Transient wall shear stress measurements at a high Reynolds number

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Abstract
Wall shear stress measurements utilizing hot-film anemometry in a high Reynolds number laboratory pipe flow $(Re = 1.7 \times 10^6)$ retarding to a complete rest have been presented. The purpose of the measurements was to investigate the level of unsteadiness in the wall shear stress during a Gibson (hydropower flow rate) measurement. The ensemble-averaged wall shear stress from 44 repetitions has been compared with the results obtained from both the 'standard', the 'unsteady' transient Gibson method, and a quasi-steady Gibson approach. The results show very small differences between the measured wall shear stress and the estimated values. The small deviations between the standard Gibson method and the measured values are encouraging. For, if transient modeling of the wall shear stress can be avoided, this would be beneficial because any empirical parameter entering a transient formulation must, inevitably, be experimentally calibrated at Reynolds numbers much smaller than those encountered in a typical Gibson measurement, thus introducing uncertainties regarding the extrapolation of such models to high Reynolds numbers. Albeit being a large Reynolds number in the lab scale, $Re = 1.7 \times 10^6$ is still one or two orders of magnitude smaller than that typically encountered in hydropower. Thus, direct transfer of the present result to a full scale measurement cannot be done without further studies. More detailed processing of the present data, as well as studies of five other sets of measurements are underway and will be presented elsewhere.

1. Introduction
The pressure-time method, also known as Gibson's method in memorial of the pioneering work by N.R. Gibson [1], is a cost-efficient and commonly utilized method to measure the flow rate in closed conduits. The principle of the method is derived by realizing that the pressure force acting on a decelerating enclosed volume of water is proportional to the rate of change of momentum and the frictional losses. By integrating this force balance, it is straightforward to show that the discharge, Q_0, before the commencement of the transient is given by

$$Q_0 = \frac{A}{\rho L} \int_0^t (\Delta p + \xi) dt + q, \tag{1}$$

where $A, \rho, L, \Delta p, \xi, q$ and t are the cross-sectional area, the fluid density, the measuring length, the differential pressure, the frictional losses, the discharge after the transient, and the upper integration limit, respectively.

The differential pressure is, in many instants, straightforward to measure with high accuracy when performing a pressure-time measurement in a full-scale hydropower plant. The frictional losses do, however, require modeling. In the 'standard' Gibson method described in IEC41 [2], ξ is calculated by

$$\xi = CQ(t)^2. \tag{2}$$

C is a constant loss-coefficient, extracted from the pre-transient pressure loss, and Q(t) is the instantaneous value of the time-varying flow rate. Jonsson et al. [3] found a systematic error of the calculated flow rate that was dependent on the initial Reynolds number. When the initial Reynolds number was increased from 0.6×10^6 to 1.7×10^6, the error decreased from positive to negative. Jonsson et al. [4] suggested that the discrepancy originated from an inaccurate formulation of the frictional losses (eq. 2), and to overcome this deficiency, the authors introduced an 'unsteady' Gibson method. In the unsteady formulation, the instantaneous value of ξ is estimated by calculating the value that would prevail if the flow was in equilibrium, obtained from the steady state friction factor, plus a contribution due to the unsteadiness of the flow, modeled by the time dependent part of the Brunone friction factor

$$f_b = \frac{kD}{U|U|}\frac{\partial U}{\partial t} \tag{3}$$

The convective part of the formulation has been neglected. D is the pipe diameter, and U is the instantaneous bulk flow velocity. The coefficient k is determined through an empirical relation (see Jonsson et al. [4], for details on the determination of k and the justification of neglecting convective effects). With the inclusion of unsteady friction, the systematic error was shown to decrease and the calculated flow rate error compared to the 'standard' Gibson approach decreased.

In the present paper, selected findings from wall shear stress measurements performed in a laboratory during a Gibson flow rate measurement are presented. The ensemble-averaged wall shear stress, obtained from many repeated runs, was measured using hot-film anemometry. The measured wall shear stress is compared with estimates from the 'standard', as well as, the 'unsteady' Gibson method. The objective of the work is to assess how the current means of modeling frictional losses in the Gibson method compare with direct wall shear stress measurements. Measurements of differential pressure have been performed as well; however, these results will show up only in the wall shear stress estimations. The effects of varying L and Q_0 have been described elsewhere (Jonsson et al. [3]), and will thus not be reconsidered in here.

2. Experimental apparatus

The experiments were performed in a 27 m long straight pipe of internal diameter D=300 mm. Water was supplied to the system from a 9.75 m high, constant-head tank, as shown schematically in figure 1. The flow rate retardation was realized by closing a computer-controlled, hydraulically driven knife gate. The gate closing-time was set to approximately 4 s, and only realizations that were repeatable within ± 0.15 s were kept for ensemble-averaging of the wall shear stress. The Reynolds number, based on the bulk velocity, pipe diameter and fluid viscosity, $Re = \frac{UD}{\nu}$, before the commencement of the retardation was approximately 1.7×10^6. Pressure measurements were performed 37D and 50D away from the inlet, respectively, whereas the wall shear stress was measured at 52D. The hydrodynamic entry length L_H can be estimated from the empirical relation $L_H/D = 1.359Re^{1/4}$ (Kasar et al. [5]), which takes a value of 49. The pressure and the wall shear stress are known to be flow quantities that develop quicker than, e.g., the turbulent fluctuations of the velocity (see He et al. [6]), and thus, the entry length in the present study is judged to be sufficient for inlet effects to be small.

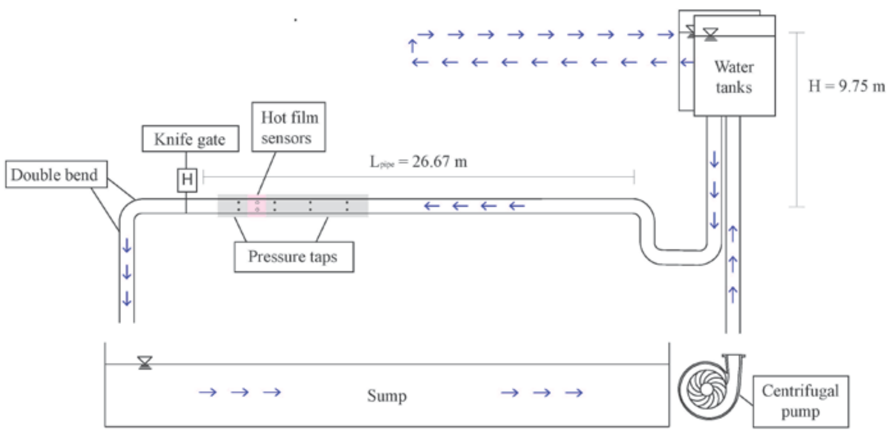

Figure 1. Schematic illustration of the experimental set up.

2.1 Measurement methods

For reference, the bulk flow rate was measured using a Krohne optiflux electromagnetic flow meter. Two UNIK 5000 absolute pressure sensors (0-5 bar) separated by a distance of L=4 m were used to measure the differential pressure. Wall shear stress measurements were performed by using three hot-film sensors. The hot-film sensors were mounted evenly around the circumference at one axial location. Calibration of the hot-film sensors were performed *in-situ* before and after the measurement by varying the Reynolds number between 1.7×10^6 and 0 in 12 steps. The flow rate at each Reynolds number was kept constant for 70 s, but only the data acquired during the last 50 s were used for the hot-film-voltage to wall shear stress calibration curve. At each Reynolds number, an estimate of the wall shear stress was obtained from the friction factor, *f*, the bulk flow velocity and the fluid density through $\tau = \frac{1}{8}\rho f U^2$. The friction factor was extracted from the empirical relation $\frac{1}{\sqrt{f}} = 1.901 \log_{10}(\mathrm{Re}\sqrt{f}) + 0.432$, suggested by Zagarola & Smits [7]. The expression for the friction factor has been derived from measurements over the range $31 \times 10^3 < \mathrm{Re} < 35 \times 10^6$, and is believed to be the most accurate representation of the friction factor. In the present setup, for $Re > 6 \times 10^5$, the friction factor calculated from the pressure drop was within 2% of the empirical formula. For $Re < 6 \times 10^5$, however, the differential pressure was small and thus difficult to measure with sufficient accuracy. The approach of calibrating the hot-film versus the pressure drop was, therefore, abandoned altogether (although this is more direct) to have a single calibration procedure over the entire range of Reynolds numbers. For $Re > 2 \times 10^5$ the hot-film voltages, *E*, aptly fitted the estimated wall shear stresses by a logarithmic expression, $\log_{10}(\tau) = AE + B$, where A and B are calibration constants. For $Re < 2 \times 10^5$, the 'standard' relation $\tau^{1/3} = A'E^2 + B'$ was used. In addition to flow rate, pressure and wall shear stress, the gate position and the water temperature were recorded. The signals were acquired simultaneously at a sampling rate of 2 kHz. The ensemble-averaged wall shear stress presented in the results section is from 44 repeated runs.

3. Results and discussion

3.1 Preliminaries

For $Re < 4.5 \times 10^4$, the tendency of the wall shear stress in a linearly decelerating flow to deviate from the quasi-steady value, i.e., the value that would prevail for a steady-state flow at the instantaneous Reynolds number, has been shown to depend on the relative importance of turbulence dynamics and inertia (see Ariyaratne et al. [8]). A quantitative measure of this tendency is derived from the ratio of the turbulence time scale, $\nu/u_{\tau 0}^2$, to the time scale of the inertia, $U_0\left(\frac{dU}{dt}\right)^{-1}$, i.e., through a non-dimensional parameter

$$\delta = \frac{\nu}{u_{\tau 0}^2}\frac{1}{U_0}\frac{dU}{dt}. \tag{4}$$

Where $u_{\tau 0}$ and U_0 are the friction and the bulk velocities before the commencement of the deceleration, and $\frac{dU}{dt}$ is the (constant) bulk flow deceleration. For $\delta < 10^{-3}$, the transient wall shear stress shows negligible deviations from quasi-steady values ($Re < 4.5 \times 10^4$). In the present work, the rate of deceleration is not constant, however, a 'mean' δ can be defined based on the closing time and the initial bulk velocity (which is known from the flow meter). This mean δ takes a value of 5.7×10^{-6}, and, if the results for low-Reynolds-number flows can be directly transferred to high-Reynolds-number flows, deviations from the quasi-steady values should be negligible.

3.2 Results

The time-developments of the ensemble-averaged mean wall shear stresses measured by the three hot-film sensors are plotted in figure 1a. Data is plotted from the instant that the gate starts to close, but, owing to the fact that the knife gate does not alter the flow rate until it has reached approximately 70% of its initial position, the wall shear stress remain unaltered until t > 1 s. The flow shows no tendency to develop asymmetrically in space, but large fluctuations are observed in the measured signals. The large fluctuations are a result of the high turbulence intensity close to the wall; it has been shown in numerous studies (see Alfredsson et al. [9], e.g.) that $\frac{\tau'_{rms}}{\tau} \approx 0.4$, in steady state. Owing to the circumferential symmetry of the wall shear stress, the mean value of the signals measured from the three sensors is employed for the results presented henceforth (which dampens the fluctuations as shown in figure 1b).

To judge whether the transient wall shear stress deviates from the estimated values, the instantaneous bulk velocity is required. The flow meter utilized in the present study is not suitable to extract this information, as the response time is too slow. The bulk velocity has therefore been approximated by the cumulative velocity distribution obtained from the integral in eq. 1.

Figure 2c-e displays the wall shear stresses estimated from a typical realization of a Gibson measurement together with the measured values. Initially, when the rate of deceleration is low, the measured and estimated values overlap. With time, the deceleration rate increases, and so does the flow unsteadiness. As a result, small discrepancies between the measured and estimated values are observed, especially for the quasi-steady values, see figure 2 e. However, it should be noted that the exact trend between the measured value and the various estimates differ somewhat between each realization, owing to the spread in closing time among the repetitions (±0.15s). The fact that the measured and estimated values do not differ appreciably is encouraging for the Gibson method. For, the results show that accurate estimates of the wall shear stress can be obtained without advanced modeling of the losses. Furthermore, the turbulence time scales in full scale hydropower are even smaller than those in the present study owing to the high Reynolds numbers. Thus, the effect of the imposed deceleration is expected to be even smaller in an in-site measurement, especially since the deceleration time is usually fairly long (>30s). The short time scale of the turbulence in comparison to the imposed deceleration is in favor of a quasi-steady approach for evaluating the losses (although this approach displays the worst agreement in figure 2). The quasi-steady formulation takes into account the increase in friction factor, as opposed to the standard approach, yet being simple because it incorporates available formulae for the friction factor (see, e.g., the expression of Zagarola and Smits [7]).

The good agreement between the measured and the estimated wall shear stresses are from a single case. If this result is a general trend must be investigated in detail by analyzing more data sets. Further measurements, incorporating different closing times, and closing curves, as well as a lower initial Reynolds number have been performed in conjunction to the present measurement. The processing of this data is under way, and is therefore not reported in here.

As a final remark, it should be noted that the hot-film measurements are not without errors. Uncertainty estimates presented by Sundstrom et al. [13] and He et al. [6], indicates that the uncertainty of unsteady wall shear stress measurements by hot-film anemometry are of the order of ±10%.

4. Conclusions

Wall shear stress measurements using hot-film anemometry have been presented for a transient turbulent pipe flow. An initially steady flow at a Reynolds number of 1.7×10^6 was brought to a complete rest by closing a knife gate over a time-period of 4 s. The ensemble-averaged mean wall shear stress calculated from 44 repeated runs has been compared with estimates obtained from a Gibson flow rate measurement. It has been shown that both the measured wall shear stress and the wall shear stress calculated with either the standard Gibson approach, the recently proposed ‘unsteady’ Gibson method, and a quasi-steady Gibson method all produce wall shear stress traces that are very similar. Such result is promising because it shows that the standard Gibson method, despite its simplicity, accurately predicts the losses in the evaluation procedure. Avoiding transient modeling of the wall shear stress in the Gibson flow rate evaluation is practical, because the empirical parameters entering such formulations can rarely be tuned at the high Reynolds numbers encountered in typical hydropower measurements. However, the direct applicability of the present results to a full scale measurement should be imposed with caution because the Reynolds number, albeit large in the lab scale, is still one or two orders of magnitude smaller than that encountered in a hydropower plant. Thus, the generality of the results cannot be guaranteed. Further measurements with different closing times, and closing sequences, as well as a different initial Reynolds number have been undertaken in conjunction to the currently presented measurements. The results from these measurements are, however, still under processing and will be presented elsewhere.

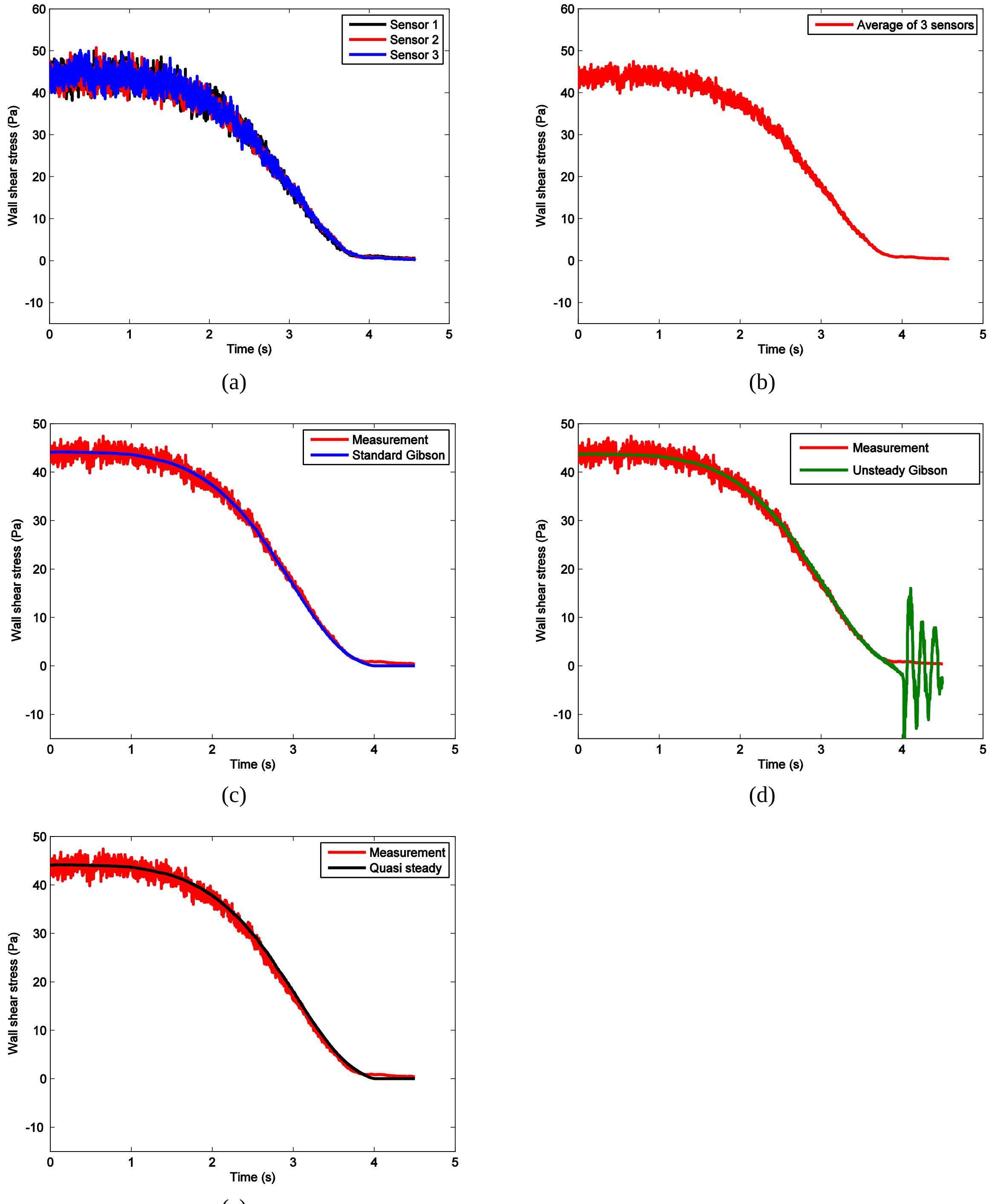

Figure 2. Time-development of the wall shear stress during gate closure. a) and b), measurements performed at three circumferential positions and the average of these measurements, respectively; c) standard Gibson; d), 'unsteady' Gibson; e) quasi steady.

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