A COMPARISON OF DISCHARGE CALCULATION METHODS

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Abstract

Ott meter data collected at Manitoba Hydro Pine Falls generating station is used to compute discharge according to the German and ASME codes regarding the performance testing of hydraulic turbines. In addition, the Gauss Quadrature technique used in acoustic methods of performance testing is examined. The results indicate that the estimates of discharge given by the German and ASME codes vary in this case by approximately 16%. However a 5 point Gauss Quadrature provides estimates within 0.7% of that given by the German code. Therefore, the position, not just the number of metering points, is central to the accuracy of discharge calculations.

Résumé

Les données recueillies de moulinets Ott furent utilisées pour calculer les débits selon les normes allemande et ASME pertinentes à la mesure des performances des turbines hydrauliques. Est examinée de plus, la technique de quadrature de gauss utilisée pour la méthode acoustique lors des essais de performance. Les résultats montrent que les débits calculés par les techniques des normes allemande et ASME varient d'environ 16% dans le cas présent. La technique de la quadrature de gauss conduit cependant à des débit sconcordant à 0.7% près, avec ceux calculés selon la norme allemande. La position des points de mesurage est donc tout aussi importante que leur nombre pour établir les débits avec justesse.

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Introduction

The velocity-area method is a well-established technique for metering low-head plants. This technique involves traversing an array of propeller-type flow meters (*i.e.*, Ott Type A) mounted on a rigid horizontal frame across the flow at the intake stoplog guides. The appropriate summation of the grid of point measurements of velocity times the corresponding area within the metering plane, yields an estimate of the discharge. This process is repeated for numerous power settings, and in the case of variable pitch turbines, blade angles, to determine the head-power-discharge relationship for a turbine.

The positioning and summation of the velocity measurements can be carried out in various ways. The American Society of Mechanical Engineers (ASME) uses a log-linear method which is based on a weighted sum of appropriately spaced velocity measurements to estimate discharge (ASME PTC 18, 1992). The German code for hydraulic testing bases its discharge estimate on representative flow areas within the intake to integrate across the velocity profile (Abnahmeversuche an Wasserturinen, DIN 1948). Another method of estimating discharge, which is not based on testing code but on a numerical method, is the Gauss Quadrature technique. This numerical integration method requires functional values (*i.e.*, velocity) at non-uniformly spaced points in the plane of integration in order to estimate the volumetric rate of flow.

Although each of these methods has its merits in terms of ease of application, the most important consideration is accuracy of discharge estimate as a small error could result in substantial revenue losses for hydroelectric utilities. The subject of the following analysis is a comparison of these techniques. Each method will be used to estimate discharge over a range of wicket gate settings. These estimates will then be compared to a measurement of plant discharge based largely on the method outlines in the German code to assess the relative difference between the calculation methods.

Data

The data for this study was collected at the Pine Falls Generating Station during Ott Meter testing conducted by Manitoba Hydro in March 1994. These tests were conducted in accordance with German hydraulic testing code. Ott (type A) meters were traversed across the flow to establish the velocity profile for various gate settings; these settings were 40, 60, 70, 80, 85, 90, 95, and 100 percent gates. Velocity measurements are based on (an average from) a two minute sample. Data herein are confined to the results for unit 5, intake A. Unit 5 is an end unit, and intake A is the intake farthest from the end of the forebay deck. The discharge estimates obtained using the three different integration methods were compared for all gate settings. The complete set of available Pine Falls discharge data was used, instead of confining the analysis to one gate setting to identify the average deviation from the German code based estimate.

The Pine Falls Generating Station is part of the Manitoba Hydro Winnipeg River system. Pine Falls has 5 turbine units and a plant capacity of 82 megawatts. Each turbine unit has three intake openings. Each of the intakes has an area of approximately 38 m^2 and measures 4.4 m wide by 8.1 m high. The distribution of metering points across the intake is in accordance with the German code for hydraulic efficiency testing which requires between $20\sqrt[3]{A}$ *to* $40\sqrt[3]{A}$ metering points to measure the velocity profile, where A is the cross-sectional area of the intake in m^2. Eighty-four measurement points (7 horizontal by 12 vertical) were used to establish the velocity profile at the Pine Falls Generating Station. The vertical and horizontal spacing was set assuming the inflow profile can be approximated by the $1/7^{th}$ power law for fluid flow in a closed conduit. As a result, the density of velocity measurements is greatest near the flow boundaries where the variation of velocity (shear) is the largest. Spacing between traverse levels is greater through the center of the intake where the velocity is assumed to be the most uniform. The vertical and horizontal spacing of metering points are given in Table 1.

Table 1: Measurement positions

Vertical [m]	Horizontal [m]
0.1524	0.1524
0.6400	0.5730
1.2801	1.2832
2.0726	2.2860
2.8651	3.2888
3.6881	3.9989
4.4806	4.4196
5.2730	
6.0655	
6.8580	
7.5590	
7.9553	

The ASME log-linear method and the Gauss Quadrature method require velocity measurements at points close to, but not coincident with the measurements used by Manitoba Hydro. In order to obtain the appropriate velocity measurement for the log-linear and Gauss Quadrature techniques it was necessary to interpolate between the measured points. MATLAB™ algorithms were used to perform 2-D cubic spline interpolation on the original velocity point measurements. Figure 1 shows the results of the interpolation.

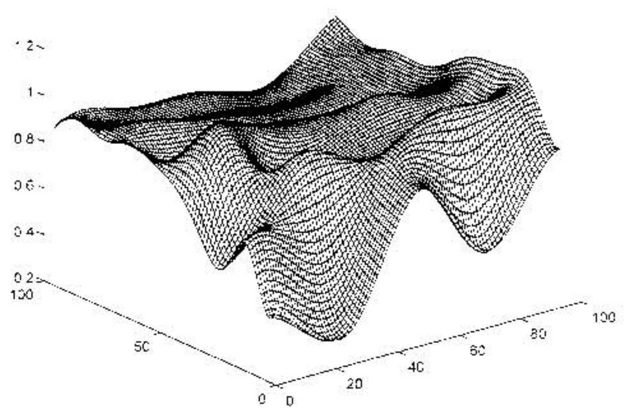

Figure 1. Cubic spline interpolated surface mesh of velocity profile at Pine Fall Generating Station (unit 5A) at 60% gate. The vertical axis shows the interpolated velocity [m/s].

German Code

The German code for hydraulic efficiency testing specifies calculation of discharge using three representative elements in the flow profile. These elements are interior, edge, and corner elements. Discharge through an interior element is calculated by using the average of four corner velocities, multiplied by the element area to estimate the discharge. Flow at the boundaries is computed based on

$$Q = \int_{y=0}^{H} \int_{x=0}^{W} \left[\frac{w}{x}\right]^{1/7} \cdot V_e \, dxdy \quad \textbf{(1a)}$$

$$Q = \frac{7}{8} W \cdot V_e \cdot H \quad \textbf{(1b)}$$

where W is the width of the element, H is the height of the element, and $V_e = (v_1+v_2)/2$ is the average edge velocity; v_1 and v_2 are measured point velocities.

Discharge through corner elements is based on the formula for edge flow, expanded to account for an additional edge influence. It is calculated by

$$\frac{1}{2} Q = \int_{y=0}^{W} \int_{x=0}^{yW/H} \left[\frac{x}{W}\right]^{1/7} \cdot V_c \, dxdy \quad \textbf{(2a)}$$

$$Q = \frac{49}{60} H \cdot W \cdot V_c \quad \textbf{(2b)}$$

where V_c is the measured point velocity near the corner. Total discharge is computed based on a summation of the discharge through all elements.

The German hydraulic testing code recommends $20\sqrt[3]{A}$ *to* $40\sqrt[3]{A}$ metering points across a rectangular cross-section. Minimum horizontal spacing of metering points is one propeller diameter between any metering point and the flow boundaries as well as at least one propeller diameter between adjacent meters.

Using approximately $30\sqrt[3]{A}$ metering points to determine the discharge is estimated by Manitoba Hydro to be accurate to 2% (Abnahmeversuche an Wasserturinen, DIN 1948). Thus, the method outlined in the German code, can provide a relatively accurate estimate of discharge although it is somewhat labor intensive given the necessary density of measurement points. Possible further analysis could address the effect of reducing the number of metering points used for the German code method.

A weakness in the German code is the reliance on an idealized velocity profile to determine the distribution of metering points across the flow. Figure 2 shows a measured profile. Clearly, an idealized $1/7^{th}$ power law would not adequately represent the measured profile. Further, large variations in the measured profile occur through the center of the intake, where the idealized profile assumes the velocity to be most uniform. Reliance on an idealized profile results in a distribution of measuring levels which may over-sample the slowly varying velocity profile in proximity to the flow boundaries, while providing too few measurements where there is considerable variation.

A comparison of the measured velocity profiles for the range of gate settings reveals that the deviation from an idealized profile is consistent throughout the range of discharges. Based on this observation, it is possible to infer that a more accurate measurement of volumetric flow rate may be obtained by modifying the metering locations within the traverse to account for the properties of the actual velocity profile. Such a re-distribution of vertical measurement positions could result in a more efficient measurement of the velocity profile by increasing the number of points in areas of high velocity variability, while decreasing measurements where the velocity profile is more uniform.

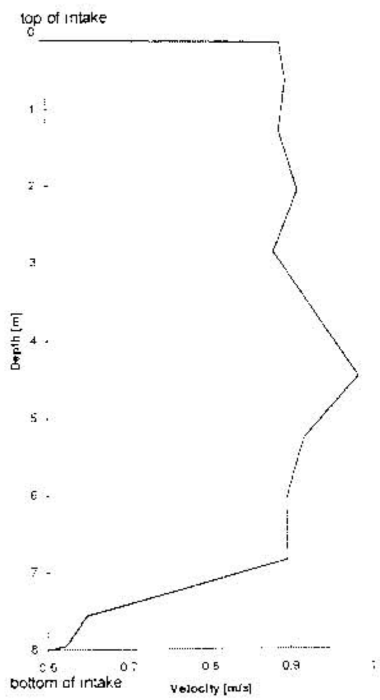

Figure 2. Measured velocity profile unit 5A, 60% gate.

ASME Code

The American Society of Mechanical Engineers (ASME) code for the performance testing of hydraulic turbines (PTC 18) recommends a minimum of 26 measurement points in a rectangular metering section. The spacing of these points, as well as the weighting scheme used to calculate discharge is based on a log-linear method of integration. Weights are assigned to velocity points to give more weight to the measurement points at the center of the intake and place less emphasis on velocity measurement points that are influenced by proximity to top and bottom flow boundaries. Figure 3 shows the distribution of the metering points and their corresponding weights.

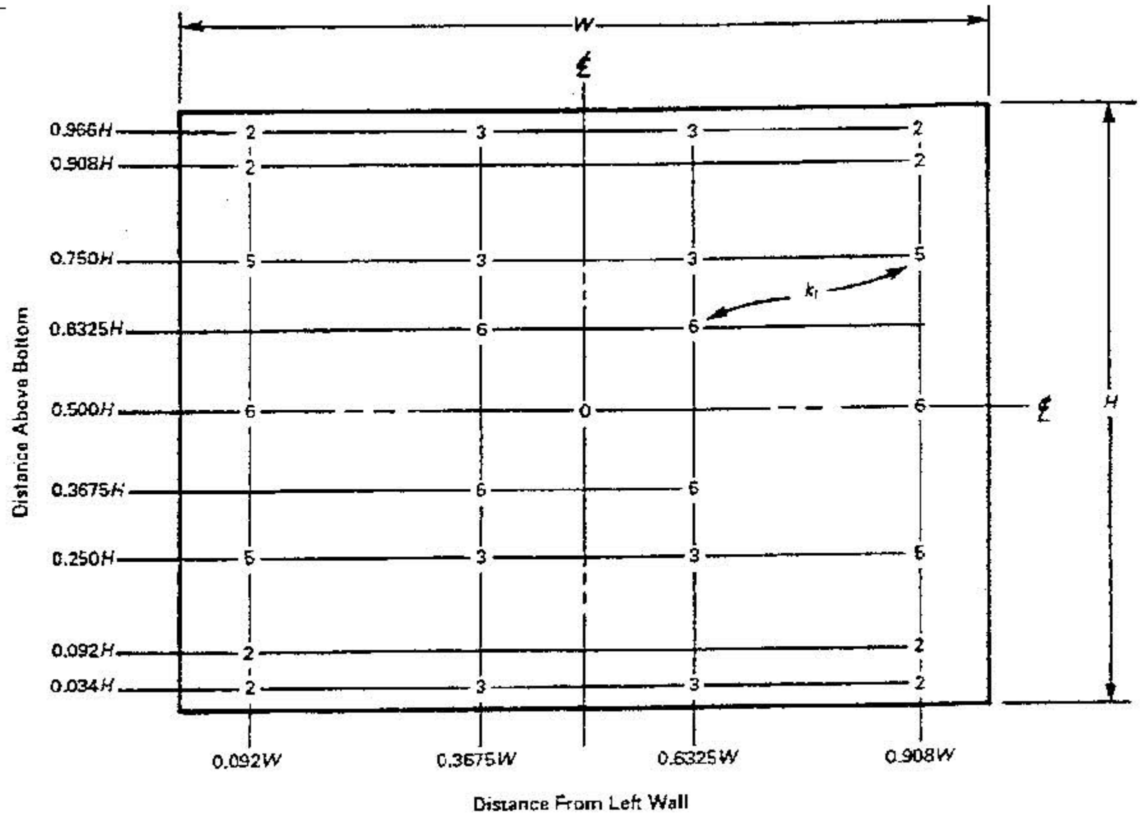

Figure 3: Distribution of points and weights for the ASME log-Linear method (ASME PTC 18, 1992).

The measured flow rate is calculated based on the average velocity over the measurement area. Average velocity is calculated according to

$$\overline{V} = \frac{\sum k_i v_i}{\sum k_i} = \frac{\sum k_i}{96}, \tag{3}$$

where k_i is the weight given in Figure 3, v_i is the corresponding velocity, and $\overline{V}$ is the average weighted velocity. The corresponding discharge is calculated from

$$Q = \overline{V} \cdot A. \tag{4}$$

The primary advantage of applying the log linear method is the relatively small number of metering points required to estimate discharge as compared to the German testing code requirement.

Gauss Quadrature Method

The Gauss Quadrature method of integration is used in acoustic discharge measurement techniques because of its accuracy and efficiency (Voser *et al.*, 1996; ASME, 1992). Spacing of measurement points according to the Gaussian method allows a complex velocity profile to be determined using a minimal number of points; 5 point Gauss Quadrature, with 25 measurements can approximate an 9^{th} order polynomial. The basis for the Gauss Quadrature integration method requires function evaluations at non-uniformly spaced points across the integration plane. Because the functional form of the velocity profile is not known *a priori*, it is necessary to interpolate between the known velocity points in order to establish a more detailed representation of the velocity profile across the intake. Cubic spline interpolation (described above) was used to compute the requried velocity profiles.

Five point gauss quadrature was performed in both the horizontal and vertical dimensions across the intake to estimate the discharge. The function arguments and weights used in this integration method are given in Table 2; the function arguments are given as a percentage of the distance above or below the center line of the intake.

Table 2 Gauss Quadrature Weights and Function Arguments (from Chapra & Canale, 1988)

	Weights		Function Arguments
c_0	0.236926885	x_0	-0.906179846
c_1	0.478628670	x_1	-0.538469310
c_2	0.568888889	x_2	0.0
c_3	0.478628670	x_3	0.538469310
c_4	0.236926885	x_4	0.906179846

Among the benefits of using the Gauss quadrature method of integration is the small number of point velocity measurements required to obtain an accurate estimate of the volumetric flow rate. Since five point Quadrature is sufficient to approximate a 9^{th} order function, a velocity profile that is assumed to follow the $1/7^{th}$ power law could be adequately approximated using 25 measurement points, 5 in each of the horizontal and vertical dimensions. This number of measurement points is less than the ASME minimum requirement of 26 points for rectangular sections. and is considerably less than the German minimum requirement of $20\sqrt[3]{A}$ measurement points.

Discussion

A summary of the discharge estimates obtained using each of the methods described is given in Table 3. Comparison of the three integration methods reveals that the Gauss Quadrature numerical integration technique provides values closest to that estimated using the German integration method. The log-linear method, proposed by the ASME, yields an estimate that significantly underestimates is substantially higher (16% greater than the Manitoba Hydro estimate).

Comparison of the Gauss Quadrature and ASME log-linear methods shows that although they are similar in method, (each applies a weighting scheme to the distribution of velocity measurements), and number of velocity measurements required, the discharge estimates differ significantly. The discharge estimate calculated by the log-linear method is considerably lower than the German code discharge, while the 5-point Quadrature is within reasonable limits of accuracy. Based on these observations it is possible to infer that the distribution of velocity measurements and the corresponding weighting scheme are important factors in determining the accuracy of discharge estimate.

Table 3 Summary of discharge estimates

% Gate	German Code [cms]	ASME Code [cms]	5 Point G.Q. [cms]	% Difference wrt German Code G.Q.	ASME
40	23.840	20.475	24.139	1.237	16.435
60	33.080	28.377	34.643	4.512	16.574
70	39.390	33.700	39.429	0.099	16.883
80	43.870	37.680	43.900	0.068	16.429
85	45.570	39.063	45.674	0.228	16.659
90	46.830	40.151	46.831	0.002	16.634
95	48.410	41.506	48.414	0.008	16.633
100	49.760	42.719	49.764	0.008	16.482

Conclusion

Ott meter velocity measurements obtained from Manitoba Hydro's Pine Falls generating station were used to compare the relative accuracy of discharge measurements computed using the German code, ASME code, and the Gauss Quadrature methods of estimating discharge. The results indicate that the estimates of discharge given by the German and ASME codes vary in this case by approximately 16%. However a 5 point Gauss Quadrature provides estimates within 0.7% of that given by the German code.

Comparison of the values obtained using the log-linear method with the Gauss Quadrature technique reveals that although one more point is used in the log-linear method, the relative accuracy of the final result is (apparently) significantly lower than that obtained through Gauss Quadrature integration. The discharge estimate using the log-linear method was consistently higher than the Manitoba Hydro discharge estimate, with a mean difference of 16%, which suggests that too much emphasis may have been placed on measurement points which provide a velocity significantly higher than the mean velocity. This leads one to conclude that the distribution of metering points as well as the weighting scheme used in the log-linear method does not adequately sample the velocity profile across the intake. These results indicate that it is the distribution of velocity measurement, not the number, which most significantly affects the accuracy of discharge estimate.

Research will be conducted at the Hydraulics Research and Testing Facility at the University of Manitoba to investigate the effect of the different distributions of measurement points on the estimate of volumetric flow rate. The measurement positions specified by the Gauss Quadrature integration technique will be compared to both an increased, as well as decreased number of measurements to determine the optimal number of points required to both adequately represent the velocity profile, and accurately estimate discharge. The ultimate goal of this testing program will be to determine the minimum number of metering positions that can be used to accurately estimate discharge.

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