.015	-0.015	-0.015	7.585	7.585	7.585
	-6.613	-6.613	-6.613	0.010	0.010	0.010	7.612	7.612	7.612
	-6.589	-6.589	-6.589	0.035	0.035	0.035	7.638	7.638	7.639

Using the data in Table 2, first the analysis of variance is performed according to table 1, in order to determine the parameters having significant influence over the flow rate estimation error. Also, the critical value of the Fisher-Snedecor distribution estimator, f_{cr} , are determined for each influence parameter, for a confidence level interval 1-α = 95%. The results are presented in table 3.

Table 3. The analysis of variance for data in table 2

Source of variation	Sum of squares	Degrees of Freedom	Mean Square	F_0	f_{cr}
Reynolds number (A)	0.072479301	2	0.03623965	**26.449**	**3.17**
Wave speed (B)	2925.233383	2	1462.616692	**1067468**	**3.17**
Measuring length (C)	1.098E-06	2	5.48999E-07	0.0004	3.17
AB	3.737866523	4	0.934466631	**682**	**2.55**
AC	6.67897E-07	4	1.66974E-07	0.0001	2.55
BC	3.72037E-08	4	9.30093E-09	6.788E-06	2.55
ABC	2.68912E-08	8	3.36139E-09	2.453E-06	2.12
Error	0.073989396	54	0.001370174		
Total	2929.11772	80	36.61397151		

As the F_0 estimator is larger than the critical value, f_{cr}, only for two analysed parameters, Reynolds number, *Re*, and pressure wave speed, *a*, and for their interaction (highlighted in table 3), it can be stated that only those have an important influence over the flow rate estimation error, thus over the developed accuracy.

Further, a fitted regression model will be tested for this case having the following equation:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \varepsilon \quad (7)$$

where y represents the observed response (flow rate estimation error obtained with the developed method), x_1 is the Reynolds number, x_2 is the pressure wave speed, ε represents the residual of the model and parameters β_j, j = 0, 1, ..., k, are called the regression coefficients

The model corresponds to a multilinear regression and its parameters, β_i, are determined using the least square method. The obtained equation for flow rate estimation error in function of the influence parameters: Reynolds number and pressure wave speed is:

$$\varepsilon_Q[\%] = 92.5262 + 7.0091 \cdot 10^{-7} \cdot \mathrm{Re} - 0.1025 \cdot a - 7.7260 \cdot 10^{-10} \cdot \mathrm{Re} \cdot a \quad (8)$$

In figure 3 the model and the data are shown and in figure 4 the residuals of the model are presented as differences between the computed and the observed values. It can be seen that the best fit of the multilinear regression model is obtained for the higher values of the flow rate estimation error, while for the lower values the model returns residuals with the same magnitude as the evaluated response. In figure 4 it can be seen that the regression model overestimates the flow rate error for the higher values and underestimates the lower values. Further, a more complex regression model could be developed, in order to correct this over-underestimation.

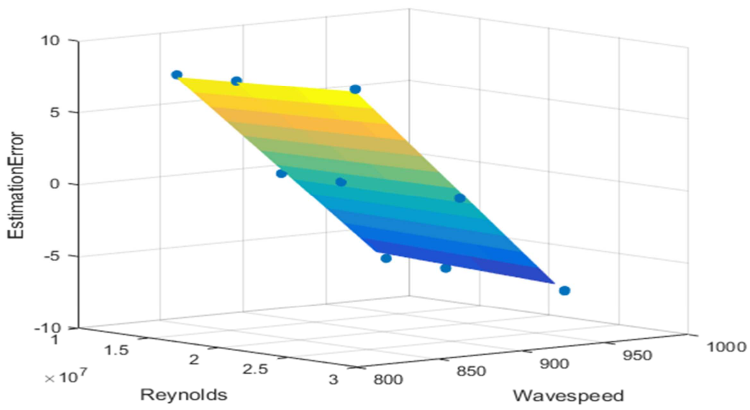

Fig. 3. Multilinear regression model with interaction and observed data.

In table 3 it can be seen that the influence of the pressure wave speed correct estimation is the most significant in the developed method accuracy, and it is confirmed in figure 3.

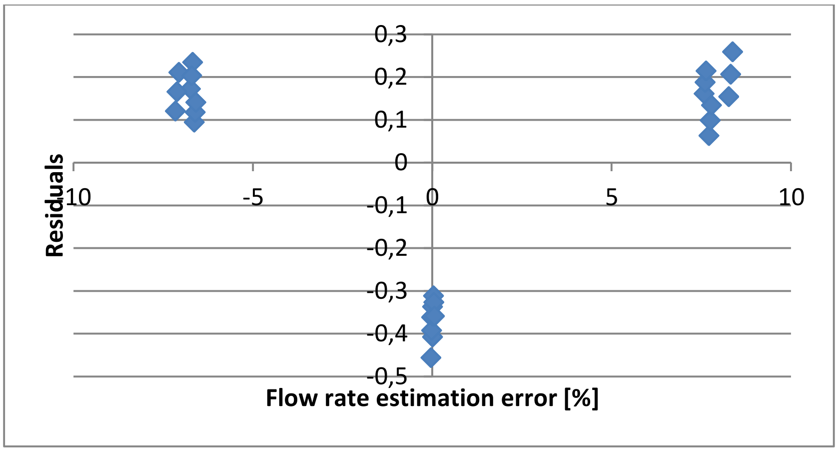

Fig. 4. Residuals, ε, between the model and the observed values.

5. Conclusions

The present paper presents the sensibility analysis of a new developed method for flow rate determination based on differential pressure measurements, as the pressure-time method. The focus is set on analysing the influence of three parameters used in the flow rate estimation method over the method accuracy. The analysed parameters are: the Reynolds number of the flow, *Re*, the pressure wave speed, *a*, and the measuring length, *L*, between two cross-sections used for pressure measurement.

As sensibility analysis method, a 3^3 factorial design is employed. First, in order to determine which parameter has a significant influence over the observed response a statistical analysis in made.

The result of the statistical analysis showed that only the Reynolds number, the pressure wave speed and their interaction influence the method accuracy. Thus, it was decided to determine a multilinear regression model with two parameters and with interaction, for flow rate estimation error (for the developed method's accuracy).

The model captures the tendency of the flow rate estimation error to vary in function of the analysed parameters, but still, the residuals shown in figure 4 are important compared to the desired method accuracy (lower than 0.1%).

Further another regression model should be determined to obtain a better fit for the lower values of flow rate estimation error. After, the regression models will be validated using laboratory data, obtained in the Waterpower Laboratory from the Norwegian University of Science and Technology, Norway.

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