Diagnosis of Acoustic Transit Time Data based on the Area Flow Function

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Abstract

While commissioning an acoustic transit time (ATT)-installation for discharge measurement in a hydro power plant, it is proposed to check the data on the basis of the area flow function (AFF) with the purpose to detect potential outliers in the path velocities. Such outliers could be caused by extremely distorted velocity distributions, by faulty path velocity measurements or by mistakes in the parameter settings of the measuring device.
In the following, the area flow function is discussed for a series of simulated velocity distributions in power plants, where distorted velocity distributions were expected. Furthermore, selected data of field measurements are presented in non-dimensional form. The simulated AFFs are compared to the Gauss-Jacobi and the OWICS AFF and the integration errors of the two methods are evaluated. Where available, simulated data are also compared to measured data.

1. Introduction

A first proposal to use acoustic of ultrasonic methods for flow measurements in hydropower plant stems from [1]. However, the proposed method of antenna-type-transducers crossing the entire conduit was soon abandoned. The first application of the acoustic transit time (ATT) measurement in a hydropower plant in 1957 is described by [2]. With the developing of the electronic semiconductor devices and the possibility for accurate time of flight measurements new products for discharge measurement in hydro power plants came into the market in the 1970th, [3], [4].
In an early study, Hastings [4] demonstrated that the accuracy of the flow rate evaluation can be increased by multiplying path velocities with the width of a circular cross section at the elevation of each individual path before interpolating and integrating. By doing so, he set the basis for the idea of the area flow function. This concept was further developed by Voser [5]. In his thesis he describes in detail the Gauss-Jacobi quadrature procedure and comes to the conclusion that a higher integration accuracy of fully developed velocity distributions in conduits of circular cross section can be achieved be slightly adapting the weighting function of the quadrature formula. He called this modified procedure the Optimized Weighted Integration Method for Circular Sections (OWICS).
In the following the basic equations and parameter definitions are introduced, which allows to evaluate flow rates as well as to compare results of case studies in a normalized way. Advantages of the OWICS method with respect to accuracy are pointed out.

Numerically the flow rate Q can be approximated by summing up the partial flow rates ΔQ_i for each horizontal strip, as displayed in Figure 1.

$$Q = \lim_{N\to\infty} \sum_{i=1}^{N} \Delta Q_i = \lim_{N\to\infty} \sum_{i=1}^{N} \bar{v}_{ax}(z_i) \cdot b(z_i) \cdot \Delta z \quad \left[\frac{m^3}{s}\right] \tag{1}$$

The area flow function F(z) (AFF) describes the distribution of the partial flow rates on the strips and is expressed by

$$Q = \int_{-\frac{D}{2}}^{\frac{D}{2}} F(z) dz = \lim_{N\to\infty} \sum_{i=1}^{N} \Delta Q_i \tag{2}$$

The area flow function *F(z)* at height *z* is then defined as the product between the averaged axial velocity and the width *b(z)* of the conduit at height *z*. For an arbitrary *z*, *F(z)* can be written as:

$$F(z) = \bar{v}_{ax}(z) \cdot b(z) \left[\frac{m^2}{s}\right] \tag{3}$$

Therefore, the AFF at a given position zi is:

$$F(z_i) = \bar{v}_{ax}(z_i) \cdot b(z_i) \tag{4}$$

Due to the finite number of measured paths, it is not possible to calculate the sum of the right hand side of the equation (2). For a finite number of paths the coefficient C_1 is introduced, which however is irrelevant for the determination of the positions and weights. The dimension of C_1 is $[m]$.

$$Q = \int_{-\frac{D}{2}}^{\frac{D}{2}} F(z)dz \cong C_1 \cdot \sum_{i=1}^{N} \bar{v}_{ax}(z_i) \cdot b(z_i) \tag{5}$$

Accordingly, the integral is approximated by a finite sum using weighting factors $w_1, \ldots w_N$.
For circular cross sections the flow rate is calculated as:

$$Q = \frac{D}{2} \cdot \sum_{i=1}^{N} w_i \cdot \bar{v}_{ax}(z_i) \cdot b(z_i) \tag{6}$$

The width $b(z_i)$ can be determined from the measured path length L_i projected on the y-z-plane (Fig. 2)

$$b(z_i) = L_i \cdot \sin(\varphi)$$

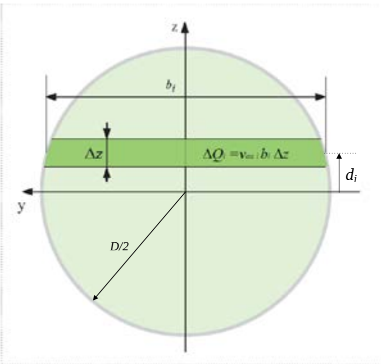

Fig. 1: Integration by summing up (d_i = abs (z_i))

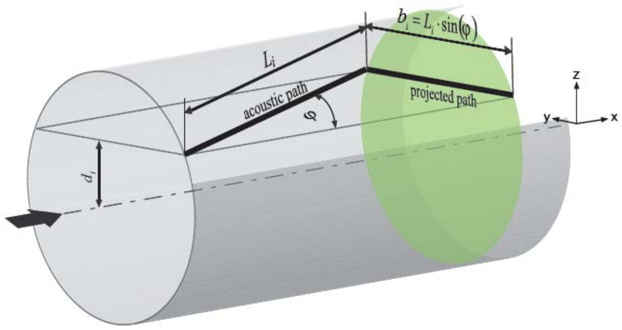

Fig. 2: Projection of the acoustic path of the partial flow rates on the y-z-plane (d_i =abs(z_i))

The different integration weights of the Gauss-Jacobi and the OWICS method have their origin in different assumption on the shape of the reference area flow function, respectively velocity distribution. They can be distinguished by a single parameter κ. The assumed AFF for the circular cross section is given by:

$$F_{ref}(z) = C_2 \cdot \left(1 - \frac{z^2}{\left(\frac{D}{2}\right)^2}\right)^{\kappa} \left[\frac{m^2}{s}\right] \tag{7}$$

Parameter κ for circular sections: Gauss-Jacobi: 0.5 OWICS: 0.6

In the following only circular cross-sections are considered. In order to compare AFFs of different flow rates and AFFs of different case studies the AFF is brought into non-dimensional form by introducing:

$$\zeta = \frac{z}{D/2}, \qquad \beta = \frac{b}{D}$$

The normalized AFF can be accordingly be written as:

$$F_{ref\ norm}(\zeta) = (1 - \zeta^2)^{\kappa} \quad [-] \tag{8}$$

The integration of this normalized area flow function results in:

$$I_{GJ} = \int_{-1}^{1} (1-\zeta^2)^{0.5} d\zeta = \frac{\pi}{2} = 1.57080 \tag{9}$$

$$I_{OWICS} = \int_{-1}^{1} (1-\zeta^2)^{0.6} d\zeta = 1.51336 \tag{10}$$

In order to compare data of measured or simulated path velocities of different installations in hydro plants, the path velocities are also normalized.

$$\bar{v}_{ax\,norm\,GJ}(\zeta_i) = \frac{\bar{v}_{ax}(z_i)}{Q} \frac{I_{GJ} D^2}{2} \tag{11}$$

and

$$\bar{v}_{ax\,norm\,OWICS}(\zeta_i) = \frac{\bar{v}_{ax}(z_i)}{Q} \frac{I_{OWICS} D^2}{2} \tag{12}$$

And accordingly, the normalized product of velocity and width becomes:

$$\bar{v}_{ax\,norm\,GJ}(\zeta_i) \cdot \beta(\zeta_i) = \frac{\bar{v}_{ax}(z_i) \cdot I_{GJ} \cdot D \cdot b(z_i)}{Q \cdot 2} \tag{13}$$

$$\bar{v}_{ax\,norm\,OWICS}(\zeta_i) \cdot \beta(\zeta_i) = \frac{\bar{v}_{ax}(z_i) \cdot I_{OWICS} \cdot D \cdot b(z_i)}{Q \cdot 2} \tag{14}$$

The two area flow functions differ slightly. The Gauss-Jacobi AFF corresponds to a half circle. The OWICS AFF was developed by [5] to give a better fit of AFFs to velocity distributions to be typically expected for fully developed velocity distributions at high Reynolds numbers.

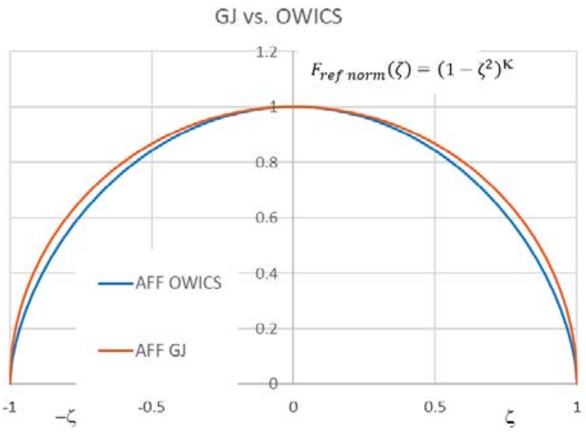

Fig. 3: Normalized Gauss-Jacobi and OWICS AFFs

The deviation of simulated or measured data on the individual paths from the reference AFF is an important quantity to judge whether the velocity distribution in the conduit is heavily distorted or if some of the path readings might be in error for any reason.
For this purpose, we define the following quantities:

$$\Delta F_i = \bar{v}_{ax\,norm\,GJ}(\zeta_i) \cdot \beta(\zeta_i) - F_{ref\,norm}(\zeta_i) \tag{15}$$

Depending on the normalization method we will distinguish $\Delta F_{i\,GJ}$ and $\Delta F_{i\,OWICS}$

Of interest are on the one hand the absolute maximum deviation

$$\Delta F_{max} = \max|\Delta F_i| \tag{16}$$

and, on the other hand statistical quantities such as a weighted mean deviation or a weighted squared deviation

$$\mu = \frac{\sum_{i=1}^{N} |w_i \cdot \Delta F_i|}{\sum_{i=1}^{N} w_i} \tag{17}$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} w_i \cdot \Delta F_i^{\,2}}{(\sum_{i=1}^{N} w_i) - 1}} \tag{18}$$

Again, we will have to distinguish between μ_{GJ}, σ_{GJ} and μ_{OWICS}, σ_{OWICS}

2. Fully developed pipe flow

In order to demonstrate this normalization of the OWICS AFF and the Gauss-Jacobi AFF, the fully developed flow in a conduit was simulated using numerical flow simulation (CFD) at a Reynolds number of Re = 2 10^7.

The simulations were performed with ANSYS CFX 18. The mesh of the penstock is a manually generated structured hexahedral mesh. The inlet boundary comprises about 5000 hexahedral elements. Minimum angles were >50°, maximum angles <142°, and the maximum volume ratio were 2.8. The dimensionless wall distance (y+) is on average about 100.
The simulations were performed in steady state and with translational periodic boundary conditions. The solutions are based on the SST (shear stress transport) turbulence model. The calculations are solved with the high resolution advection scheme and a physical time scale factor of 1 s. The RMS residuals were at $2.4 \cdot 10^{-10}$, the maximum residuals at $3.3 \cdot 10^{-8}$, and the imbalance at $1.2 \cdot 10^{-6}$. The simulations required about 150 iterations to reach this convergence.

From the numerical data the AFF of the simulated flow field as well as path velocities at the Gauss-Jacobi elevations. The normalized AFFs are displayed in Fig. 4. We observe that the simulated data are considerably closer to the OWICS AFF (Fig. 4a) than to the Gauss-Jacobi AFF (Fig. 4b).

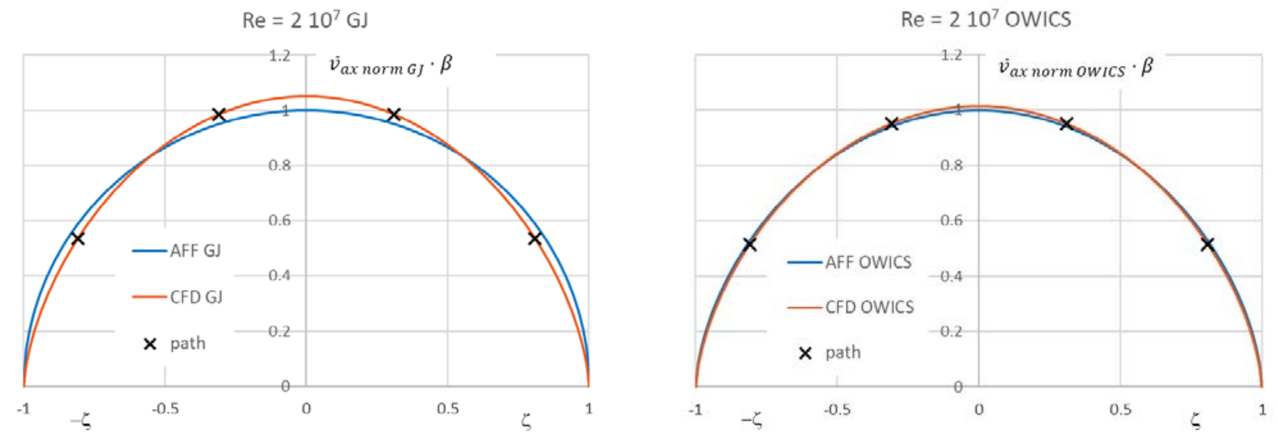

Fig. 4: Fully developed a) normalized CFD AFF vs. Gauss-Jacobi AFF, b) normalized CFD AFF vs. OWICS AFF

As a further test, flow rates were integrated in accordance with the two methods proposed in IEC 60041 and in [5] based on path velocities exctracted from the CFD data for four layer ATT. The used weights are listed in Table 1.

Table 1: Integration error of Gauss-Jacobi and OWICS method of the CFD test case

position ζ_i	w_i GJ	w_i OWICS	error GJ [%]	error OWICS [%]
-0.809017	0.369316	0.365222	$100\,\frac{Q_{GJ}-Q_{CFD}}{Q_{CFD}} = 0.21\%$	$100\,\frac{Q_{OWICS}-Q_{CFD}}{Q_{CFD}} = 0.07\%$
-0.309017	0.597566	0.598640		
0.309017	0.597566	0.598640		
0.809017	0.369316	0.365222		

3. Case studies with distorted velocity distributions

In the first case study, case 1, the flow in the conduit at the position of the flow meter (2x4 paths on crossed planes) was heavily distorted by an inflow from as side branch. This flow was simulated by CFD, measured path were not available.

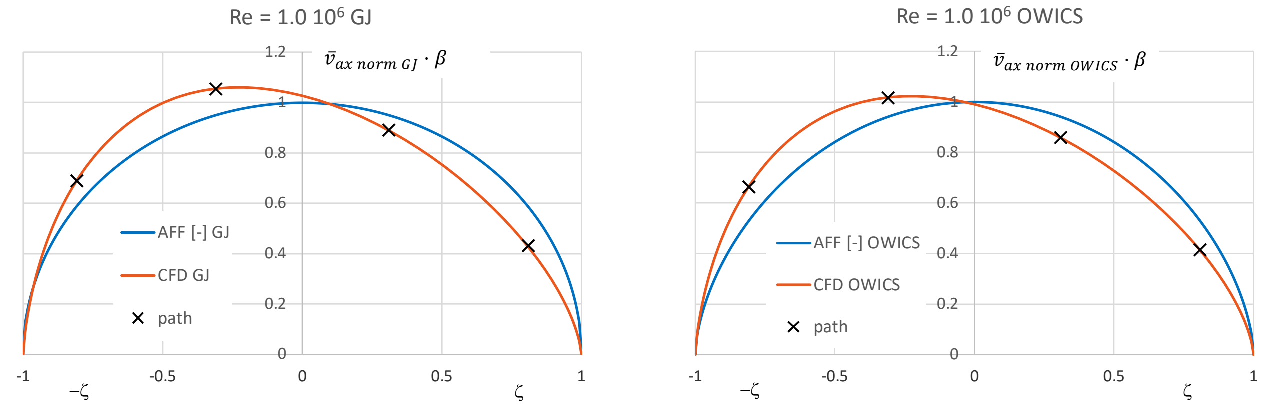

Fig. 4: Case I, a) normalized CFD AFF vs. Gauss-Jacobi AFF, b) normalized CFD AFF vs. OWICS AFF

In contrast to the fully developed pipe flow, hardly any advantage of the OWICS method can be detected, however the statistical values collected in Table 2, still show data slightly in favor of the OWICS method. In this example the maximum deviations ΔF_i in the order of 0.15 GJ and 0.11 OWICS are very high but the integration error of -0.28 % GJ and -0.13 % OWICS are rather small. This is due to the fact that the deviation of the distorted area flow function from the assumed one can be well approximated with a low order polynomial (maximum 7th order for a 4-layer installation) [6].

A further case study is especially insightful since measured path velocities were made available [7]. In case 2 the flow was disturbed by an upstream bifurcation feeding three turbines with short penstocks.

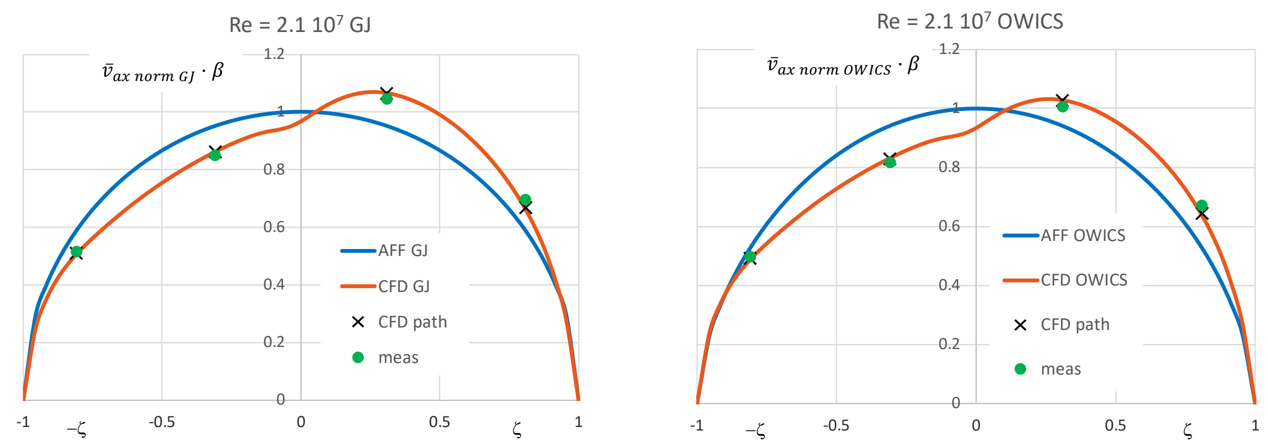

Fig. 5: Case 2, a) normalized CFD AFF vs. Gauss-Jacobi AFF, b) normalized CFD AFF vs. OWICS AFF

The green points in figure 5 stem from measured path velocities. The agreement of the simulated and measured data is excellent [7]. Looking at the statistical data in Table 2 we observe that measured and simulated data give very consistent results. While in the other cases the OWICS method showed clear advantages over the Gauss-Jacobi method, in case 2 no improvement of the results was found. The integration uncertainty (error defined in Table 1) is quite large in this case because of the local deficit in the simulated AFF in the centre. This information is missed with the four-layer measurement and accordingly the flow rates are predicted too high. With the proof that the simulated velocities agree well with the measured ones, the measured flow rate could be theoretically corrected. However, due to the varying plant operation such a correction is not feasible since the velocity distribution in the measuring section varies as a function of the operation mode of the other turbines.

Hulse et al. [8] presented data of an 18 path (2x9) measurement at Grand Coulee Dam These measurements are adopted in case 3. The data of four, slightly varying, flow rates are displayed in Fig. 6. The normalized measured data show an excellent repeatability and the data points are very close to the OWICS AFF. The velocity distribution, as reported in [8], are only distorted to a minor degree in spite of an upstream elbow. The statistical values of this case 3 are small, in contrast to the cases 1 and 2 with distorted flow conditions.

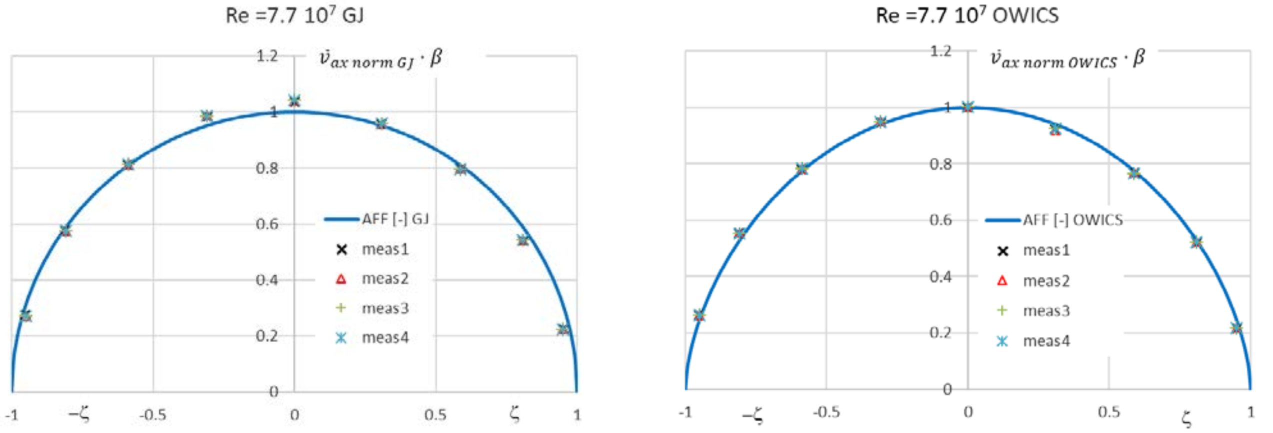

Fig. 6: Case 3, normalized measured AFF data, a) vs. Gauss-Jacobi AFF, b) vs. OWICS AFF

Case 4 stems again from a CFD study. Measured path velocities are not yet available. In contrast to the other cases, the flow is here distorted by smaller size flow structure. The evaluation location only a little more than one conduit diameter downstream of a lattice type butterfly valve. The wakes of the lattice is still present in the measuring section. For this reason, the acoustic paths have to be oriented vertically. With a vertical orientation, the path velocity averages the wakes, while with horizontal orientation the path velocities will be heavily affected by the wakes of the lattice type butterfly valve.

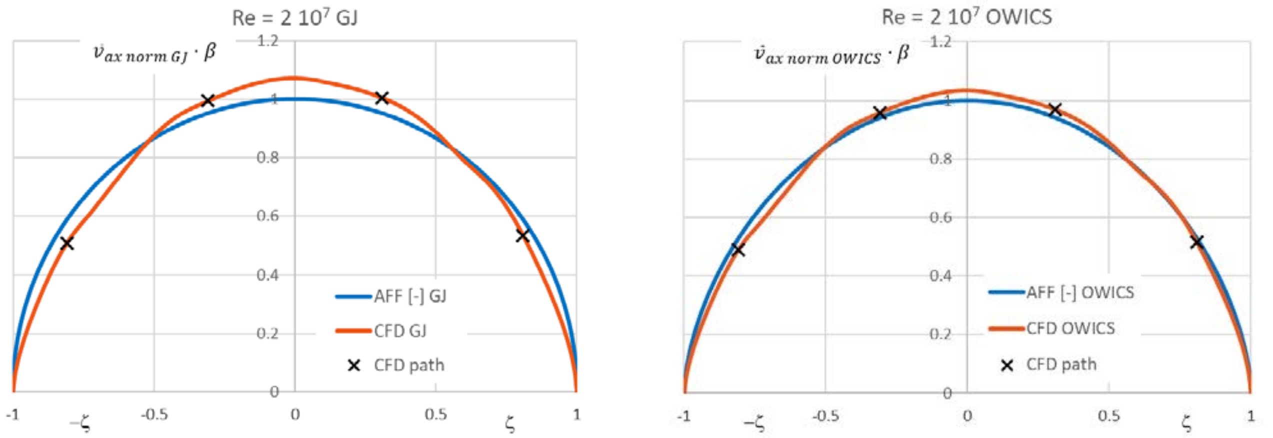

Fig. 7: Case 4, a) normalized CFD AFF vs. Gauss-Jacobi AFF, b) normalized CFD AFF vs. OWICS AFF

In order to demonstrate how data could look like in a normal case with sufficient distance from an upstream elbow, case 5 is introduced. This case was an installation of the transducers at the OWICS positions [5] and evaluation with position corrected OWICS weights, based on an exact measurement of the installed transducers. The data stem from recent measurements performed by Hydrovision GmbH [9]. Figure 8 confirms that the measured points lie very close to the OWICS AFF. The statistical data shown at the end of Table 1 come close to the OWICS values of the simulated fully developed pipe flow.

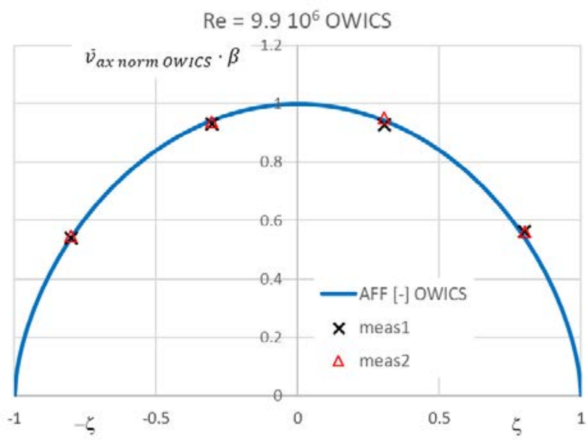

Fig. 8: Case 5, normalized measured AFF data vs. OWICS AFF

Table 2: Statistical evaluation

	ΔF_{max}	μ	σ	error [%]
Fully developed				
Gauss-Jacobi	0.053	0.042	0.029	0.212
OWICS	0.013	0.010	0.007	0.068
Case 1				
Gauss-Jacobi	0.157	0.100	0.072	0.284
OWICS	0.135	0.096	0.067	0.125
Case 2 CFD				
Gauss-Jacobi	0.114	0.093	0.074	0.931
OWICS	0.115	0.090	0.073	0.755
Case 2 Measured				
Gauss-Jacobi	0.108	0.095	0.073	
OWICS	0.142	0.092	0.075	
Case 3				
Gauss-Jacobi	0.049	0.033	0.055	
OWICS	0.046	0.019	0.039	
Case 4				
Gauss-Jacobi	0.080	0.055	0.040	0.558
OWICS	0.039	0.023	0.018	0.423
Case 5				
OWICS	0.021	0.011	0.009	

4. Conclusions

Analysing measured path velocities by comparing the data with the AFF gives an indication of the level of distortion of the velocity distribution in conduits. Such a comparison can be done visually by plotting the data and by evaluation of statistical data. Distorted ATTs are not necessary sign for an increased uncertainty of the flow rate integration, but the distortion should be explainable with the upstream flow conditions. If only one single path deviates from the AFF, then it should be checked if the configuration of this path is correctly implemented. The above analysis can be done for averaged path velocities of crossed planes, but of course, it might be also very useful to perform such an analysis for each plane separately. In case of ATT installations downstream of flow disturbing elements, where a CFD simulation is carried out in order to check the expected integration uncertainty, the AFF can be plotted and checked for its smoothness and the integration uncertainty of such an installation can be estimated.

We propose to check the AFF for all new ATT installations to analyse the path velocities and the associated points in the AFF distribution for different flow rates in normalized form. This procedure can be carried out for a number of acoustic paths, but it is most informative for higher numbers of layers, e.g. 9 layers. Deviations from the theoretical AFFs are a sign for distorted velocity distributions or by mistakes in the parameterization of the instrument or other human errors. Concerning the statistical data evaluation, the deviations should be small comparable to the fully developed pipe flow or the data of case 5 in Table 2. From the collected cases, there is an indication that the OWICS procedure gives better results, especially for well-developed flows.

The steps of such and analysis can summarized as follows:

- Evaluate the flow rates with Gauss-Jacobi and/or OWICS weights,
- Normalize product of measured velocities and width according to Eq. (13) and (14),
- Plot the reference AFF of Eq. (8) and add the normalized product of measured velocities and width,
- Evaluate the maximum deviation ΔF_{max} and the statistical quantities μ and σ,
- Check for outliers,
- Check for distorted velocity distributions, for faulty path velocity measurements or for mistakes in the parameter settings.

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