Basic concept and first case study of an in-situ calibration method for secondary flow meters on bifurcated penstocks

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Abstract

This paper reveals the feasibility to calibrate flow meters, which are installed within a network of conduits having ≥ 3 branches involved. The proposed evaluation method introduces the terms pressure function π and flow function φ to approximate measured pressure losses and to provide estimates of the actual branch flow rates, respectively. The minimum number of branches is required to include one or more correlation term(s), which comprise the values of several or all branch flow rate estimates, in the pressure functions. This method opens up the possibility to calibrate flow meters under real conditions and, for instance, under extensively high Reynolds numbers, which remain unattainable in the laboratory.
A case study on a bifurcated penstock is presented, where the Winter-Kennedy flowmeters have been undergone this calibration procedure. The discharges of the calibrated flowmeters deviate by $-1.1 \dots 1.4\%$ from the reference flow rates of the 8-path ATT flow meters.
If the biasing factors are known this in-situ calibration method will be contemplated to have an extended uncertainty of $U(Q) \leq 2\%$ $(k = 2)$ under favorable measurement conditions. This is sufficient for field applications but poor for laboratory use. Neither the metrological requirements, which distinguish between "good" and "bad" measurement quality nor the conditions for an appropriate numerical algorithm have been sufficiently known so far. Further investigation is essential.

1 Introduction

Hydraulic test campaigns on multi-branch pipings such as irrigation systems are time-consuming and costly when using absolute flow measurement methods in accordance with relevant standard test codes [1] [2]. In many cases, the available measurement conditions do not comply with the recommendations given in these test codes. There is a huge need of simple and cheaply applicable methods or techniques in the energy generating business and in the pump industry. Recently, it could be shown that the use of secondary flow meters on branched pipes and the introduction of flow functions is a powerful tool to calibrate the individual flow meters relatively to each other [3].
Based on this concept the next section evolves the theory in combining the ideas of flow functions and pressure functions. The latter denotes approximations of measured pressure losses between a measurement section (MS) in the main branch of a hydraulic network and other MSs in the side branches. The hydraulic losses in the main branch play a key role in this calibration method because they introduce one or several correlation terms, which involve all flow functions, in the pressure function. This is subject to a non-linear regression analysis, which yields the desired calibration coefficients of the flow metering devices in use.

The case study in section 3 shows a practical approach in calibrating the Winter-Kennedy flow meters of two neighboring Francis turbine units which have separate penstocks but share a common headrace tunnel. Therefore, real data have been used.

Section 4 discusses the problems and challenges of this methodology based on the current status.

2 Concept

Let us consider the main conduit which splits into $\mu \in \{\mathbb{N} | \mu > 1\}$ side branches as shown in Figure 1. There is no need to restrict the type of conduits. That is, we may consider open channels or closed pipes or a combination of both, which are charged with the same uniphase, Newtonian liquid. Without loss of generality, we let the fluid propagate from the main conduit into the side branches, each charged by the mass flow rate $\dot{m}_i$ where $i \in \{\mathbb{N} | i \leq \mu\}$. The main conduit carries $\dot{m} = \sum_{i=1}^{\mu} \dot{m}_i$, consequently. Any additional inflow or outflow with respect to the reference region is forbidden. The reference region is defined between measurement section MS_0, which is located in the main conduit, and the measurement sections MS_i, which are located in the contributing side conduits. We measure simultaneously the differential pressure between MS_0 and MS_i, i.e., Δp_i, and an independent flow parameter x by a relative flow meter or a flow meter to be calibrated for each branch flow. Depending on the type of flow meter the independent parameter x can

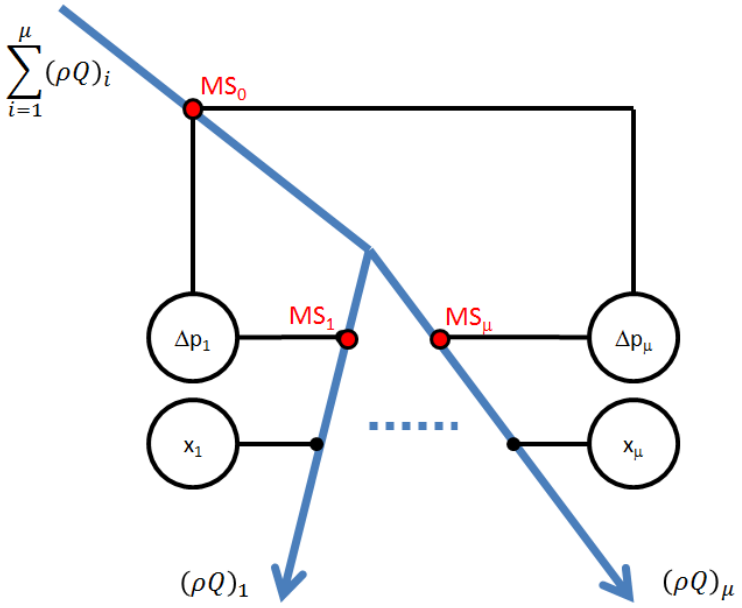

Figure 1: Flow scheme and parameters to be measured

be associated with a differential pressure, for instance, when using some kind of orifice or with pressure losses along a part of the branch. A flow meter appropriate for this calibration method requires at least good reproducibility and repeatability and it needs to provide a bijective transfer function. However, we see that we have to record $2 \cdot \mu$ signals in total.

We imply stationary flow conditions and we approximate the differential pressure Δp_i by a general pressure function

$$\Delta p_i \cong \pi_i\left(Q_1, \ldots, Q_\mu;\ \kappa_0, \ldots, \kappa_\nu\right) \tag{1}$$

where $\nu \in \{\mathbb{N} | \nu \geq \mu\}$. The independent parameter Q_i denotes the volumetric flow rate at MS_i and multiplied by the fluid density ρ_i it provides the mass flow rate

$$\dot{m}_i = \rho_i \cdot Q_i = (\rho Q)_i \tag{2}$$

The number of constants κ_i is related to the type of flow (i.e., laminar, turbulent, intermediate), to the location of the MSs at hand and to the desired accuracy level. We subsequently facilitate the syntax of $\pi_i\left(Q_1, \ldots, Q_\mu;\ \kappa_0, \ldots, \kappa_\nu\right)$ by $\pi_i(Q; \kappa)$.
A case in point: assuming turbulent flow conditions in the flow scheme of Figure 1 we may favor the subsequent model function

$$\pi_i(Q; \kappa) = \kappa_0 \cdot \left(\sum_{j=1}^{\mu} Q_j\right)^2 + \kappa_i \cdot Q_i^2 \tag{3}$$

The coefficients κ_0 and κ_i in the previous equation comprise terms with physical and geometrical constants whose informational value is not of interest with respect to the scope of this paper.[1]

So far, we have used the actual branch flow rate Q_i. However, the flow meter at hand can only provide a more or less rough estimate of Q_i by the independent flow parameter x_i. We thus need to approximate the actual -- and unknown -- branch flow rate by the subsequent general flow function

$$Q_i \cong \varphi_i\left(x_i;\ \lambda_{i,1}, \ldots, \lambda_{i,\nu_i}\right) \tag{4}$$

with $\nu_i \in \mathbb{N}$. Here again, we write $\varphi_i(x; \lambda)$ instead of the right side of Equation (4) to simplify its representation.
The number of constants λ_i takes into consideration the type of flow measurement (e.g., ultrasonic flowmeter), the flow measurement conditions and the desired accuracy level. An arbitrarily extendible list of typical model functions $\varphi(x; \lambda)$ reveals Table 1.
Inserting Equation (4) into Equation (1) gives

[1] Technically relevant expansions are, for instance, $\kappa_0 = k_0 - \alpha_0\rho_0/(2\,A_0^2)$ and $\kappa_i = k_i + \alpha_i\rho_i/(2\,A_i^2)$, where the parameters k_0 and k_i denote the coefficients considering the pressure losses between MS_0 and the manifold and between the manifold and MS_i, respectively. The second term on the right side takes into account the dynamic pressure.

Table 1: Typical flow functions

type of flow measurement	**independent flow parameter x**	$\boldsymbol{\varphi(x;\lambda)}$
flow meter (general)	flow rate q	$\sum_i \lambda_i \cdot q^i$
(C)ATT (single-path, direct transmission)	axial velocity v	$\sum_i \lambda_i \cdot v^i$
(C)ATT (single-path, reflect transmission)	axial velocity v	$\lambda_1 \cdot v$
(C)ATT (multiple paths/elevations)	flow rate q	$\lambda_1 \cdot q$
orifice flowmeter	differential pressure p	$\lambda_1 \cdot \left(\frac{p}{1\ \mathrm{Pa}}\right)^{0.5}$
pressure losses (turbulent flow)	differential pressure p	$\lambda_1 \cdot \left(\frac{p}{1\ \mathrm{Pa}}\right)^{0.5}$
Winter-Kennedy method	differential pressure p	$\lambda_1 \cdot \left(\frac{p}{1\ \mathrm{Pa}}\right)^{\lambda_2}$

$$\Delta p_i \cong \pi_i\big(\varphi_1(x;\lambda), \dots, \varphi_\mu(x;\lambda);\ \kappa_0, \dots, \kappa_\nu\big) \quad (5)$$

We also facilitate the notation above by

$$\Delta p_i \cong \pi_i(\varphi(x;\lambda);\kappa) \quad (6)$$

or

$$\Delta p_i \cong \pi_i(x;\lambda,\kappa) \quad (7)$$

respectively. We conclude that a total number of

$$\mathrm{m} = \nu + 1 + \sum_{\mathrm{i}=1}^{\mu} \nu_i \quad (8)$$

coefficients ($m \in \mathbb{N}$) need to be determined experimentally. Since we gain μ equations per measuring point (i.e., one pressure function for each branch and measuring point $\Delta p_i \cong \pi_i$), the required number of measuring points $n \in \mathbb{N}$ must satisfy the criterion

$$\mathrm{n} > \frac{\mathrm{m}}{\mu} \quad (9)$$

to obtain an overdetermined system of equations. Only then we are able to investigate the statistical significance of the coefficients in the pressure functions by means of a non-linear regression analysis. The system of non-linear equations in the coefficients can be solved by any sophisticated optimization procedure.
Returning to our example: If we used orifice flowmeters on each side branch a relevant model function of our pressure function would yield

$$\pi_i(Q;\kappa) = \kappa_0 \cdot \left(\sum_{j=1}^{\mu} \lambda_{j,1} \cdot \left(\frac{p_j}{1\ \mathrm{Pa}}\right)^{0.5}\right)^2 + \kappa_i \cdot \left(\lambda_{i,1} \cdot \left(\frac{p_i}{1\ \mathrm{Pa}}\right)^{0.5}\right)^2 \quad (10)$$

With Equations (8) and (9) we obtain the number of regressors to be determined ($m = 2\mu + 1$) and the minimum number of measuring points ($n_{min} = 3$). Although in this example the required number of measuring points is low it makes sense to increase this number from a statistical point of view since with $n = n_{min}$ it only remains $d_f = n\mu - m = \mu - 1$ degrees of freedom ($d_f \in \mathbb{N}$) left. Consequently, the significance of the calculated regressor values needs to be questioned.

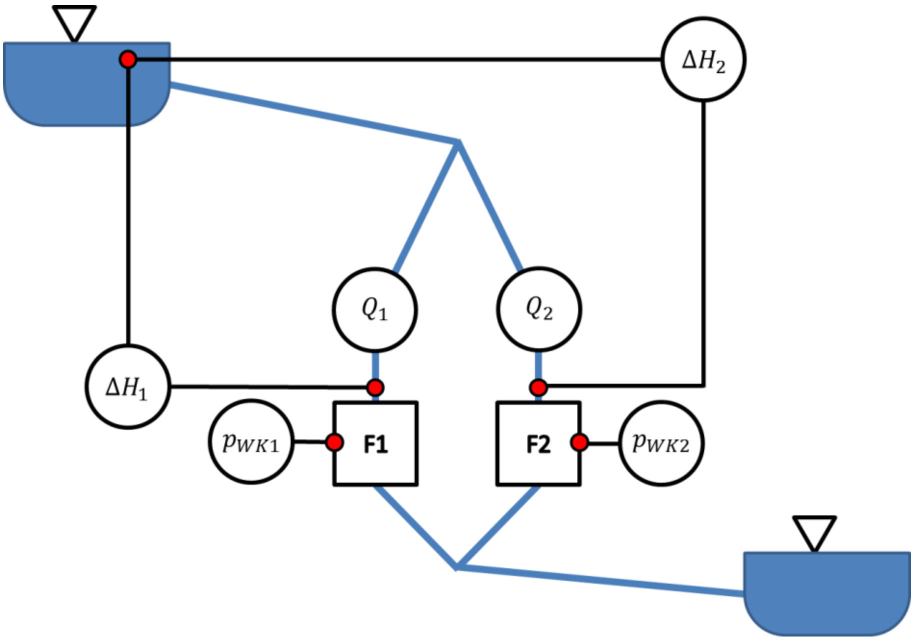

Figure 2: Schematic representation of waterway and test setup: Francis turbine Fi, head losses ΔH_i, ATT flow measurement Q_i and Winter-Kennedy differential pressure p_{WKi} for $i \in \{1,2\}$

Table 2: Measurement data

MPNR (-)	ΔH_1 (m)	Q_1 (m^3/s)	p_{WK1} (Pa)	ΔH_2 (m)	Q_2 (m^3/s)	p_{WK2} (Pa)	OUTLIER
35	17.851	19.853	4195	22.548	44.234	20942	x
36	18.142	19.885	4187	22.853	44.137	20937	x
37	18.043	19.830	4181	22.759	44.248	20923	
38	19.186	27.329	8010	21.145	36.873	14753	
39	19.220	27.422	8096	21.191	36.856	14739	
40	19.142	27.428	8098	21.118	36.930	14730	
41	20.472	35.077	13256	19.621	29.754	9574	x
42	20.812	35.107	13090	19.971	29.666	9531	
43	20.826	35.207	13290	19.975	29.649	9468	
44	22.643	41.596	18774	19.383	23.796	6092	
45	22.673	41.674	18768	19.417	23.805	6124	
46	22.670	41.836	18868	19.383	23.757	6149	

3 Case study: Calibration of two Winter-Kennedy flow meters

In 2013 I executed performance tests on two Francis turbine units simultaneously. At this hydropower plant, the water is taken from an upstream lake (see Figure 2). It flows through a headrace tunnel of several kilometers and it passes a surge chamber. Downstream the surge chamber the waterway bifurcates into two separate penstocks of several hundred meters in length, which feed a single Francis turbine unit each. The penstocks are permanently equipped with 8-path acoustic transit time flow meters measuring the volumetric flow rates Q_1 and Q_2, respectively. On the turbine units, the Winter-Kennedy differential pressures p_{WK1} and p_{WK2} are also recorded for reasons of discharge indication.
Fortunately, I recorded all necessary parameter signals which now makes it now possible to calibrate these Winter-Kennedy flow meters based on the theoretical concept of the previous section and to compare them with results of the acoustic flow measurements. Table 2 shows the mean parameter values of the individual measuring points $MPNR$. It contains the data of four operating conditions, which have been recorded three times each. Three measuring points are considered to be outliers because the transients show undamped penstock pressure oscillations, which bias the parameter estimates.
Let us define the flow functions (Winter-Kennedy) and the pressure (head) functions by

$$\varphi_i(x;\lambda) = \lambda_i \cdot \left(\frac{p_{WKi}}{1\ \text{Pa}}\right)^{0.5} \tag{11}$$

and

$$\pi_i(x;\lambda,\kappa) = \frac{\left[\kappa_0 \cdot \left(\varphi_1(x;\lambda) + \varphi_2(x;\lambda)\right)^2 + \kappa_i \cdot \left(\varphi_i(x;\lambda)\right)^2\right]}{\left[1\frac{m^3}{s}\right]^2} \tag{12}$$

These model functions look simple and require the determination of only $m = 5$ regressors (see Equation (8)). We can use $n = 9$ valid measuring points from Table 2 to meet the inequality criterion (9) and end up with a system of $2 \cdot n$ equations, which are nonlinear in the coefficients λ, κ. Consequently, we may express the unweighted cost function to be minimized by

$$\chi^2 = \frac{1}{\mu \cdot n - m} \cdot \sum_{i=1}^{\mu} \sum_{j=1}^{n} (\Delta H_i[j] - \pi_i[j])^2 \tag{13}$$

Without loss of accuracy, I use here the equivalence of pressure head and pressure keeping in mind the direct proportionality between both physical parameters.
A local minimum of Equation (13) can be obtained, for instance, by using the Levenberg-Marquardt algorithm. Depending on the type of cost function and on the choice of the initial solution vector $(\kappa_0, \kappa_1, \kappa_2, \lambda_1, \lambda_2)^T$, the global minimum can be reached. The initial vector in Table 3 has been chosen based on plausibility and expectations. The table also contains the final estimates of the regressors which provide a standard deviation of $s = \sqrt{\min \chi^2} = 0.094\, m$. The new estimates of the regressors λ_1 and λ_2 together with Equation (11) allow us to calculate the discharge values, which are associated with the *calibrated* Winter-Kennedy flowmeters. The relative deviation to the reference discharge yields $(Q_{WKi} - Q_i)/Q_i = -1.1 \ldots 1.4\%$ as can be seen in Figure 3. We additionally obtain the loss coefficients of the waterway (i.e., κ_0, κ_1, κ_2) as a byproduct.

4 Discussion

This rudimental case study above reveals the possibility of an in-situ calibration of Winter-Kennedy flowmeters by standard measurements of head losses along the waterway. The comparative results in Figure 3 are promising but this calibration procedure is far away from applying it instead of any accepted primary flow measurement method. There are

Table 3: Regressor values

# iterations	κ_0	κ_1	κ_2	λ_1	λ_2
(-)	(m)	(m)	(m)	(m^3/s)	(m^3/s)
0 (initial)	0.004000	0.0030000	0.003000	0.300000	0.300000
4 (final)	0.004112	0.002875	0.002955	0.303364	0.306971

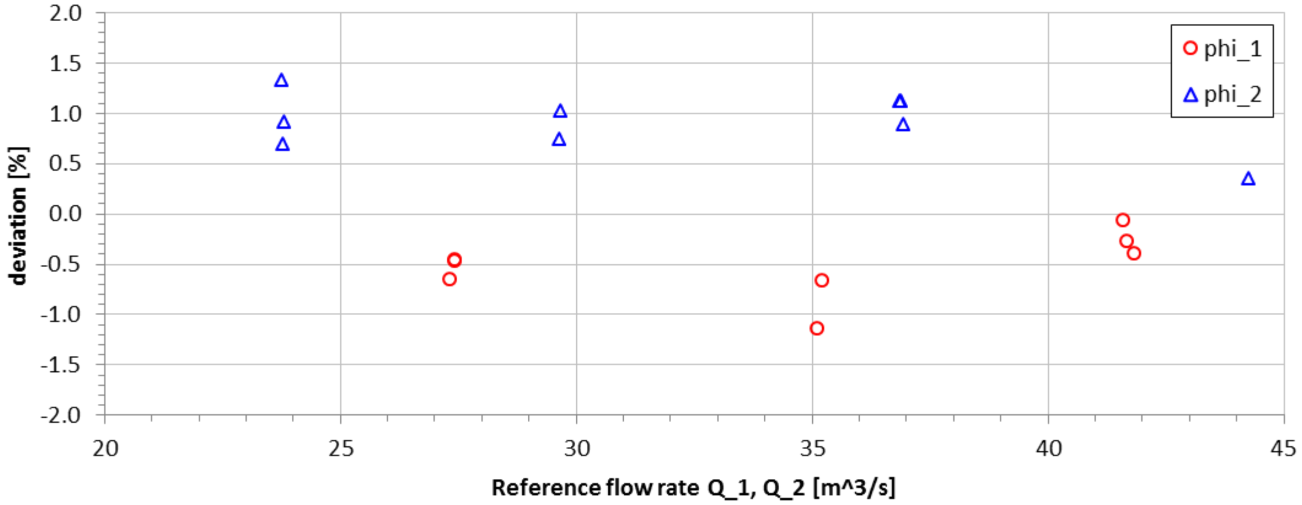

Figure 3: Deviation of calibrated Winter-Kennedy discharge from reference discharge ($Q_{WKi}/Q_i - 1$)

a couple of influencing factors, which have to be considered and analyzed in detail:

4.1 Pressure function

The model function should describe the actual behavior accurately enough by a minor number of coefficients to avoid a numerical meltdown. It is necessary to be cautious in applying constant loss coefficients in branch connections since they may vary under changing load balance conditions. Scientific data collections, e.g., [4], or CFD simulations should help to define the application range of the chosen model function.

4.2 Flow function

The complexity of this model function depends mainly on the type and method of the secondary flow metering. For instance, the applicability of a Winter-Kennedy flow meter calibrated in the optimum range can be questionable whenever using it under turbine part load conditions [1].

4.3 Numerical stability

The type of algorithm to minimize the cost function is crucial for the success of this procedure. I used here a Levenberg-Marquardt algorithm knowing that the quality of the result depends mainly on the choice of the initial solution vector. A case in point: Let us increase the λ_i values of the initial vector by 1% compared to that one in Table 3 and we obtain a significantly different solution vector as can be seen in Table 4. That is unsatisfactory. Consequently, an appropriate type of optimization procedure has to be applied to reach the global minimum of the cost function.

Table 4: Regressor values resulting from a different initial vector

# iterations	$\boldsymbol{\kappa_0}$	$\boldsymbol{\kappa_1}$	$\boldsymbol{\kappa_2}$	λ_1	λ_2
(-)	**(m)**	**(m)**	**(m)**	**(m^3/s)**	**(m^3/s)**
0 (initial)	0.004000	0.0030000	0.003000	0.303000	0.303000
4 (final)	0.004077	0.002851	0.002930	0.304657	0.308280

4.4 Metrological aspects

Investigations for the minimum requirements of the measurement quality (e.g., signal-to-noise ratio, relative and absolute magnitude of branch losses and collector pipe losses, operating conditions) are pending. They are necessary to define the instrumentation setup and to estimate the applicability of this calibration procedure at all.

5 Conclusions

The author presented the basic ideas to calibrate flow meters in-situ by measuring the pressure losses within a flow scheme of multiple branches and by obeying the continuity law. The calibration procedure is applicable to flow schemes with different conduit types (e.g. open channels, partly or fully wetted conduits) and to different flow regimes, i.e., under laminar, intermediate or turbulent flow conditions and its execution on site is simple.

Although this method seems to provide an interesting cost-benefit ratio for the manufacturing industry and for the energy business future work on this subject becomes inevitable. Numerical stability and metrological requirements need to be investigated rigorously to provide guidelines for a successful application of this method.

6 Bibliography

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