UNCERTAINTY IN EXPERIMENTAL EVALUATION OF CHEZY-STRICKLER-MANNING AND COLEBROOK-WHITE ROUGHNESS COEFFICIENTS IN CIRCULAR PIPES

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ABSTRACT

This paper shows how to estimate the uncertainty of the experimental measurements of the Chézy-Strickler-Manning and Colebrook-White roughness coefficients in circular pipes, on the basis of the uncertainties of the measurements of diameter, discharge and headloss by which such roughness coefficients are evaluated. In fact the importance of Chézy-Strickler-Manning and Colebrook-White friction formulas is fundamental in both theoretical and applied Hydraulics; therefore, it is often very relevant to estimate also the uncertainty rates of their roughness coefficients, like in the problems related to the verification of the flow conveyance capacity. The relationships among those uncertainties are hereby summarized, also in adimensional forms. Finally, a numerical example of a realistic application is shown, regarding the individuation of the Chézy-Strickler-Manning and Colebrook-White roughness coefficients and their uncertainty in a circular pipe.

INTRODUCTION

A suitable and reliable evaluation of the roughness coefficient and of is uncertainty directly affects the different aspects of theoretical and applied Hydraulics, for example in the analyses of water hammer damping and water supply conveyance capacity; in particular, about this second field of application, there are essentially two problems:

- verification of the flow conveyance capacity Q, given the pipe length L, its diameter D, its roughness coefficient and the available head Y;
- design of the minimum diameter D required to convey a fixed flow Q, given the pipe length L, its roughness coefficient and the available head Y.

Indeed, the second of these problems is slightly less important, since the minimum diameter theoretically required is in practice replaced by the immediately bigger size available on the market. The experimental assessment of the roughness coefficient can be reached through the main scheme reported in Figure 1, where a pipe with a diameter D is fed by a constant flow Q and the headloss Y is measured by a piezometric gauge which has its two sensors set at a distance L along the pipe.

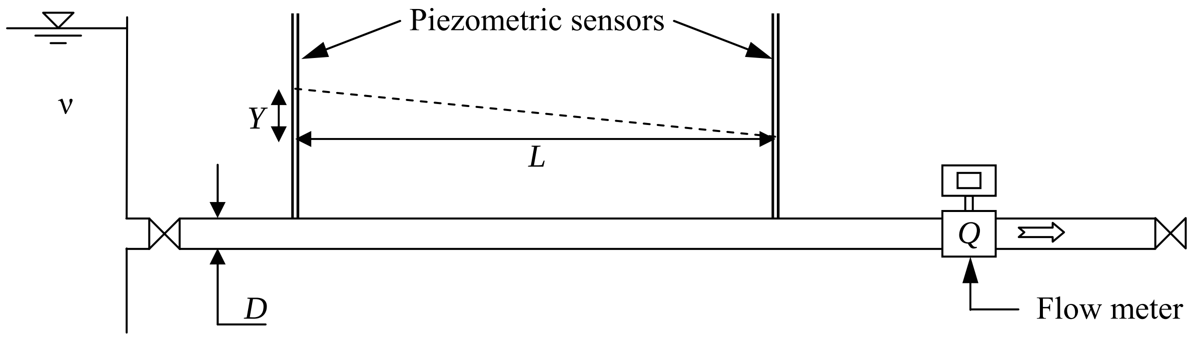

Figure 1 – General scheme of an experimental device to evaluate the roughness coefficient by means of measurements of pipe diameter D, flow Q, headloss Y and length L between the piezometric sensors.

1 CHÉZY-STRICKLER-MANNING FRICTION FORMULA

1.1 Generalities

Remembering that, by definition:

$$J = Y / L \tag{1}$$

the estimation of the friction loss per unit of pipe length J through the Chézy-Strickler formula (Chézy, 1776; Strickler, 1923) is very popular because of its monomial structure:

$$J = \frac{V^2}{{K_S}^2 \cdot (D/4)^{4/3}} = \frac{10.29 \cdot Q^2}{{K_S}^2 \cdot D^{16/3}} \tag{2}$$

that allows to find out in an explicit form each one of the involved physical variables, which are the diameter D, the Chézy-Strickler roughness coefficient K_S and the flow Q, or the mean velocity V:

$$V = \frac{Q}{A} = \frac{Q}{(\pi / 4) \cdot D^2} \tag{3}$$

By the way, the dimensional factor 10.29 is related to the normal choice of considering the units of Q, K_S and D according to the international units system (m^3/s for Q, $m^{1/3}/s$ for K_S and m for D).
Especially in Anglo-Saxon scientific literature and technical habits, it's very common to meet the so-called Chézy-Manning formula (Manning, 1891; Yen, 1991), where, instead of the Chézy-Strickler roughness coefficient K_S, there is the Chézy-Manning roughness coefficient:

$$n = 1 / K_S \tag{4}$$

Of course it's very easy to apply eq. (2) to solve any kind of pipe verification or design problem. Nevertheless, it must be remarked that it is valid only if the flow regime is fully turbulent, that is the condition under which J becomes completely independent from the density ρ and the dynamic viscosity μ of the liquid. This is expressed by the inequality (Nikuradse, 1932; Moody, 1944):

$$Re^* = \sqrt{\frac{\lambda}{8}} \cdot \frac{\rho \cdot V \cdot \varepsilon}{\mu} > 70 \tag{5}$$

where g is the acceleration due to gravity, ε is the size representing the pipe roughness (see, further below, the description of the Colebrook-White friction formula), and λ is the adimensional friction number of the Darcy-Weisbach expression (Weisbach, 1845; Darcy, 1857; Darcy and Bazin, 1865):

$$J = \lambda \cdot \frac{V^2}{2 \cdot g \cdot D} \tag{6}$$

1.2 Individuation of the Chézy-Strickler-Manning roughness coefficient and of its uncertainty

Starting from eq. (2), the Chézy-Strickler coefficient K_S can be drawn as:

$$K_S = 0.25^{-5/3} \cdot \pi^{-1} \cdot Q \cdot D^{-8/3} \cdot J^{-1/2} = 0.25^{-5/3} \cdot \pi^{-1} \cdot Q \cdot D^{-8/3} \cdot (Y/L)^{-1/2} \tag{7}$$

Under the hypothesis that L gives a uncertainty contribute which is negligible in comparison with the ones coming from D, Q and Y, the uncertainty of K_S can be estimated, on the basis of the theory of uncertainty propagation for independent variables (Taylor, 1997; EA-4/02, 1999), as:

$$u(K_S) = \sqrt{\left(\frac{\partial K_S}{\partial Q}\right)^2 \cdot u^2(Q) + \left(\frac{\partial K_S}{\partial D}\right)^2 \cdot u^2(D) + \left(\frac{\partial K_S}{\partial Y}\right)^2 \cdot u^2(Y)} \tag{8}$$

where the sensitivity coefficients of K_S regarding D, Q and Y are, after deriving eq. (7):

$$\frac{\partial K_S}{\partial D} = -\frac{8}{3} \cdot 0.25^{-5/3} \cdot \pi \cdot Q \cdot D^{-11/3} \cdot (Y/L)^{-1/2} = -\frac{8}{3} \cdot \frac{K_S}{D} \tag{9}$$

$$\frac{\partial K_S}{\partial Q} = 0.25^{-5/3} \cdot \pi \cdot D^{-8/3} \cdot (Y/L)^{-1/2} = \frac{K_S}{Q} \tag{10}$$

$$\frac{\partial K_S}{\partial Y} = -\frac{1}{2} \cdot 0.25^{-5/3} \cdot \pi \cdot Q \cdot D^{-8/3} \cdot Y^{-1} \cdot (Y/L)^{-1/2} = -\frac{1}{2} \cdot \frac{K_S}{Y} \tag{11}$$

Remembering eq. (4), the uncertainty of the corresponding Chézy-Manning roughness coefficient is:

$$u(n) = \sqrt{\left(\frac{\partial n}{\partial K_S}\right)^2 \cdot u^2(K_S)} = \frac{1}{K_S^2} \cdot u(K_S) = n \cdot \frac{u(K_S)}{K_S} \tag{12}$$

1.3 Numerical example

To show a numerical example of an individuation of the Chézy-Strickler roughness coefficient K_S and its uncertainty, it can be considered an experimental laboratory test characterized by:

diameter	D = 50 mm = 0.050 m	$u(D)$ = 0.500 mm	$u_R(D) = u(D)/D$ = 1.00%
flow	Q = 2 l/s = 0.002 m³/s	$u(Q)$ = 0.040 l/s	$u_R(Q) = u(Q)/Q$ = 2.00%
headloss	Y = 0.25 m = 250 mm	$u(Y)$ = 1.000 mm	$u_R(Y) = u(Y)/Y$ = 0.40%
length	L = 4 m	$u(L) \cong 0$	$u_R(L) = u(L)/L \cong 0\%$

where u_R are the adimensional uncertainties.
Complying with the previous eq. (1), eq. (3), eq. (7) the results are:

velocity V = 1.02 m/s		
friction loss per unit of pipe length	J = 0.0625 m/m	J = 6.25%
Chézy-Strickler roughness coefficient	K_S = 75.65 m$^{1/3}$/s	

and the corresponding uncertainty for K_S, according to eq. (8), is:

$u(K_S)$ = 2.53 m$^{1/3}$/s $u_R(K_S) = u(K_S)/K_S$ = 3.34%

It is also interesting to highlight that, according to eq. (9), eq. (10) and eq. (11) the heaviest contribute comes from the uncertainty of the diameter D, followed by the one of the flow Q, while the contribute due to is the headloss Y is the smallest. In fact such three contributes are, respectively:

$$\frac{\partial K_S}{\partial D} \cdot u(D) = -2.02 \text{ m}^{1/3}/\text{s} \qquad \frac{\partial K_S}{\partial Q} \cdot u(Q) = 1.51 \text{ m}^{1/3}/\text{s} \qquad \frac{\partial K_S}{\partial Y} \cdot u(Y) = -0.15 \text{ m}^{1/3}/\text{s}$$

It could be seen, through many others similar examples, that in general this is the kind of ranking that comes out of the experimental tests, according to both the usual levels of uncertainty of such quantities and the fact that the modules of their exponents in eq. (7) are respectively 8/3, 1 and 1/2.

1.4 Adimensional approach

The uncertainty of the Chézy-Strickler roughness coefficient can be shown in an adimensional form:

$$\frac{u(K_S)}{K_S} = \sqrt{\frac{\left(\frac{\partial K_S}{\partial Q}\cdot Q\right)^2 \cdot \left(\frac{u(Q)}{Q}\right)^2 + \left(\frac{\partial K_S}{\partial D}\cdot D\right)^2 \cdot \left(\frac{u(D)}{D}\right)^2 + \left(\frac{\partial K_S}{\partial Y}\cdot Y\right)^2 \cdot \left(\frac{u(Y)}{Y}\right)^2}{K_S{}^2}} \tag{13}$$

that is:

$$\frac{u(K_S)}{K_S} = \sqrt{1^2 \cdot \left(\frac{u(Q)}{Q}\right)^2 + \left(-\frac{8}{3}\right)^2 \cdot \left(\frac{u(D)}{D}\right)^2 + \left(-\frac{1}{2}\right)^2 \cdot \left(\frac{u(Y)}{Y}\right)^2} \tag{14}$$

and, defining the corresponding adimensional uncertainties $u_R(Q)$, $u_R(D)$ and $u_R(Y)$ for Q, D and Y:

$$\frac{u(K_S)}{K_S} = u_R(K_S) = \sqrt{u_R^2(Q) + \frac{64}{9}\cdot u_R^2(D) + \frac{1}{4}\cdot u_R^2(Y)} \tag{15}$$

Besides, a second step can consist in summarizing the ratio between the adimensional uncertainties of K_S and Q as function of both the ratio between the adimensional uncertainties of D and Q and the ratio between the adimensional uncertainties of Y and Q, in order to concentrate eq. (15) in one group of adimensional curves (Figure 2):

$$\frac{u_R(K_S)}{u_R(Q)} = \sqrt{1 + \frac{64}{9}\cdot\left[\frac{u_R(D)}{u_R(Q)}\right]^2 + \frac{1}{4}\cdot\left[\frac{u_R(Y)}{u_R(Q)}\right]^2} \tag{16}$$

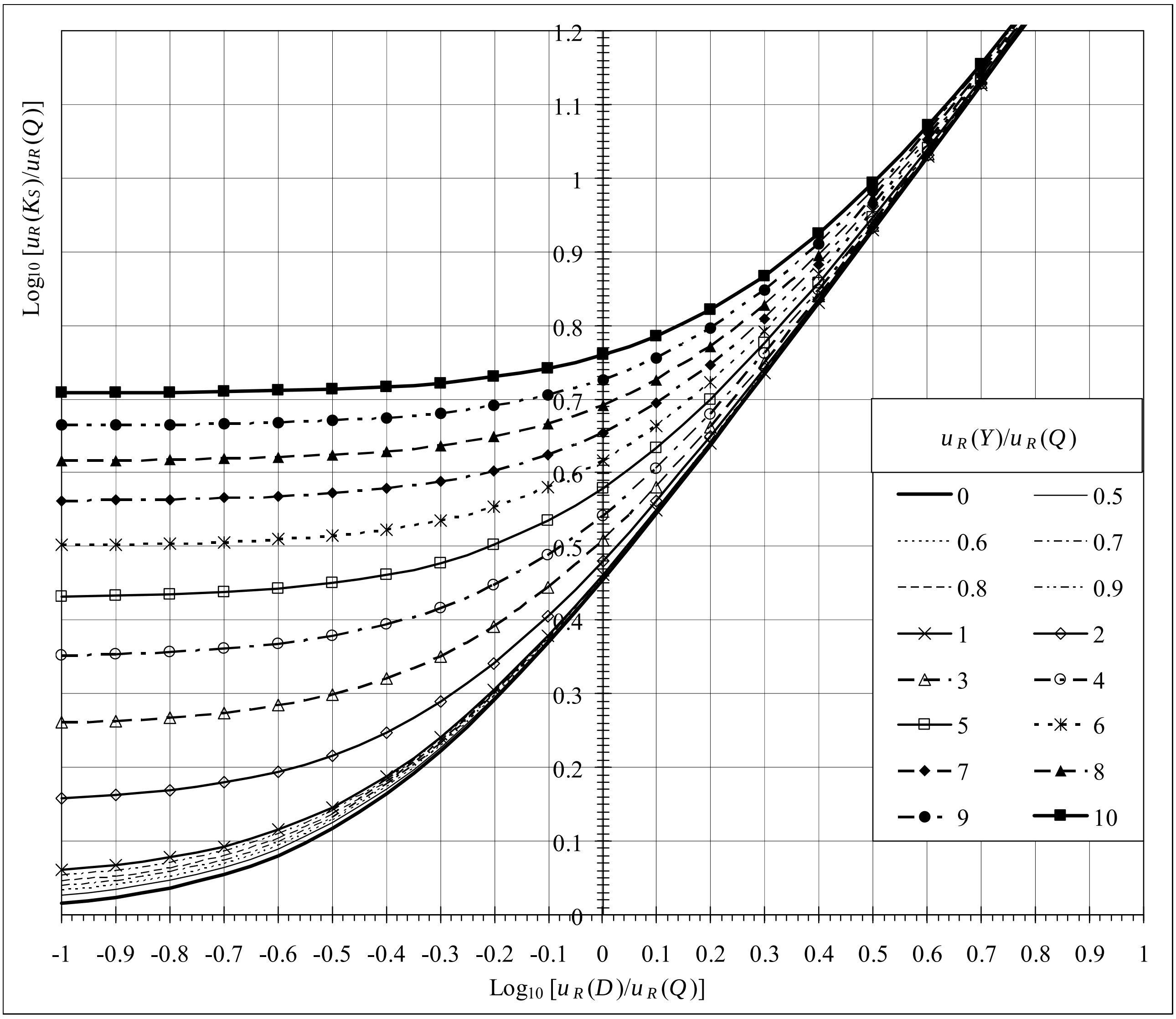

Figure 2 – The group of curves representing eq. (16) in a logarithmic plan.

1.5 Individuation of the pipe conveyance capacity and of its uncertainty

Starting from the Chézy-Strickler friction formula written as:

$$Q = 0.25^{5/3} \cdot \pi \cdot K_S \cdot D^{8/3} \cdot J^{1/2} = 0.25^{5/3} \cdot \pi \cdot K_S \cdot D^{8/3} \cdot (Y/L)^{1/2} \tag{17}$$

it can be derived, following the same procedure of above:

$$\frac{u(Q)}{Q} = \sqrt{1 \cdot \left(\frac{u(K_S)}{K_S}\right)^2 + \left(\frac{8}{3}\right)^2 \cdot \left(\frac{u(D)}{D}\right)^2 + \left(\frac{1}{2}\right)^2 \cdot \left(\frac{u(Y)}{Y}\right)^2} \tag{18}$$

that can be concentrated) in one group of adimensional curves (Figure 3):

$$\frac{u_R(Q)}{u_R(K_S)} = \sqrt{1 + \frac{64}{9} \cdot \left[\frac{u_R(D)}{u_R(K_S)}\right]^2 + \frac{1}{4} \cdot \left[\frac{u_R(Y)}{u_R(K_S)}\right]^2} \tag{19}$$

It’s easy to see that the group of adimensional curves reported in Figure 3 has, exactly, the same shape of the group of adimensional curves reported in Figure 3, because eq. (19) has the same structure and the same numerical coefficients of eq. (16).

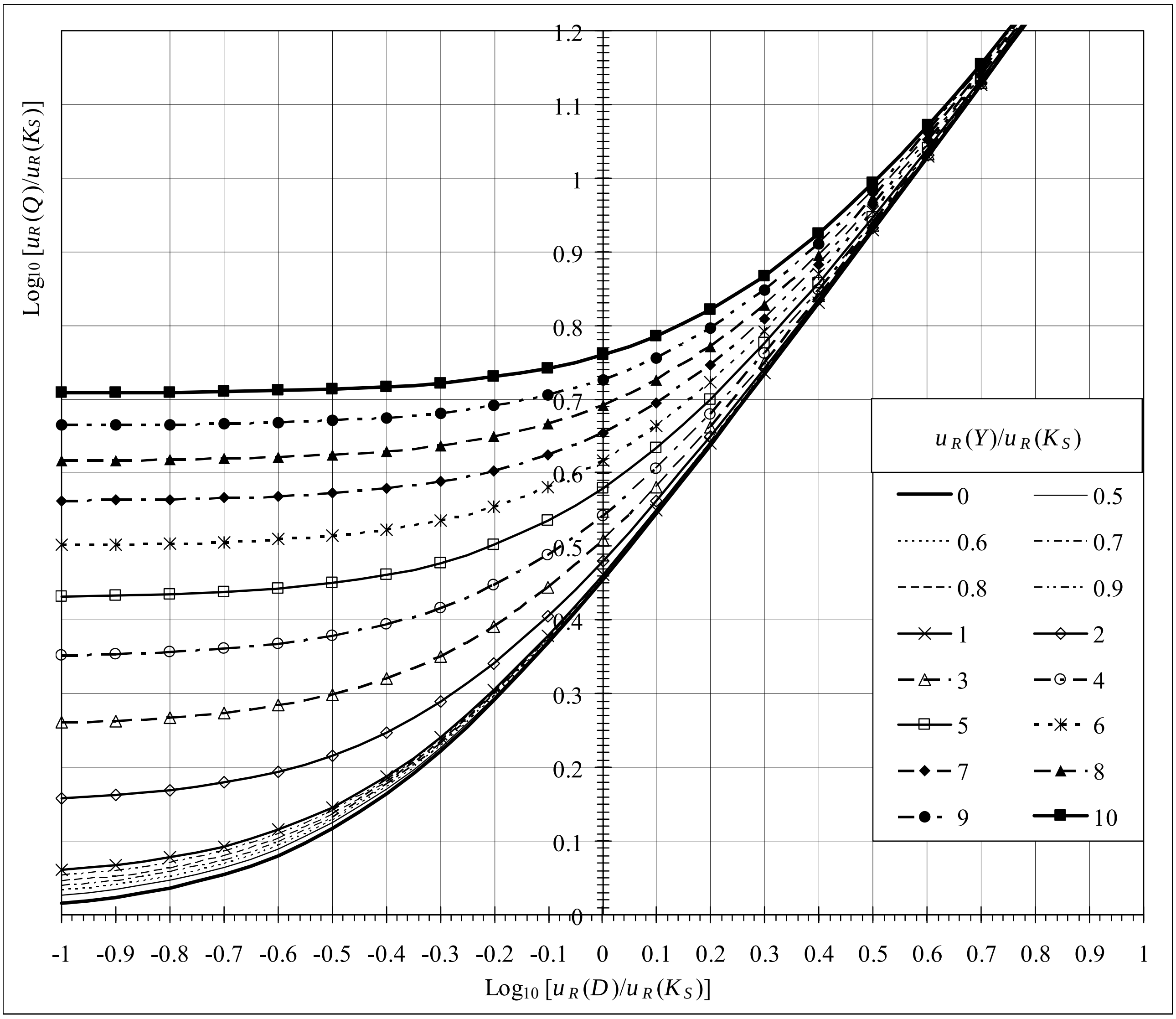

Figure 3 – The group of curves representing eq. (19) in a logarithmic plan.

2 COLEBROOK-WHITE FRICTION FORMULA

2.1 Generalities

The technical evolution has progressively increased the importance, in real practical applications, of smoother and smoother pipes and viscous liquids (i.e. oil), that make the flow regime drop into the zone of critical transition. In such conditions, the relationships among the geometric, kinematic and dissipative variables cannot be simply traced back to the Darcy formula given by eq. (6), but the adimensional friction number λ depends on both the pipe roughness and the Reynolds number *Re*.
Usually, the friction formula applied for these conditions is the Colebrook-White one (Colebrook and White, 1937a and 1937b; Colebrook, 1939):

$$\frac{1}{\sqrt{\lambda}} = -2 \cdot \mathrm{Log}_{10}\left(\frac{2.51}{Re \cdot \sqrt{\lambda}} + \frac{1}{3.71} \cdot \frac{\varepsilon}{D}\right) \tag{20}$$

where the adimensional friction number λ still represents the adimensional friction number of the Darcy-Weisbach eq. (6).
From now on, the uncertainties of the acceleration due to gravity g and the kinematic viscosity of the liquid ν will be neglected, under the condition that they are accurately evaluated.

The acceleration due to gravity g can be estimated through formulas that keeps into account the latitude ϕ (°) and the altitude h (m above sea level), like the following one, which is conventionally prescribed by law for the Italian territory (Decreto 19 maggio 1999):

$$g = 9.780318\cdot[1 + 0.0053024\cdot \mathrm{sen}^2\phi - 0.0000058\cdot \mathrm{sen}^2(2\cdot\phi)] - 0.000003085\cdot h \quad (\mathrm{m/s^2}) \tag{21}$$

where the range of variation is lower than 1‰.
About the kinematic viscosity of the liquid ν, it must be remarked that to reach a negligible level of uncertainty in its evaluation the dependence on temperature T (°C) (Streeter, Wylie, Bedford, 1998) is absolutely essential (Figure 4). It can be expressed for example like (Citrini and Noseda, 1987):

$$\nu = \frac{\mu}{\rho} = \frac{1.773\cdot 10^{-3}\cdot(1+0.0337\cdot T + 0.00022\cdot T^2)^{-1}}{999.457\cdot(1+0.000052939\cdot T - 0.0000065322\cdot T^2 + 0.00000001445\cdot T^3)} \quad (\mathrm{m^2/s}) \tag{22}$$

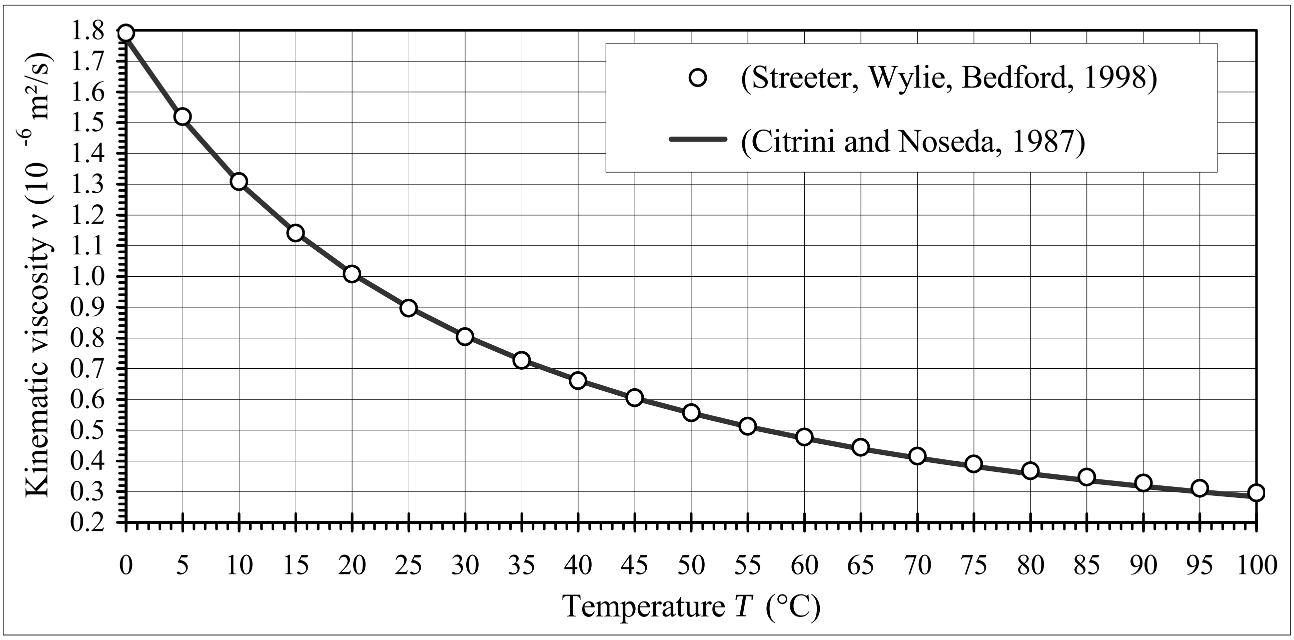

Figure 4 – Kinematic viscosity ν versus temperature T for pure water.

2.2 Individuation of the Colebrook-White roughness coefficient and of its uncertainty

Starting from eq. (21), the Colebrook-White roughness coefficient ε can be drawn as:

$$\varepsilon = 3.71\cdot D\cdot\left(10^{-\frac{1}{2\cdot\sqrt{\lambda}}} - \frac{2.51}{Re\cdot\sqrt{\lambda}}\right) \tag{23}$$

Remembering eq. (1), eq. (3) and eq. (6) and the definitions of kinematic viscosity and Reynolds adimensional number, the following expressions can be written:

$$\nu = \rho/\mu \tag{24}$$

$$Re = \frac{\rho\cdot V\cdot D}{\mu} = \frac{V\cdot D}{\nu} = \frac{Q}{\pi/4\cdot D\cdot\nu} = \frac{1.273\cdot Q}{D\cdot\nu} \tag{25}$$

$$\lambda = \frac{2\cdot g\cdot D\cdot J}{V^2} = \frac{2\cdot(\pi/4)^2\cdot g\cdot D^5\cdot J}{Q^2} = \frac{2\cdot(\pi/4)^2\cdot g\cdot D^5\cdot Y}{Q^2\cdot L} = \frac{1.234\cdot g\cdot D^5\cdot Y}{Q^2\cdot L} \tag{6'}$$

Hence eq. (23) becomes:

$$\varepsilon = 3.71 \cdot D \cdot \left(10^{-\frac{\sqrt{2 \cdot L} \cdot Q}{\pi \cdot D^{5/2} \cdot \sqrt{g \cdot Y}}} - \frac{2.51 \cdot \nu \cdot \sqrt{L}}{\sqrt{2 \cdot g \cdot Y} \cdot D^{3/2}} \right) \tag{23'}$$

Under the hypothesis that L, g and ν give uncertainty contributes which are negligible (see above) in comparison with the ones coming from D, Q and Y, the uncertainty of ε can be estimated as:

$$u(\varepsilon) = \sqrt{\left(\frac{\partial \varepsilon}{\partial D} \right)^2 \cdot u^2(D) + \left(\frac{\partial \varepsilon}{\partial Q} \right)^2 \cdot u^2(Q) + \left(\frac{\partial \varepsilon}{\partial Y} \right)^2 \cdot u^2(Y)} \tag{26}$$

where the sensitivity coefficients of ε regarding D, Q and Y are, after deriving eq. (23'):

$$\frac{\partial \varepsilon}{\partial D} = \frac{\varepsilon}{D} + 3.71 \cdot D \cdot \left(\frac{5}{2} \cdot \frac{\sqrt{2 \cdot L} \cdot Q}{\pi \cdot D^{7/2} \cdot \sqrt{g \cdot Y}} \cdot \ln(10) \cdot 10^{-\frac{\sqrt{2 \cdot L} \cdot Q}{\pi \cdot D^{5/2} \cdot \sqrt{g \cdot Y}}} + \frac{3}{2} \cdot \frac{2.51 \cdot \nu \cdot \sqrt{L}}{\sqrt{2 \cdot g \cdot Y} \cdot D^{5/2}} \right) \tag{27}$$

$$\frac{\partial \varepsilon}{\partial Q} = -3.71 \cdot \ln(10) \cdot 10^{-\frac{\sqrt{2 \cdot L} \cdot Q}{\pi \cdot D^{5/2} \cdot \sqrt{g \cdot Y}}} \cdot \frac{\sqrt{2 \cdot L}}{\pi \cdot D^{3/2} \cdot \sqrt{g \cdot Y}} \tag{28}$$

$$\frac{\partial \varepsilon}{\partial Y} = \frac{3.71}{2} \cdot \frac{\sqrt{L}}{\sqrt{2 \cdot g \cdot D} \cdot Y^{3/2}} \cdot \left(\frac{2 \cdot Q}{\pi \cdot D} \cdot \ln(10) \cdot 10^{-\frac{\sqrt{2 \cdot L} \cdot Q}{\pi \cdot D^{5/2} \cdot \sqrt{g \cdot Y}}} + 2.51 \cdot \nu \right) \tag{29}$$

2.3 Numerical example

To show a numerical example of an individuation of the Colebrook-White roughness coefficient ε and its uncertainty, it can be considered an experimental laboratory test characterized by the same data already exploited for the previous example on the Chézy-Strickler roughness coefficient K_S. Complying with the previous eq. (23'), it comes that:

Colebrook-White roughness coefficient $\varepsilon = 1.59$ mm

and the corresponding uncertainty for ε, according to eq. (26), is:

$u(\varepsilon) = 0.26\ \mathrm{m}^{1/3}/\mathrm{s}$ $\qquad u_R(\varepsilon) = u(\varepsilon)/\varepsilon = 16.4\%$

Again, it is also interesting to highlight that, according to eq. (27), eq. (28) and eq. (29) the heaviest contribute comes from the uncertainty of the diameter D, followed by the one of the flow Q, while the contribute due to is the headloss Y is the smallest. In fact such three contributes are, respectively:

$$\frac{\partial \varepsilon}{\partial D} \cdot u(D) = 0.209 \text{ mm} \qquad \frac{\partial \varepsilon}{\partial Q} \cdot u(Q) = -0.154 \text{ mm} \qquad \frac{\partial \varepsilon}{\partial Y} \cdot u(Y) = 0.015 \text{ mm}$$

It could be seen, through many others similar examples, that in general this is the kind of ranking that comes out of the experimental tests, according to both the usual levels of uncertainty of such quantities and the fact that the structure of eq. (20).

More, the ratios among these contributes are extremely close to the corresponding ones for the case of the Chézy-Strickler roughness coefficient K_S (since the respective contributes were $-2.02\ \mathrm{m}^{1/3}/\mathrm{s}$ for D, $1.51\ \mathrm{m}^{1/3}/\mathrm{s}$ for Q and $-0.15\ \mathrm{m}^{1/3}/\mathrm{s}$ for the headloss Y). In particular, the sign of each contribute is exactly the opposite, because K_S raises and ε decreases when roughness decreases.

Nevertheless, the adimensional uncertainty $u_R(\varepsilon)$ appears bigger than the corresponding $u_R(\varepsilon)$, which was equal to for the Chézy-Strickler roughness coefficient K_S.

Finally, remembering eq. (5):

$$Re^* = 139 > 70$$

Therefore the previous use of the Chézy-Strickler friction formula was allowable to this case study.

2.4 Adimensional approach

Starting from the Colebrook-White friction formula given by eq. (20) and eq. (23), an adimensional form of the Colebrook-White roughness coefficient ε can be defined as $K_{CW} = \varepsilon/D$:

$$K_{CW} = \frac{\varepsilon}{D} = 3.71 \cdot \left(10^{-\frac{1}{2\cdot\sqrt{\lambda}}} - \frac{2.51}{Re \cdot \sqrt{\lambda}} \right) \tag{23"}$$

The uncertainty of K_{CW} is:

$$u(K_{CW}) = \sqrt{\left(\frac{\partial K_{CW}}{\partial \lambda}\right)^2 \cdot u^2(\lambda) + \left(\frac{\partial K_{CW}}{\partial Re}\right)^2 \cdot u^2(Re)} \tag{30}$$

where the sensitivity coefficients of K_{CW} regarding Re and λ are, after deriving eq. (23"):

$$\frac{\partial K_{CW}}{\partial Re} = \frac{3.71 \cdot 2.51}{Re^2 \cdot \sqrt{\lambda}} \tag{31}$$

$$\frac{\partial K_{CW}}{\partial \lambda} = 3.71 \cdot \left[\frac{1}{4\cdot\lambda^{3/2}} \cdot \ln(10) \cdot 10^{-\frac{1}{2\cdot\sqrt{\lambda}}} + \frac{2.51/2}{Re \cdot \lambda^{3/2}} \right] = \frac{3.71}{2\cdot\lambda^{3/2}} \cdot \left(\frac{\ln(10)}{2} \cdot 10^{-\frac{1}{2\cdot\sqrt{\lambda}}} + \frac{2.51}{Re} \right) \tag{32}$$

Again remembering eq. (1), eq. (3), eq. (24), eq. (25) and eq. (6'), and under the hypothesis that L, g and ν give uncertainty contributes which are negligible in comparison with the ones coming from D, Q and Y, the uncertainty of Re and λ can be estimated as:

$$u(Re) = \sqrt{\left(\frac{\partial Re}{\partial D}\right)^2 \cdot u^2(D) + \left(\frac{\partial Re}{\partial Q}\right)^2 \cdot u^2(Q)} \tag{33}$$

$$u(\lambda) = \sqrt{\left(\frac{\partial \lambda}{\partial D}\right)^2 \cdot u^2(D) + \left(\frac{\partial \lambda}{\partial Q}\right)^2 \cdot u^2(Q) + \left(\frac{\partial \lambda}{\partial Y}\right)^2 \cdot u^2(Y)} \tag{34}$$

The sensitivity coefficients of Re regarding D and Q are, after deriving eq. (25):

$$\frac{\partial Re}{\partial D} = -\frac{Q}{\pi/4 \cdot D^2 \cdot \nu} = -\frac{Re}{D} \tag{35}$$

$$\frac{\partial Re}{\partial Q} = \frac{1}{\pi/4 \cdot D \cdot \nu} = \frac{Re}{Q} \tag{36}$$

while the sensitivity coefficients of λ regarding D, Q and Y are, after deriving eq. (6'):

$$\frac{\partial \lambda}{\partial D} = \frac{10 \cdot (\pi/4)^2 \cdot g \cdot D^4 \cdot Y}{Q^2 \cdot L} = 5 \cdot \frac{\lambda}{D} \tag{37}$$

$$\frac{\partial \lambda}{\partial Q} = -\frac{4 \cdot (\pi/4)^2 \cdot g \cdot D^5 \cdot Y}{Q^3 \cdot L} = -2 \cdot \frac{\lambda}{Q} \tag{38}$$

$$\frac{\partial\lambda}{\partial Y} = \frac{2\cdot(\pi/4)^2\cdot g\cdot D^5}{Q^2\cdot L} = \frac{\lambda}{Y} \tag{39}$$

Finally, the uncertainty of $\varepsilon = D\cdot K_{CW}$ can be drawn as:

$$u(\varepsilon) = \sqrt{\left(\frac{\partial\varepsilon}{\partial D}\right)^2\cdot u^2(D) + \left(\frac{\partial\varepsilon}{\partial K_{CW}}\right)^2\cdot u^2(K_{CW})} \tag{40}$$

where the sensitivity coefficients of ε regarding D and K_{CW} are, simply:

$$\frac{\partial\varepsilon}{\partial D} = K_{CW} \tag{41}$$

$$\frac{\partial\varepsilon}{\partial K_{CW}} = D \tag{42}$$

2.5 Individuation of the pipe conveyance capacity and of its uncertainty

Again starting from the Colebrook-White friction formula given by eq. (20), an auxiliary variable ξ can be defined as:

$$\xi = Re\cdot\sqrt{\lambda} = \frac{V\cdot D}{\nu}\cdot\sqrt{\frac{2\cdot g\cdot D\cdot J}{V^2}} = \frac{\sqrt{2}\cdot D^{3/2}\cdot\sqrt{g\cdot J}}{\nu} = \frac{\sqrt{2}\cdot D^{3/2}\cdot\sqrt{g}\cdot\sqrt{Y}}{\nu\cdot\sqrt{L}} \tag{43}$$

that doesn't imply any dependence on velocity V and, therefore, on flow Q.

Hence, the Colebrook-White friction formula given by eq. (20) becomes, still considering the adimensional rate $K_{CW} = \varepsilon/D$ instead of the Colebrook-White roughness coefficient ε:

$$\frac{1}{\sqrt{\lambda}} = -2\cdot\mathrm{Log}_{10}\left(\frac{2.51}{\xi} + \frac{K_{CW}}{3.71}\right) \tag{20'}$$

that is:

$$\lambda = \left[-2\cdot\mathrm{Log}_{10}\left(\frac{2.51}{\xi} + \frac{K_{CW}}{3.71}\right)\right]^{-2} \tag{20''}$$

The uncertainty of λ is:

$$u(\lambda) = \sqrt{\left(\frac{\partial\lambda}{\partial\xi}\right)^2\cdot u^2(\xi) + \left(\frac{\partial\lambda}{\partial K_{CW}}\right)^2\cdot u^2(K_{CW})} \tag{44}$$

where the sensitivity coefficients of λ regarding ξ and K_{CW} are, after deriving eq. (44):

$$\frac{\partial\lambda}{\partial\xi} = -\frac{4\cdot 2.51}{\xi^2}\cdot\mathrm{Log}_{10}(e)\cdot\left(\frac{2.51}{\xi} + \frac{K_{CW}}{3.71}\right)^{-1}\cdot\left[-2\cdot\mathrm{Log}_{10}\left(\frac{2.51}{\xi} + \frac{K_{CW}}{3.71}\right)\right]^{-3} \tag{45}$$

$$\frac{\partial\lambda}{\partial K_{CW}} = \frac{4}{3.71}\cdot\mathrm{Log}_{10}(e)\cdot\left(\frac{2.51}{\xi} + \frac{K_{CW}}{3.71}\right)^{-1}\cdot\left[-2\cdot\mathrm{Log}_{10}\left(\frac{2.51}{\xi} + \frac{K_{CW}}{3.71}\right)\right]^{-3} \tag{46}$$

The uncertainty of ξ can be written, again under the hypothesis that L, g and ν give uncertainty contributes which are negligible in comparison with the ones coming from D and Y, as:

$$u(\xi) = \sqrt{\left(\frac{\partial \xi}{\partial D}\right)^2 \cdot u^2(D) + \left(\frac{\partial \xi}{\partial Y}\right)^2 \cdot u^2(Y)} \tag{47}$$

The sensitivity coefficients of ξ regarding D and Y are, after deriving eq. (43):

$$\frac{\partial \xi}{\partial D} = \frac{3}{2} \cdot \frac{\sqrt{2} \cdot D^{1/2} \cdot \sqrt{g} \cdot \sqrt{Y}}{\nu \cdot \sqrt{L}} = \frac{3}{2} \cdot \frac{\xi}{D} \tag{48}$$

$$\frac{\partial \xi}{\partial Y} = \frac{D^{3/2} \cdot \sqrt{g}}{\sqrt{2} \cdot \nu \cdot \sqrt{L} \cdot \sqrt{Y}} = \frac{1}{2} \cdot \frac{\xi}{Y} \tag{49}$$

Finally, from eq. (6') it can be seen that:

$$Q = \frac{\sqrt{2} \cdot (\pi/4) \cdot \sqrt{g} \cdot D^{5/2} \cdot \sqrt{J}}{\sqrt{\lambda}} = \frac{\sqrt{2} \cdot (\pi/4) \cdot \sqrt{g} \cdot D^{5/2} \cdot \sqrt{Y}}{\sqrt{\lambda} \cdot \sqrt{L}} = \frac{\sqrt{2} \cdot (\pi/4) \cdot \sqrt{g} \cdot D^{5/2} \cdot \sqrt{Y}}{\sqrt{\lambda} \cdot \sqrt{L}} \tag{50}$$

and the corresponding uncertainty of Q is, under the hypothesis that L and g give uncertainty contributes which are negligible in comparison with the ones coming from D, Y and λ:

$$u(Q) = \sqrt{\left(\frac{\partial Q}{\partial D}\right)^2 \cdot u^2(D) + \left(\frac{\partial Q}{\partial Y}\right)^2 \cdot u^2(Y) + \left(\frac{\partial Q}{\partial \lambda}\right)^2 \cdot u^2(\lambda)} \tag{51}$$

where the sensitivity coefficients of Q regarding D, Y and λ are, after deriving eq. (50):

$$\frac{\partial Q}{\partial D} = \frac{5 \cdot (\pi/4) \cdot \sqrt{g} \cdot D^{3/2} \cdot \sqrt{Y}}{\sqrt{2} \cdot \sqrt{\lambda} \cdot \sqrt{L}} = \frac{5}{2} \cdot \frac{Q}{D} \tag{52}$$

$$\frac{\partial Q}{\partial Y} = \frac{(\pi/4) \cdot \sqrt{g} \cdot D^{5/2}}{\sqrt{2} \cdot \sqrt{\lambda} \cdot \sqrt{L} \cdot \sqrt{Y}} = \frac{1}{2} \cdot \frac{Q}{Y} \tag{53}$$

$$\frac{\partial Q}{\partial \lambda} = -\frac{(\pi/4) \cdot \sqrt{g} \cdot D^{5/2} \cdot \sqrt{Y}}{\sqrt{2} \cdot \lambda^{3/2} \cdot \sqrt{L}} = -\frac{1}{2} \cdot \frac{Q}{\lambda} \tag{54}$$

3 CONCLUSIONS

The paper shows how the EA-4/02 could be applied to estimate the uncertainty of the experimental measurements of the Chézy-Strickler-Manning and Colebrook-White roughness coefficients in circular pipes. Of course, such coefficients are obtained through the measurements of diameter size, discharge and headloss for the pipes which are experimentally tested in laboratory, solving the above mentioned Chézy-Strickler-Manning or Colebrook-White equations respect to its own corresponding roughness coefficient. In few words, the uncertainty of the roughness coefficients looks mainly affected by their sensitivity to the uncertainty of diameter, while the uncertainty of discharge is slightly less important, and the uncertainty of headloss become easily negligible on condition that the pipe trunk between the pressure gauges is long enough. The relationships among those uncertainties can be summarized by adimensional relationships, that can be a useful support for designing the experimental tests, in order to allow them to achieve the expected level of uncertainty for the roughness coefficients. In the particular case of the Chézy-Strickler-Manning, those uncertainties can be summarized also graphically by adimensional curves.
More, the paper investigates also the effects of the uncertainty of the Chézy-Strickler-Manning and Colebrook-White coefficients on the uncertainty of the flow conveyance of circular pipes, comparing it with the other contributing factors due to the uncertainty of, respectively, diameter

size and available head. Also to this aim, an adimensional approach is described to evaluate the weight of their different contributions of uncertainty on the uncertainty of the pipe flow conveyance.

NOMENCLATURE

A	cross section area	(m^2)
D	diameter	(m)
g	acceleration due to gravity	(m/s^2)
h	altitude	(m above sea level)
J	friction loss per unit of pipe length	(m/m)
K_{CW}	ε/D	(-)
K_S	Chezy-Strickler roughness coefficient	($m^{1/3}/s$)
L	pipe length	(m)
Q	flow	(m^3/s)
Re	Reynolds number	(-)
$u(\ldots)$	uncertainty	(…)
$u_R(\ldots)$	adimensional uncertainty	(-)
T	temperature	(°C)
V	mean velocity	(m/s)
Y	headloss	(m)
ε	Colebrook-White roughness coefficient	(m)
ϕ	latitude	(°)
λ	adimensional friction number	(-)
μ	dynamic viscosity	(kg/m/s)
ν	kinematic viscosity	(m^2/s)
ξ	$Re\cdot\sqrt{\lambda}$	(-)
ρ	density	(kg/m^3)

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