Volumetric gauging method vs Gibson method - comparative analysis of the measurement results of discharge through pomp-turbine in both operation modes

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Abstract

The measurements of discharge through a reversible machine were carried out using both the volumetric gauging method and the pressure-time method (the Gibson method) for turbine and pump operation modes.

Accurate measurements of the water level in the head water reservoir of the pump-storage power plant and the dedicated solution of taking into account waves on the water surface in that reservoir was used during the discharge measurements by means of the volumetric gauging method.

Instead of a classic version, based on a straight segment of the penstock with constant diameter, the Gibson method was used in a version in which the entire penstock with geometrical irregularities, such as bends, cones and bifurcation are taken into consideration. The impact of these irregularities on the results of discharge measurements has been taken into account by utilizing special calculation procedure based on CFD analysis.

The solutions implemented to the both methods presented in the paper are discussed basing on the comparison of the discharge measurement results obtained using these methods. Very high rate of convergence between these results were obtained for turbine mode of operation while for pump mode of operation the differences were slightly larger, however they were still within the range of measurement uncertainties. The most probable reasons for these results are presented in the paper.

Introduction

The volumetric gauging method is not very often used in hydropower plants, mainly because of lack of artificial reservoirs with precisely defined geometric characteristics. Usually only the pump-storage power plants are equipped with such reservoirs. In turn, just in these power plants it is possible to use the volumetric gauging method for determining the characteristics of their pump-turbines. Furthermore, and that comes naturally, in some favorable circumstances, such conditions make it possible to compare the volumetric gauging method with other methods having more factors affecting the measurement uncertainty.

In this paper the Authors present the comparison of the flow rate measurement results obtained using the volumetric gauging method and the pressure-time method (the Gibson method) in both operational modes of pump-turbine (two directions of reversible unit rotation). Comparative analysis between these two methods for pumping mode of operation is of great importance because the Gibson method currently is not recommended by the relevant international standards to use it for field acceptance tests of hydrounits.

The Authors of the paper present the examples of utilizing both of mentioned methods that were gathered during several years of own experience and used together with developed original solutions based on some new metrological elements.

The paper presents the solution used for accurate measuring the water level in the head water reservoir of the plant in the volumetric gauging method as well as the ways for taking into account waves on the water surface in that reservoir and other factors that without such solutions cause much wider range of uncertainty of the discharge measurements realized by means of this method. The standard way of measuring the head water level does not provide sufficient accuracy of this method.

In a case presented in the paper, the Gibson method is used in a version based on the entire penstock – from the head water reservoir to the inlet cross-section of the spiral case of the pump-turbine. If it is not possible to use the classic version of the Gibson method (based on a straight segment of the penstock with constant diameter) because some additional issues influencing the measuring results have to be taken into account, such as the change in diameter of the penstock along its length as well as bends, bifurcations, etc. The Authors of the paper present a solution to reduce uncertainty of this version of Gibson method used for measuring discharge in such conditions by using CFD analyzes for calculating of flow conditions in curved, conical and branched pipelines. The results of this analysis allow introducing appropriate adjustments to the measurement procedure used with the Gibson method.

The object of application of the flow measurement methods

The above-mentioned methods for flow measurement were used for tests on a reversible machine in one of the pumped-storage power plants (PSPP) in Poland. The considered PSPP is equipped with four Francis-type reversible hydraulic machines, with an installed capacity of about 140 MW each at a head of approximately 440 m.

Water from the artificial head water reservoir is delivered to pump-turbines using two underground pipelines, branching close to the intakes of the pump-turbines, prior to the shut-off ball-valves. Water behind draft tubes of the pump-turbines is flowing through outflow adit connected to a surge tank and further to the tail water reservoir. Diagram of the inlet and outlet of the PSPP with the main dimensions is shown in Fig. 1.

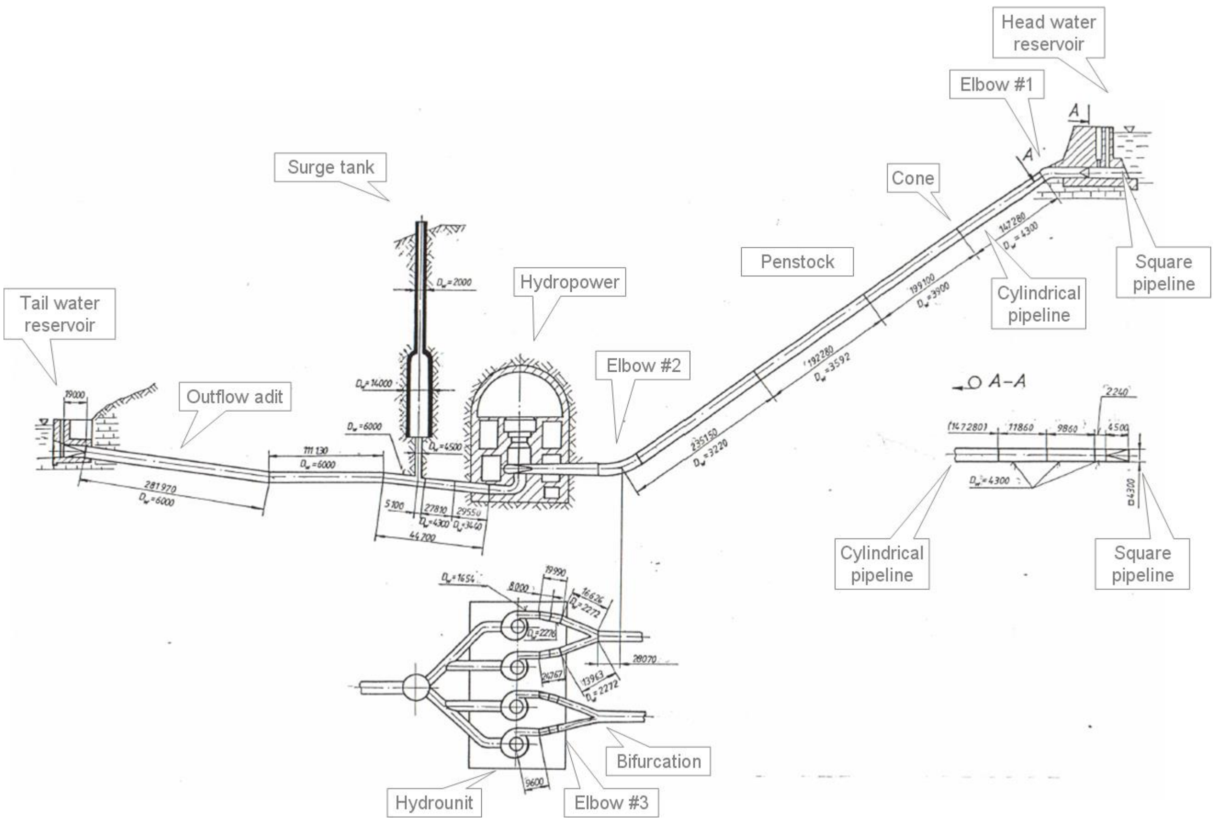

Fig. 1 A schematic view of the tested pump-turbine flow system.

The application of the volumetric gauging method

The measurement of the flow rate using volumetric gauging method relies on the determination of increase or decrease of water volume in the head water reservoir basing on direct measurement of water level changes in this reservoir referred to known relationship between water level and the water volume in the reservoir $V(z)$. Therefore, the flow rate measured by this method is given by:

$$Q = \frac{\Delta V}{\Delta t} = \frac{V[z(t_f)] - V[z(t_0)]}{t_f - t_0} \qquad (1)$$

where: ΔV [m^3] is measured increase or decrease of water volume in the head water reservoir, $\Delta t = t_f - t_0$ [s] - the time at which the increase/decrease occurred and z - water level in the reservoir.

Some of the Authors' experiences concerning application of the volumetric gauging method allowed indicating a few important issues relating to the accuracy of the relationship between water level and reservoir volume and the accuracy of measurement of water level increase (Adamkowski, 2001 and 2012, Adamkowski et al., 2006). Volume of the reservoir should be determined by precise standard geometrical measurement or by photogrammetry. In practice, the required accuracy of such measurement can be only obtained for the artificial reservoirs. Water level cannot be accurately measured using the instruments commonly used in the control system of the plants because usually they have much too high range and much too low class precision. Additionally also various disturbing effects do not ensure the required accuracy of the measurements. Therefore the change in water level has to be determined using the special method.

An effective technique for precise measurement of water level increase was successfully applied during the efficiency tests with the aid of the volumetric gauging method in the Polish PSPP including the case presented in this paper - Fig. 2. Water level increase Δz is determined by measuring the difference between pressure in the water reservoir of the plant and the constant pressure in the auxiliary tank underhung at constant height during the tests. For this purpose, the differential pressure transducer of high precision class has to be connected to the plant reservoir and auxiliary tank.

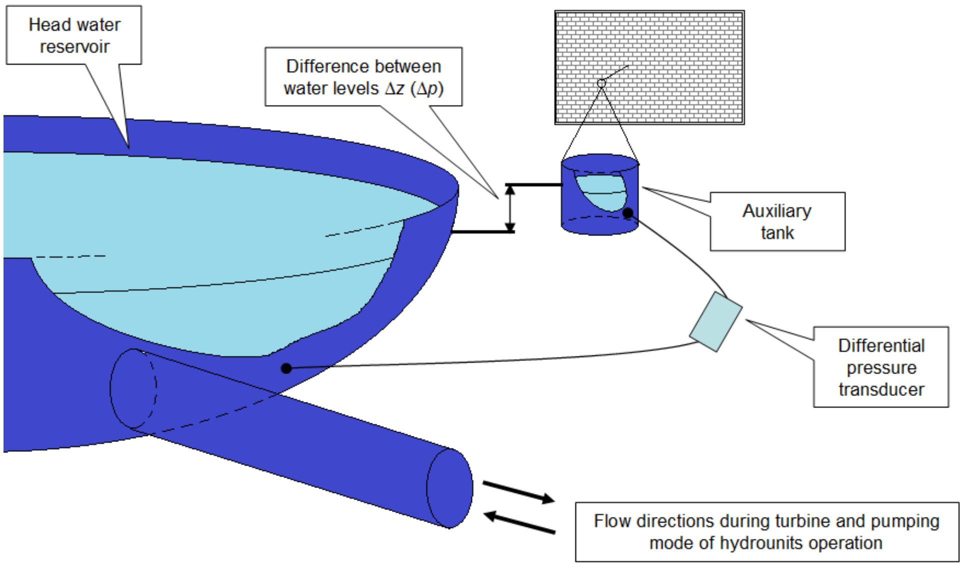

Fig. 2 Technique of water level increase measurement applied in the volumetric gauging method in a considered case (own application).

Wave motion of water surface in the reservoir is one of factors that certainly should be taken into consideration as the effect most affecting the measurement results. Properly prepared the computer data acquisition system and the regression line applied to the recorded water level values should allow eliminating the water waves disadvantageous effect – Fig. 3. Traditional readings of water level utilized in such kind of method do not give any chance to get the required accuracy of discharge measurements.

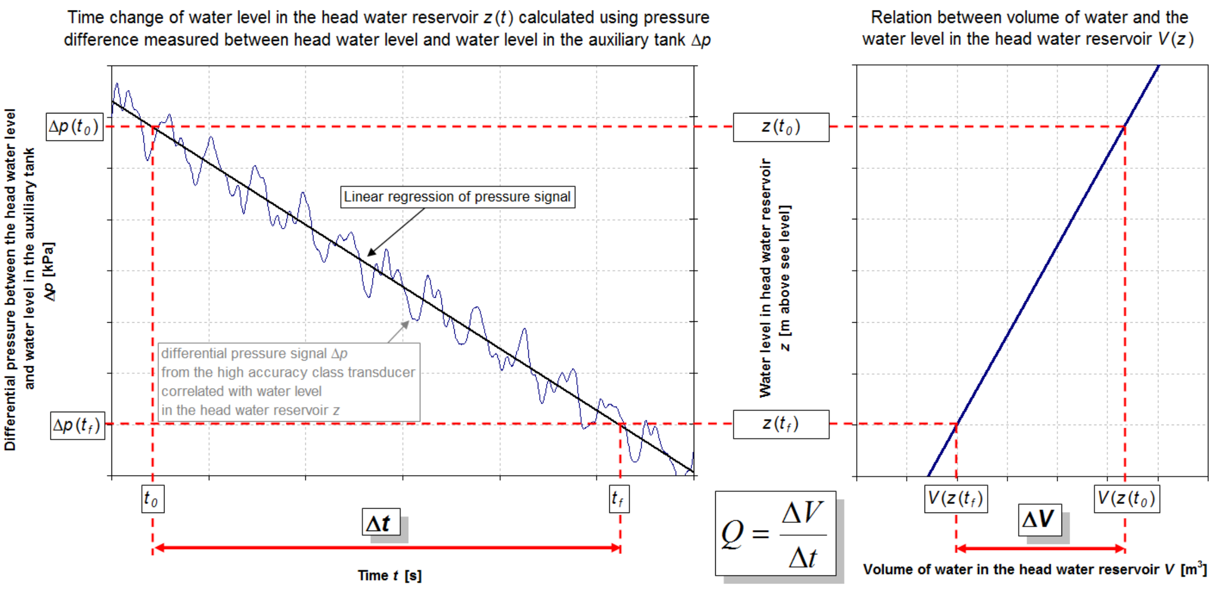

Fig. 3 Determining the flow rate using the volumetric gauging method.

Thanks to the solutions applied, the systematic uncertainty of flow rate measurement results obtained using the volumetric gauging method used in the case discussed in this paper was estimated on approximately +/-0.6%.

The application of the Gibson method

Basic information

The pressure-time method (the Gibson method) for measuring flow rate through hydraulic machines relies on shutting-off the flow through the machine and calculating initial value of flow rate (in conditions before shutting-off the flow) basing on time-changes of the pressure difference measured between chosen measuring cross sections located on the penstock segment. In detail, the value of Q that stands for flow rate at initial conditions is obtained by integrating equation (2) within time interval at which the flow velocity is changing from initial conditions to the conditions after the flow shut-off (Adamkowski 2012 ASME PTC 18-2002, IEC 41: 1991, IEC 62006: 2011):

$$Q = \frac{1}{\rho F} \int_{t_0}^{t_f} \left(\Delta p(t) + \Delta p_d(t) + \Delta P_r(t) \right) dt + Q_f \qquad (2)$$

where:

- ρ is the density of the flowing liquid, t_0 and t_f are the lower (initial) and upper (final) time-limits of integration, respectively, Q_f is the discharge under final steady-state conditions (after complete closing of the shut-off device) due to the leakage through the closed shut-off device, Δp is the pressure difference measured between the penstock measuring cross sections *B-B* i *A-A*, which geometrical center is at level z_A and z_B, respectively (Fig. 4):

$$\Delta p = p_B + \rho g z_B - p_A - \rho g z_A,$$

Δp_d is the dynamic (velocity) pressure difference between the penstock measuring cross sections with area of each section equal A_A and A_B:

$$\Delta p_d = \frac{\rho Q^2}{2A_B^2} - \frac{\rho Q^2}{2A_A^2}$$

ΔP_r is the pressure drop between the measuring cross sections caused by hydraulic resistance calculated as directly proportional to the square of flow rate and friction losses coefficient C_r:

$$\Delta P_r = C_r Q|Q|$$

The geometrical factor *F* (Eq. 2) for a pipeline segment of length *L*, consisting of *J* sub-segments with different dimensions can be calculated using the following formula:

$$F = \int_0^L \frac{dx}{A(x)} = \sum_{j=1}^{j=L} \frac{\Delta x_j}{A_j} \text{ , with } \qquad \sum_{j=1}^{j=J} \Delta x_j = L \qquad (3)$$

with Δx_j and A_j denoting the length and internal cross-sectional area of the *j*-th sub-segment, respectively.

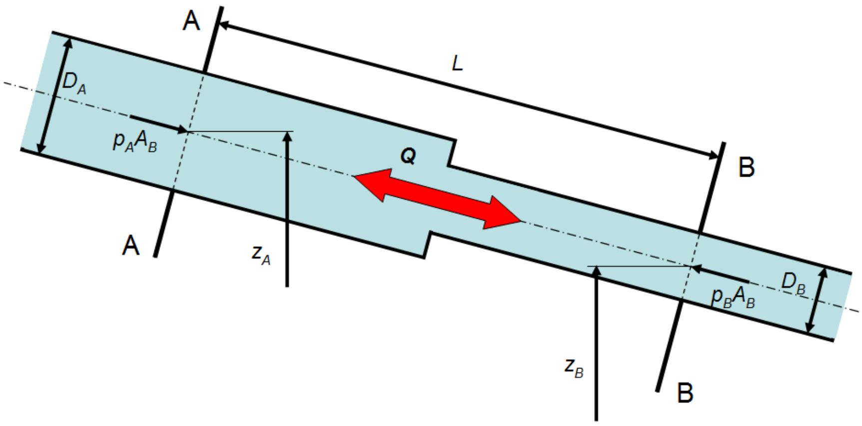

Fig. 4 Segment of the pipeline with markings.

As it is shown in equation (2) the pressure drop ΔP_r representing hydraulic losses in the pipeline segment and the dynamic pressure difference Δp_d between the measuring sections of the pipeline have to be separated from the measured changes in differential pressure between measuring cross-sections of the pipeline Δp. Remained differential pressure is associated with the forces of inertia of the liquid between measuring cross-sections of the pipeline. The values of ΔP_r and Δp_d can be calculated with good accuracy using their dependence on the square of the flow rate - see above.

In the presented case version of the Gibson method with one measuring cross-section was used for the flow rate measurement - this version is based on measuring the pressure variation in a single cross-section of the pipeline and relating these changes to the pressure exerted by the water column in an open reservoir, to which the pipeline is directly connected. For this version the geometric factor *F* has to be determined considering the entire penstock of the tested machine, starting from the inlet penstock section (*A-A*) and ending at the cross section of the spiral case inlet (*B-B*) – for this purpose four pressure taps were installed in the cross-section *B-B* and connected by impulse tubes to the manifold and a pressure transducer.

In the present case the geometry of the pipeline is quite complex. Except for the straight pipe sections with constant internal diameters the penstock consists of three elbows (two vertical and one horizontal), a number of short conical sections connecting segments of different diameters and two short branches, where one branch was remained closed during tests. In addition, in the penstock entrance area there is an inlet to the segment of the penstock with a square cross-section and further a transition section from square to the circular cross-section. These irregular penstock elements cause disturbances of flow in the form of irregular distribution of flow velocity, which should be considered for better accuracy of flow measurement. In the published work (Adamkowski et al. 2009) Authors present the relevant procedure, based on CFD, used for correction of the factor *F* for pipelines with elbows. The basis for this procedure was, except mass conservation, the assumption of equal kinetic energy resulting from the simulated and the uniform flow velocity distribution. A similar procedure, presented in details in Appendix, after appropriate extension, was used in this work to correct value of the *F* in accordance to the mentioned irregular pipeline components. The most important results of CFD calculations and adjustments to the value of *F* due to the influence of these irregular elements are presented in the following section of this paper.

Computational grid, CFD flow simulation and geometrical factor corrections

Due to its large length the analyzed pipeline was divided into three parts that are presented in Fig. 5 together with the calculated distribution of flow velocity. Solution based on such division of the calculation domain definitely shortened the time of calculations and it had insignificant effect on the calculated velocity distribution.

Commercial software *NUMECA/Hexpress* v.4 was used for computational grid generation representing the penstock geometry. The unstructured grids consisted with hexahedral elements of amounts as follows: 16M for upper part of the penstock (containing reservoir), 13.5M for its middle part (containing conical pipe segment) and 13M for lower part of the penstock (containing bifurcation and branches).

The flow boundary conditions were assumed from the operating parameters of the tested pump-turbine for both turbine and pump modes of operation. For the turbine mode of operation four values of discharge (20, 25, 30 and 35 m^3/s) and for the pump mode of operation two values of discharge (26 and 28 m^3/s) were used for computations.

The flow calculations were carried out by means of *ANSYS/Fluent v.15* commercial software. The flow was simulated by solving the steady-state RANS equations (Second Order Upwind Discretization). For closure of the flow equations system the k-ω Shear Stress Transport (*SST*) turbulence model was chosen.

The sample of CFD calculations results in the form of velocity contours in cross-sections for three analyzed flow parts of the penstock were presented in Figs 6, 7 and 8 for both flow directions. The results are presented for maximum flow rates analyzed that means 35 m^3/s for turbine regime and 28 m^3/s for pump regime.

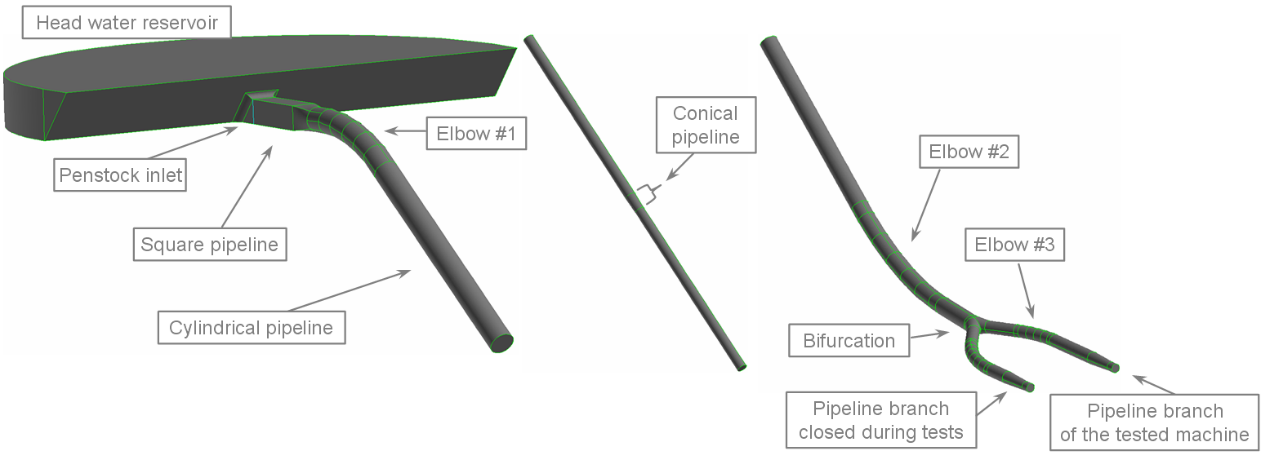

Fig. 5 Geometries of the analyzed flow domains (flow parts):
left view: upper part of the penstock and head water reservoir (reservoir→square pipeline (4.3x4.3m)→cylindrical pipeline (4.3m)), *middle view*: middle part of pipeline (cylindrical pipeline (4.3m)→conical pipeline→cylindrical pipeline (3.9m)), *right view*: lower part of pipeline (cylindrical pipeline (3.2m)→bifurcation to two pumpturbines (2.276m)→conical pipeline→cylindrical pipeline (1.654m)).

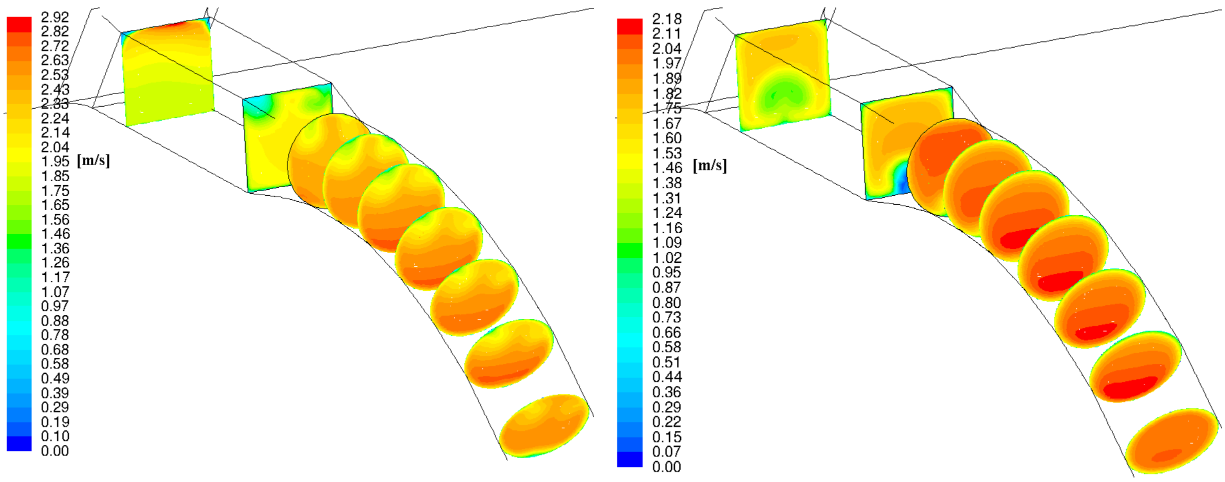

Fig. 6 The velocity distributions in the penstock inlet with first elbow for flow rate in turbine regime Q = 35 m^3/s *(left view)* and for flow rate in pump regime Q = 28 m^3/s *(right view)*.

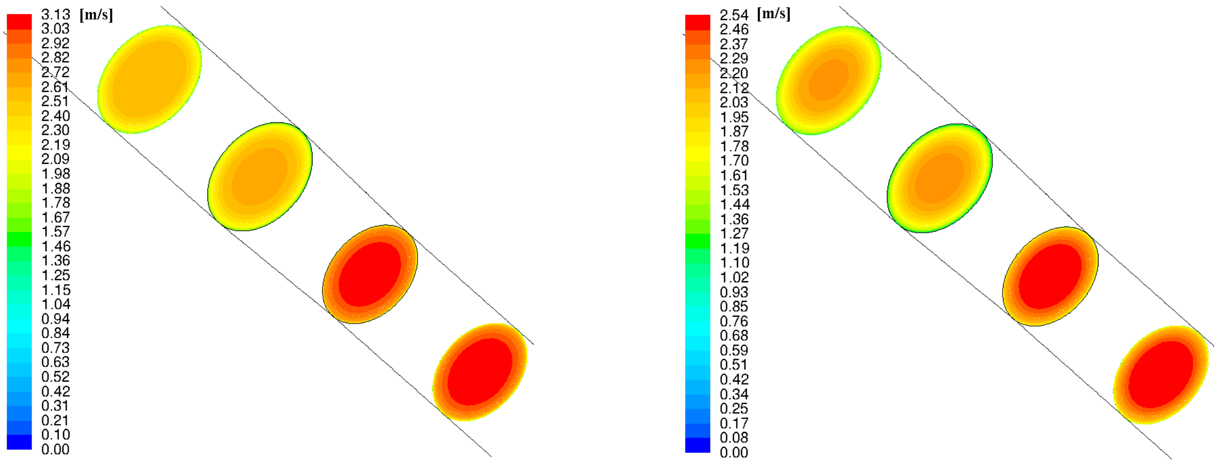

Fig. 7 The velocity distributions in the penstock containing cone for flow rate in turbine regime Q = 35 m^3/s *(left view)* and for flow rate in pump regime Q = 28 m^3/s *(right view)*.

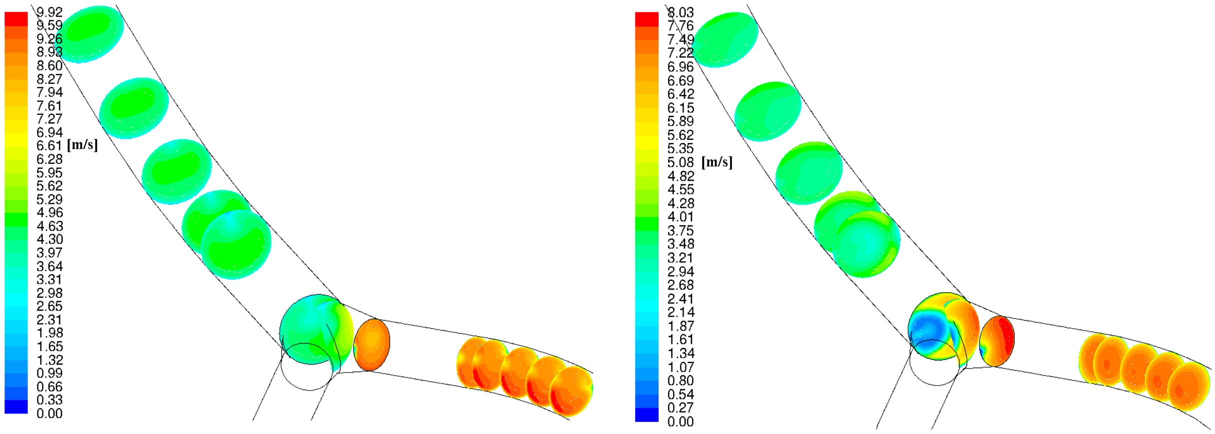

Fig. 8 The velocity distributions in the part of penstock containing bifurcation for flow rate in turbine regime Q = 35 m^3/s *(left view)* and for flow rate in pump regime Q = 28 m^3/s *(right view)*.

In Figs. 6÷8 the simulation results obtained for analyzed flow parts can be generally characterized as follows:

- Inflow or outflow of the penstock enforces a small liquid movement in the immediately adjoining area of the head water reservoir but the shape and extent of this area and the inside velocity distribution is different for turbine and pump modes of operation.
- The greatest flow disturbances are introduced in the bifurcation of the penstock, but it has only a local range. Despite the facts that it influences velocity distribution only locally the propagation of these disturbances in the direction of flow is clearly visible. The intensity of the disturbances decreases rapidly with distance. On the other hand, the least disturbances of the flow in the penstock are introduced by existing short conical pipe segments.
- The velocity distribution at elbows also differs depending on the direction of flow, which is quite obvious. For example, for turbine mode of operation the flow achieving the elbow #2 is almost uniform because of the long straight section of pipe before (looking in turbine flow direction), while in pump mode of operation similar effect takes place in the elbow #1. The elbows introduces disturbance in the flow pattern, which propagates to the next penstock elements with decreasing intensity.

The CFD results that gave the view about all flow disturbances introduced in the penstock were used to calculate the equivalent factor F_e according to the steps 4÷7 of the original procedure presented in Appendix.

In order to present the results, a deviation factor Δf was introduced. This represents a relative difference between the equivalent factor F_e (obtained by means of CFD calculations) and the geometrical factor F, calculated as follows:

$$\Delta f = \frac{F_e - F}{F}. \tag{4}$$

The values of quantity Δf determined for chosen discharge values for both flow directions is presented in Tab. 1. It can be stated that for both, turbine and pump flow directions the values of Δf are kept almost constant, however they are of different level for both considered flow directions. The average value of Δf is about +0.13% and about +0.77% for turbine and pump modes of operation, respectively. These values were used as correction ones of the geometrical factors F calculated basing only on the geometry of the penstock between *A-A* and *B-B* cross-sections.

Tab. 1 The values of deviation factor Δf for entire penstock determined for the assumed flow rates in both operation modes.

Operation mode	Flow rate Q_0	Deviation factor Δf
-	m^3/s	%
Turbine	20	0.15
Turbine	25	0.14
Turbine	30	0.13
Turbine	35	0.11
Pump	26	0.77
Pump	28	0.77

The flow rate was calculated using the own computer programme *GIB-ADAM* that was numerously tested and verified throughout years of its usage. Calculation were based on the factor F_e and the pressure difference measured between the inlet section of the spiral case (*B-B*) and the head water reservoir (cross-section (*A-A*) at the inlet of the penstock). Examples of measured values and calculation results for both pump-turbine modes of operation under investigation are shown in Fig.9[1].

The systematic uncertainty of the flow rate measurement results obtained using the Gibson method in that case was estimated to be about +/-1.3%.

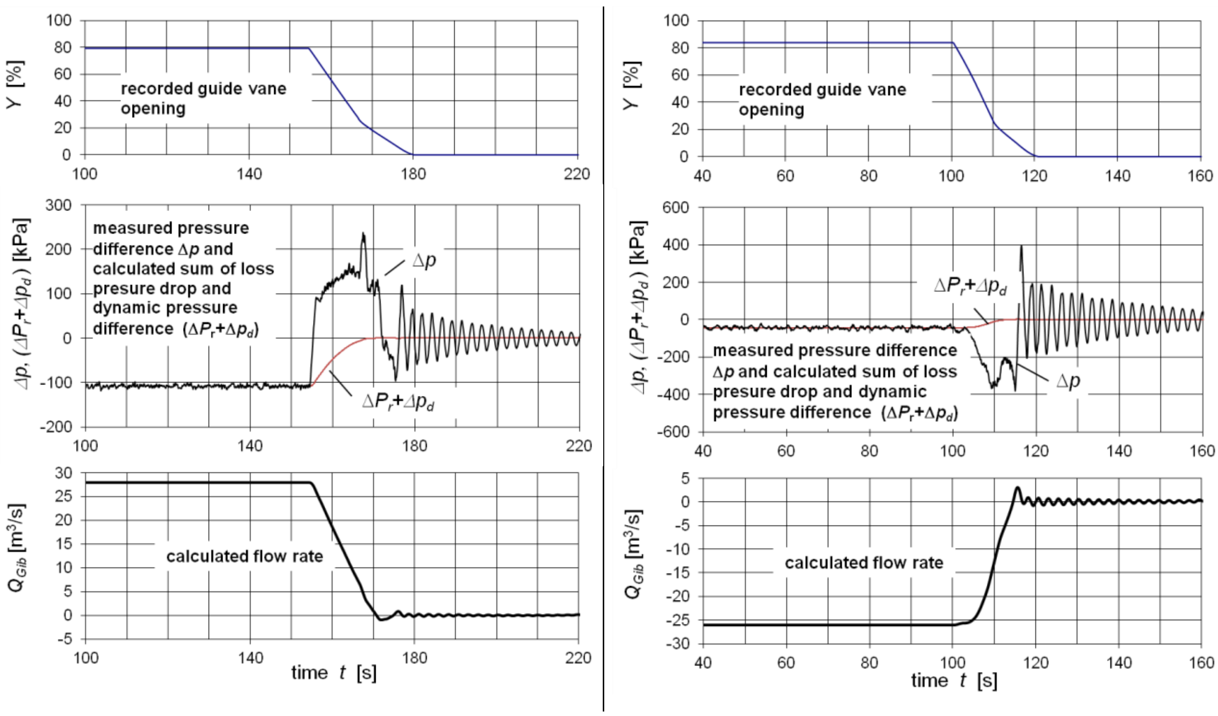

Fig. 9 Examples of measured values and calculated results of flow rate using the Gibson method - *left view*: turbine regime, *right view*: pump regime.

[1] Prior to these calculations the leakage flow rates Q_f in the conditions of closed wicket gates of the machine under tests were needed to be determined. These leakages were calculated using the pressure in the pipeline measured in the cross section near the shut-off ball-valve - for this purpose pressure changes during closing the ball-valve in conditions of closed wicket gates were registered. On this basis, and assuming that the ball-valve was perfectly leak-tight the leakages through closed wicket gates were evaluated.

Comparison of methods

Turbine mode of operation

Since it was not possible to measure flow using both methods (volumetric gauging method and Gibson method) simultaneously, the comparison of the measuring results for turbine mode of operation of tested machine was carried out using Winter-Kennedy method. According to this method that is commonly used in practice, flow measurement is based on the following relation between the volumetric flow rate Q and the pressure difference Δp_{wk} between the outer and the inner side of a spiral case of the tested machine:

$$Q = k\ \Delta p_{wk}^{n}\ , \tag{5}$$

where k is a constant coefficient determined experimentally in calibration process, and n - exponent theoretically equal to 0.5. Such a value of the exponent n was assumed for purposes of comparison. The coefficient k was determined independently on the basis of flow measurement obtained using the volumetric gauging method and the Gibson method and it is shown in Fig.10 for the tested machine. This comparison shows that the difference between values of coefficients k obtained using these two different methods is very small - it amounts up to only about 0.2%.

It should be noted that when the geometrical factor F in the Gibson method is not corrected then this difference is a little larger and amounts to about 0.33% for the tested machine. Although in the considered case this difference is not large, however, considering the various geometries of penstocks in practice, it is recommended to support the Gibson method by using CFD calculations in cases of penstock measuring segments with irregular elements causing flow disturbances.

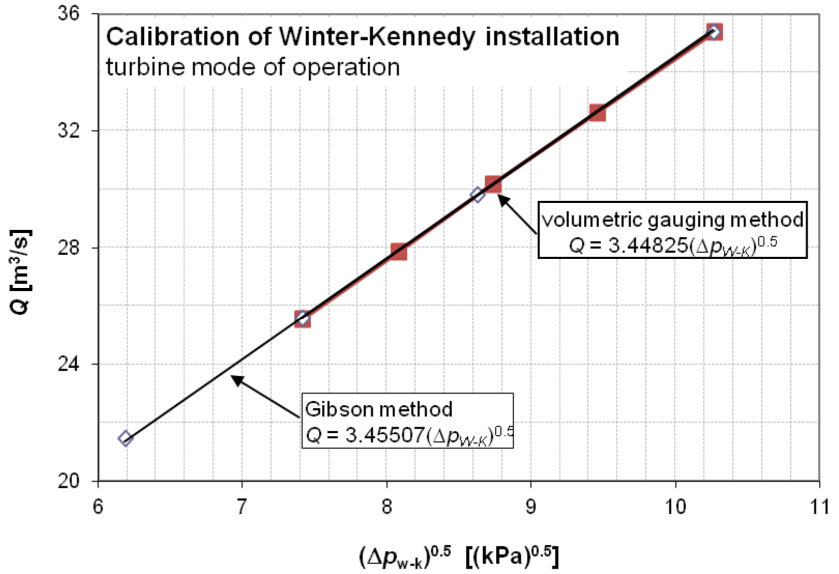

Fig. 10 Comparison of the volumetric gauging and Gibson method based on the results of the calibration of flow measuring systems used with Winter-Kennedy method installed on the tested reversible machine operated in turbine mode of operation

Pumping mode of operation

The Winter-Kennedy method is not recommended for using in pumping mode of operation of the hydraulic machines. Therefore, due to the lack of the possibility of simultaneous flow measurement using analyzed methods and the inability of regulation of the tested machine in this mode, comparison of the flow rate measurement results was made with respect to the gross head of the power plant - Fig.11. The differences between the flow rate results obtained from the volumetric gauging method and Gibson method were from -0.16% for lower head (426 m) to +0.58% for higher head (439 m). Without *F* correction the differences were more significant – their values were +0.6% and +1.35%, respectively.

These differences are much greater than those shown for turbine mode of operation of this machine, although they are still within the measurement uncertainty range characterizing compared methods. It should also be noted that the measurement of the flow rate through the hydraulic machines using the Gibson method is much more difficult to execute for pumping than for turbine mode of operation, and it may be the main reason why the current standards do not recommend using this method in the pump regime conditions of tested machines.

At this stage of the study it can not be clearly demonstrated what the cause of larger differences is for pumping mode of operation with comparing to the turbine regime. However, there are some significant differences between these conditions that can affect the results:

- Changes of the measured pressure difference is much less regular during the stopping of the pump than during flow shut-off during turbine mode of operation[2];
- During stopping the pump regime there is a higher short-term change of liquid flow direction - from pump to turbine direction;
- Pump regime, in contrast to the turbine regime, generates pressure pulsations with of a much higher level, which propagates along the penstock.

It is recommended to carry out thorough, professional investigation and analysis of these differences in order to determine their impact on the accuracy of flow measurement by means of the pressure-time method in pumping mode of operation of the hydraulic machines.

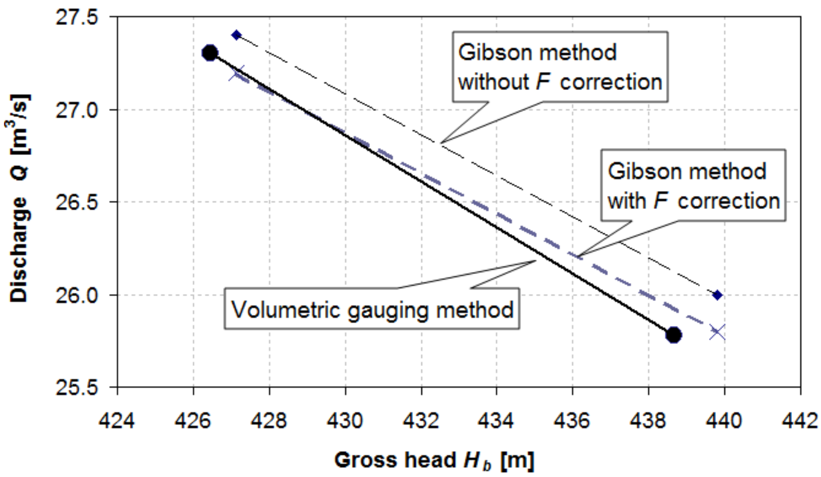

Fig. 11 Pumping mode of operation of the tested hydrounit.
Comparison of the results obtained using the volumetric gauging method and the Gibson method - relationships between the measured flow rate and the gross head.

[2] Flow rate measurement using Gibson method for turbine mode of operation was mostly made by shutting off the flow through the turbine (closing wicket gates) with the generator connected to the network during the period of test. For pumping mode of operation of the large reversible machines complete flow cut-off with the motor connected to the grid is impermissible.

Conclusions

The paper presents authors' own selected experience concerning the application of the volumetric gauging method and Gibson method complemented with developed original solutions based on some new metrological elements.
When measuring the flow rate by means of the volumetric gauging method the waves of water surface in the reservoir can be compensated by appropriate usage of a linear regression function to the measurement results of water level changes. Additionally, using a special system for measuring differential pressure can give increased accuracy of this method when measuring the increase or decrease of water level in the reservoir in comparison to the standard solution of water level measurements.
When measuring the flow rate by means of the Gibson method using version based on the entire penstock characterized by relatively complex geometry (starting from the head water reservoir to the inlet cross section of the pump-turbine spiral case), supporting calculations with CFD analysis of flow conditions in parts of the irregular pipeline (curves, branches, conical elements and inlets) can significantly reduce uncertainty of this method. The paper presents the way of utilizing the results of CFD analysis for correcting the geometrical factor F - own original calculation procedure was used for this purpose.
Very good convergence rate between the results obtained using compared methods for turbine mode of operation are clearly visible, while the larger differences between respective results are observed for pumping mode of operation. At the current stage of research there are no clear explanations of such observations, but Authors of this paper point out differences between the flow conditions for turbine and pumping modes of operation that may influence discussed results. Nevertheless there is substantial need for further research to improve the accuracy of flow measurement using the pressure-time method under conditions of pumping mode of operation of hydraulic machines. However, it should be stressed that all reported differences between compared results are within the range of uncertainty of both analyzed methods.

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Appendix: Procedure for calculating equivalent values of geometrical factor *F* in the Gibson method for irregular segments of the pipeline on the basis of the CFD analysis

The value of the geometrical factor *F*, as determined from Eq. 3, is generally correct for a straight pipeline segment with no flow irregularities. It does not account for changing the velocity profiles in an irregular pipe flow elements, such as elbows, bifurcations, cones, pipe inlets, etc. Therefore, the authors of this paper recommend using special procedure of calculation in order to take into account the influence of the irregular shape of a considered flow element on the pressure-time method results.
The following procedure is an extension of the procedure for the curved sections of the pipeline published in Adamkowski et al. 2009.

Step 1: Determine the boundary conditions (geometry of the considered pipeline flow system, discharge Q_j, etc.) and the computational control space – see Fig. A1.

Step 2: Divide the computational control flow space into *n* numerical segments using cross-sections normal to the axis of the considered *i*-th ($i = 1, 2,...,n$) pipe segment.

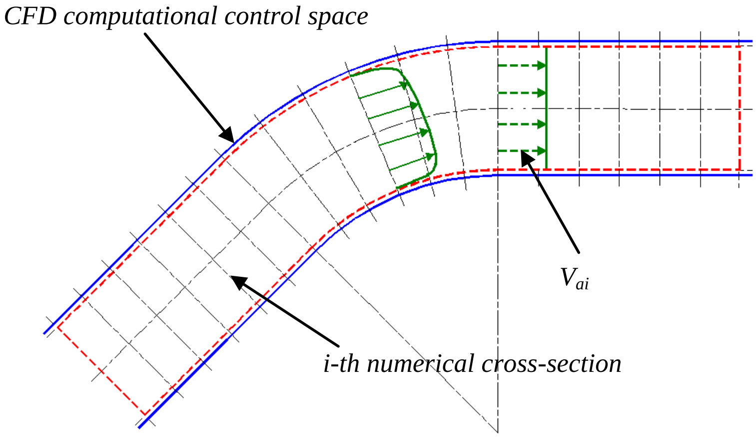

Fig. A1 A pipeline elbow with marked computational space.

Step 3: Simulate velocity distributions (velocity field $V(x,y,z)$) in the flow elements of the considered pipeline within the frame of the computational control space by means of a relevant *CFD* computer software (for instance *Fluent*™).

Step 4: Compute average values of flow velocity V_{ai} for each *i-th* numerical cross-section from the previously derived *CFD* results (step 3) and the assumption of equal kinetic energy resulting from the simulated and the uniform flow velocity distribution:

$$e_{k\,CFD\,i} = e_{k\,a\,i}\,; \quad \rho = const. \tag{A1}$$

$$\dot{e}_{k\,CFD\,i} = \frac{1}{\dot{m}}\iint_{A_i}\frac{1}{2}V_i^2\left[\rho V_i\, dA\right] = \frac{\rho}{2\dot{m}}\iint_{A_i}V_i^3\, dA; \quad \dot{m} = \rho V_{ai}\, A_i \tag{A2}$$

$$\dot{e}_{k\,a\,i} = \frac{1}{2}\, V_{ai}^2 = \frac{1}{2\dot{m}}\rho\, A_i\, V_{ai}^3 \tag{A3}$$

$$\Downarrow$$

$$V_{ai} = \left(\frac{\iint_{A_i} V_i^3 dA}{A_i}\right)^{1/3} \tag{A4}$$

where V_i denotes the flow velocity axial component (perpendicular to the *i-th* cross-section).

Step 5: Using the continuity equation $Q_j = V_{ai}\, A_{ei} = const$, compute the equivalent value of cross-sectional area A_{ei} for each numerical cross-section ($i = 1, 2, ..., n$):

$$A_{ei} = \frac{Q_j}{V_{ai}}, \quad i = 1, 2, ..., n \tag{A5}$$

Step 6: Compute coordinates of flow velocity centers in all chosen i-th numerical cross-sections, $i = 1, 2, ..., n$:

$$x_{Ci} = \frac{\iint_{A_i} xV(x,y,z)dA}{V_{ai}A_{ei}}, \qquad y_{Ci} = \frac{\iint_{A_i} yV(x,y,z)dA}{V_{ai}A_{ei}}, \qquad z_{Ci} = \frac{\iint_{A_i} zV(x,y,z)dA}{V_{ai}A_{ei}} \tag{A6}$$

Step 7: For the considered flow rate Q_j through the analyzed pipe element, compute the equivalent value of the factor F_{eQj} from the following formula:

$$F_{eQj} = \sum_{i=1}^{n-1} \frac{l_{i\to i+1}}{0.5(A_{e\,i} + A_{e\,i+1})} \tag{A7}$$

where $l_{i\to i+1}$ is the distance between the resultant velocity centers for computational sections i and $i+1$, A_{ei} and $A_{e\ i+1}$ – equivalent areas of computational cross sections i and $i+1$, respectively.

The above computation should be conducted for several discharge values (Q_j, $j = 1, 2, ..., m$) from the whole scope of its variation ($Q_{\min} < Q_j \leq Q_{\max}$). The average value of equivalent factor F_e is calculated as follows:

$$F_e = \frac{1}{m}\sum_{j=1}^{m} F_{eQj} \tag{A8}$$

and is recommended to be used in the pressure-time method.

In the calculation procedure presented above, it has been assumed that the changes in the velocity profiles are the same during steady and unsteady flow conditions. This assumption is close to reality for not very fast closure of turbine wicket gates during pressure-time method tests. Practically, such conditions occur in all hydraulic machines, because it is necessary to protect machine flow systems from the hydraulic transient destructive effects.

Taking the equivalent value of F_e instead of the value F calculated directly from the pipeline segment geometry, it is possible to increase the pressure-time method accuracy in cases when pipelines have irregular flow elements.