CHOICE OF POSITION OF FOUR ACOUSTIC PATHS IN CIRCULAR CONDUIT SECTIONS, DOWNSTREAM OF CURVES WITH INSUFFICIENT STRAIGHT LENGTH AND HIGHLY DISTURBED VELOCITY PROFILES.

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ABSTRACT

Flow control installations in hydroelectric plants are usually based on ultrasonic flowmeters with only two acoustic paths, positioned on the same plane at a distance of ±0.5R from the conduit axis. In case of a fully developed flow, i.e. for an exponential or logarithmic distribution of velocity, the average velocity along the acoustic path coincide with the average velocity in the section. On the contrary, in case of transversal components of velocity (such as downstream of curves or elbows), the velocity surveyed by the flowmeter can be very different from the axial component of velocity, e.g. approximately 10% for a 45º configuration and a 5º deviation angle. It is necessary, therefore, to resort to measurements with two crossed paths for each of the two positions (±0.5R), rather than setting up four paths on the same plane. In this essay we consider nine asymmetrical disturbed profiles, produced with Salami formula, to demonstrate that also in this case the average velocity, measured by the acoustic flowmeter in positions (±0.5R), represents with good approximation the average velocity in the section, under the hypothesis of two different installations at 90º to each other. The results are compared with two symmetrical profiles, drawn from power law and from the expression of velocity distribution in circular conduits [Grego79], starting from flow measures with current meters in optimal conditions. Isotachs of theoretical profiles, expressed in adimensional form, are compared with experimental ones, drawn from tests with current meters in highly disturbed flow conditions, to prove that they are representative of real conditions.

1) Introduction

The acoustic method for flow measurement in closed conduits is widely used in hydroelectric plants to control hydraulic machinery performances and in thermoelectric ones to check condenser performances or the thermal load of cooling systems. While a high accuracy is required for these latter applications, this is not the case in other circumstances, e.g. when monitoring discharge for civil use or treating waters for industrial use: for this reason, the customer tends to reduce the monitoring-system's cost by setting up only two acoustic paths in case of closed piping. Moreover, plant conditions very often do not offer straight lengths sufficient to assure a fully developed flow. Many installations offer only a few diameters of straight length upstream of the acoustic flowmeter, with a consequent deviation of the flow from the conduit axis, which makes it necessary to use crossed acoustic paths in order to obtain reliable results.

2) Testing

On the basis of an agreement between the supplier and the customer, the test of a Francis turbine in a hydroelectric plant was conducted through a multi-paths acoustic flowmeter, set up in the penstock, 3 m of diameter, upstream of the machine. Only a few diameters of straight length upstream of the flowmeter were available: therefore eight acoustic paths were set up, four on one plane and four on the other in a configuration at 45º to the conduit centreline. Velocity profiles, expressed in adimensional form to the section average velocity, are reported in Figure 1, together with the profile of the fully developed flow. They show a marked difference between velocities measured on the two crossed planes, with values higher than 10% for the outer paths and of 5% for inner ones, corresponding to a deviation of 5º and 1º respectively of the average velocity on the acoustic line as to the conduit axis. This means that only the use of crossed paths allows to check the possible not alignment of the velocity vector as to the conduit axis and, as a consequence, the reliability of the measurement itself.

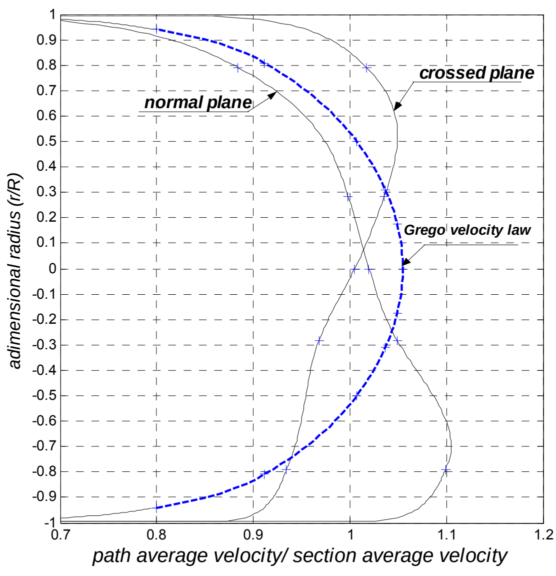


Figure 1- Velocity adimensional profiles surveyed by acoustic flowmeter

It is common to find acoustic flowmeters installed in circular penstocks with only two acoustic paths, positioned on the same plane symmetrically to the conduit centreline, at a distance of ±0.5R, even when the straight length upstream of the installation is of few diameters only. A comparison between current meters and acoustic flowmeter, conducted during tests on a Francis turbine with measurement of discharge in the feeding conduit, showed a difference ε higher than 5% between the two methods in the whole guaranteed range of the machine. The current meters (13 altogether, four for each of the four radii and one in the middle of the conduit) had at disposal an upstream straight length of 10.3 diameters and a downstream one of 8.4 diameters, while the acoustic flowmeter was positioned approximately 2.5 diameters downstream of the current meters. Upstream disturbances consisted of the intake structure, followed by a curve with a stream deviation angle of 40º. The isotachs, drawn through the velocities surveyed by the current meters, showed an asymmetrical distribution of velocity, while discharge measurement accuracy indexes (i.e. the comparison between diametric discharges and the asymmetry index) stood largely within values prescribed by the Standards.

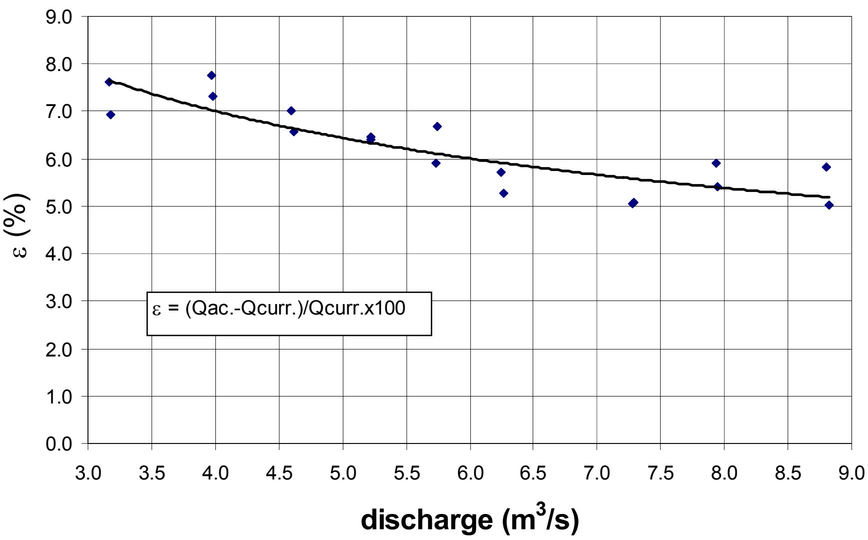


Figure 2- Comparison between acoustic flowmeter (2 paths) and current meters

Experimental results suggested a deeper analysis of results obtained through acoustic flowmeters that use acoustic paths in symmetrical position as to the axis at a distance of 0.5±R, considering two symmetrical distributions of velocity and nine asymmetrical ones, the latter deduced from the laws used by Salami [2] to test the current meters integration error in discharge measurements in circular conduits.

3) Comparison of GAUSS-JACOBI and OWICS methods in discharge measurements in circular conduits with acoustic multi-path flowmeters

3.1 Hypothesis of velocity distribution with power law : $V/V_0 = (1-r/R)^{1/9}$)

For 4 acoustic paths, the **GAUSS-JACOBI method** calculates discharge according to the following formula:
$Q = 2R^2[W_1(V_1+V_4)\cos 54°+W_2(V_2+V_3)\cos 18°]$
where $W_1 = 0.369317$ e $W_2 = 0.597566$

V_1 and V_4 are the velocities measured by the flowmeter in outer paths, V_2 and V_3 are the velocities measured in inner paths.
For 2 acoustic paths, the **GAUSS-JACOBI method** calculates discharge according to the following formula:
$Q = 2R^2[W_1(V_1)\cos 30° + W_2(V_2)\cos 30°]$
where $W_1 = W_2 = 0.906899$
Average velocity in the section $\mathbf{V_{average\ section}}$ is given, respectively, by

a) 4 paths
 $\mathbf{V_{a.s.}} = Q/\pi R^2 = 0.13820(V_1+V_4)+0.36180(V_2+V_3)$

b) 2 paths
 $\mathbf{V_{a.s.}} = Q/\pi R^2 = 0.5(V_1+V_2)$

Under the hypothesis that flow distribution for circular conduits is described by the following law:

$$\mathbf{V/V_0 = (1-r/R)^{1/9}} \qquad (1)$$

the exact value of the average velocity in the section, obtained by integration of law (1), is: $\mathbf{V_{a.s.}} = \mathbf{0.85263V_0}$, i.e. it is calculated through the following expression:

$\mathbf{V_{a.s.}} /V_0 = 2n^2/(n+1)(2n+1)$ with $n = 9$

For a flowmeter with 4 acoustic paths
$V_1/V_0 = V_4/V_0 = 0.77744$ with $\alpha = 54°$ outer paths
$V_2/V_0 = V_3/V_0 = 0.88267$ with $\alpha = 18°$ inner paths

For a flowmeter with 2 paths
$V_1/V_0 = V_2/V_0 = 0.85800$ with $\alpha = 30°$

For 4 paths the average velocity measured by the flowmeter is $\mathbf{0.85358V_0}$
For 2 paths the average velocity measured by the flowmeter is $\mathbf{0.85800V_0}$

On the other hand, for 4 acoustic paths the **OWICS method** [3]calculates discharge through the following formula:
$Q = 2R^2[W_1(V_1+V_4)\cos 53.095643° + W_2(V_2+V_3)\cos 17.684961°]$
where $W_1 = 0.371884$ e $W_2 = 0.588228$
For 2 acoustic paths the **OWICS method** calculates discharge through the following formula:
$Q = 2R^2[W_1(V_1)\cos 29.2059299° + W_2(V_2)\cos 29.2059299°]$
where $W_1 = W_2 = 0.890785$
The average velocity in the section $\mathbf{V_{average\ section}}$ is given, respectively, by

a) 4 paths
 $\mathbf{V_{a.s.}} = Q/\pi R^2 = 0.142163(V_1+V_4)+0.356780(V_2+V_3)$

b) 2 paths
 $\mathbf{V_{a.s.}} = Q/\pi R^2 = 0.494998(V_1+V_2)$

For a flowmeter with 4 paths
$V_1/V_0 = V_4/V_0 = 0.78143$ with $\alpha = 53.095643°$ outer paths

$V_2/V_0 = V_3/V_0 = 0.88319$ with $\alpha = 17.684961°$ inner paths

For a flowmeter with 2 paths
$V_1/V_0 = V_2/V_0 = 0.85993$ with $\alpha = 29.2059299°$
For 4 paths the average velocity measured by the flowmeter is: $\mathbf{0.85239V_0}$
For 2 paths the average velocity measured by the flowmeter is: $\mathbf{0.85133V_0}$
Therefore the OWICS method is more accurate than Gauss-Jacobi's one, because the difference with the exact value is smaller: for a flowmeter with 4 acoustic paths it is -0.028% compared with +0.11% of the Accusonic method; for a flowmeter with 2 acoustic paths the two values are -0.15% and +0.62%.

3.2 Hypothesis of distribution of velocity according to Grego law: *$V/V_{a.s.} = 1.1523+0.09211\ln(1-r/R)+0.1858\Phi(r/R)$*

Under the hypothesis that the flow distribution of the circular conduit is described by the law found by Grego [1]:

$$\mathbf{V/V_{a.s.} = 1.1523+0.09211\ln(1-r/R)+0.1858\Phi(r/R)} \qquad (2)$$

with $\Phi(r/R) = -0.5530347*(r/R)^4+1.6066064*(r/R)^3-1.8782031*(r/R)^2+0.6044168*(r/R)+0.0026893$

the value of the average velocity in the section, obtained by integration of law (2) $\mathbf{V_m} = \mathbf{1.00018V_{a.s.}}$ It is in practice coincident with the exact value which is 1.
For a flowmeter with 4 acoustic paths, **GAUSS-JACOBI method** gives the following values:
$V_1/V_{a.s.} = V_4/V_{a.s.} = 0.911239$ with $\alpha = 54°$ outer paths
$V_2/V_{a.s.} = V_3/V_{a.s.} = 1.036695$ with $\alpha = 18°$ inner paths

For a flowmeter with 2 paths
$V_1/V_{a.s.} = V_2/V_{a.s.} = 1.007326$ with $\alpha = 30°$
For 4 paths the average velocity measured by the acoustic flowmeter is $\mathbf{1.00202V_{a.s.}}$
For 2 paths the average velocity measured by the flowmeter is $\mathbf{1.00733V_{a.s.}}$

For a flowmeter with 4 acoustic paths the **OWICS method** gives the following values:
$V_1/V_{a.s.} = V_4/V_{a.s.} = 0.915987$ with $\alpha = 53.095643°$ outer paths
$V_2/V_{a.s.} = V_3/V_{a.s.} = 1.037299$ with $\alpha = 17.684961°$ inner paths

For a flowmeter with 2 paths
$V_2/V_{a.s.} = V_3/V_{a.s.} = 1.037299$ with $\alpha = 29.2059299°$
For 4 paths the average velocity measured by the acoustic flowmeter is: $\mathbf{1.00061V_{a.s.}}$
For 2 paths the average velocity measured by the flowmeter is: $\mathbf{0.99954V_{a.s.}}$
Therefore, also under the hypothesis of a velocity distribution calculated through Grego law, the OWICS method is more accurate than Gauss-Jacobi's one, because the difference as to exact value for a flowmeter with 4 acoustic paths is +0.04% compared with +0.18% of the Accusonic method; for a flowmeter with 2 acoustic paths the two values are -0.06% and +0.71% respectively.

4) Theoretical profiles of velocity distribution in circular conduits

As underlined in paragraph 2, when there are no straight lengths of conduit that are sufficient to assure a symmetrical and fully developed flow, it is appropriate to use crossed

paths for acoustic multi-paths flowmeters. It is plain that, in order to reduce the costs of instruments' purchase and installation, the Customer tends to reduce the number of transducers. Under the hypothesis of an installation with four acoustic paths (8 transducers), the decision on the right installation is problematic: choose one plane installation with disposition at ± 0.30902R and ± 0.80902R or two crossed planes with a disposition at ±0.5R. It is clear that the first one should reduce the integration error because 4 measuring points are available along the diameter, but there could be a high uncertainty regarding the value of the velocity in case of transversal flows; the second installation gives exact values of velocity, thanks to a crossed system, but the integration error could be high because the measuring points along the diameter are only two.
The aim of this research is to calculate the integration error of the flow distribution under the hypothesis of two measuring points only, positioned symmetrically to the axis, at the distance of ±0.5R prescribed by the IEC 60041 Standard. The research has been carried out taking into account nine distributions of the velocity, which are asymmetrical as to the conduit axis, taken from the research that Salami [2] conducted to calculate the integration error of velocity in measurements with current meters set up on a crossed frame with 4 or 8 radii.
In the analysis, we used the following expressions of the distribution law of velocity in circular penstocks:

Profile n° 1

$V/V_0=(1-r/R)^{(1/9)}-2r/R(1-r/R)^2\exp^{(-0.1\theta)}\cos(\theta)$

Profile n° 2

$V/V_0=(1-r/R)^{(1/9)}+3.32r/R(1-r/R)^2\exp^{(-0.5\theta)}\mathrm{sen}(\theta)$

Profile n° 3

$V/V_0=(1-r/R)^{(1/9)}-3.32r/R(1-r/R)^2\exp^{(-0.5\theta)}\mathrm{sen}(\theta)$

Profile n° 4

$V/V_0=1+0.5(r/R)\mathrm{sen}(\theta)$ per $r/R \leq 0.32$

$V/V_0=(1+0.5*0.32\mathrm{sen}(\theta))(((1-r/R)/(1-0.32))^{(1/9)}-0.7(r/R-0.32)(1-r/R)^{(1/9)}\exp^{(-0.5\theta)}\mathrm{sen}(\theta)/(1-0.32)^{(1/9)})$

Profile n° 5

$V/V_0=1+0.5(r/R)\mathrm{sen}(\theta)$ per $r/R \leq 0.32$

$V/V_0=(1+0.5*0.32\mathrm{sen}(\theta))(((1-r/R)/(1-0.32))^{(1/9)}+0.7(r/R-0.32)(1-r/R)^{(1/9)}\exp^{(-0.5\theta)}\mathrm{sen}(\theta)/(1-0.32)^{(1/9)})$

Profile n° 6

$V/V_0=(1-r/R)^{(1/9)}+0.7r/R(1-r/R)^{(1/9)}\exp^{(-0.5\theta)}(\mathrm{sen}(\theta))^2$

Profile n° 7

$V/V_0=(1-r/R)^{(1/9)}+0.2r/R(1-r/R)^{(1/9)}\mathrm{sen}(\theta)$

Profile n° 8

$V/V_0=(1-r/R)^{(1/9)}+0.1r/R(1-r/R)^{(1/9)}(\theta-1)(1-\cos(\theta))^2$

Profile n° 9

$V/V_0=(1-r/R)^{(1/9)}+0.05r/R(1-r/R)^{(1/9)}\theta(2\pi-\theta)$

The two symmetrical profiles previously considered for the comparison between GAUSS-JACOBI and OWICS methods have been added to these nine asymmetrical profiles.

Profile n° A

$V/V_0=(1-r/R)^{(1/n)}$ with n = 9

$V/V_m=2n^2/[(n+1)(2n+1)](1-r/R)^{(1/n)}$

Profile n° B (Grego law)

$V/V_0= 1 +0.079936\ln(1-r/R)+0.161243\Phi(r/R)$

$V/V_m= 1.1523 +0.09211\ln(1-r/R)+0.1858\Phi(r/R)$

where $\Phi(r/R) = -0.55303473*(r/R)^4+1.60660640*(r/R)^3-1.87820315*(r/R)^2+0.60441675*(r/R)+0.00268932$

5) Results of the analysis

In Figure 3, θ represents the angle of the radius as to the horizontal axis, while α represents the angle of the acoustic path as to the horizontal axis. For each of the eight radii we have calculated the values of velocity and deduced the average diametric velocities and the average section velocity. The average velocity in the acoustic horizontal paths has been calculated for the following values of the angle **α**: 5°,10°,18°,30°,40°,54°,70°,80°,85°

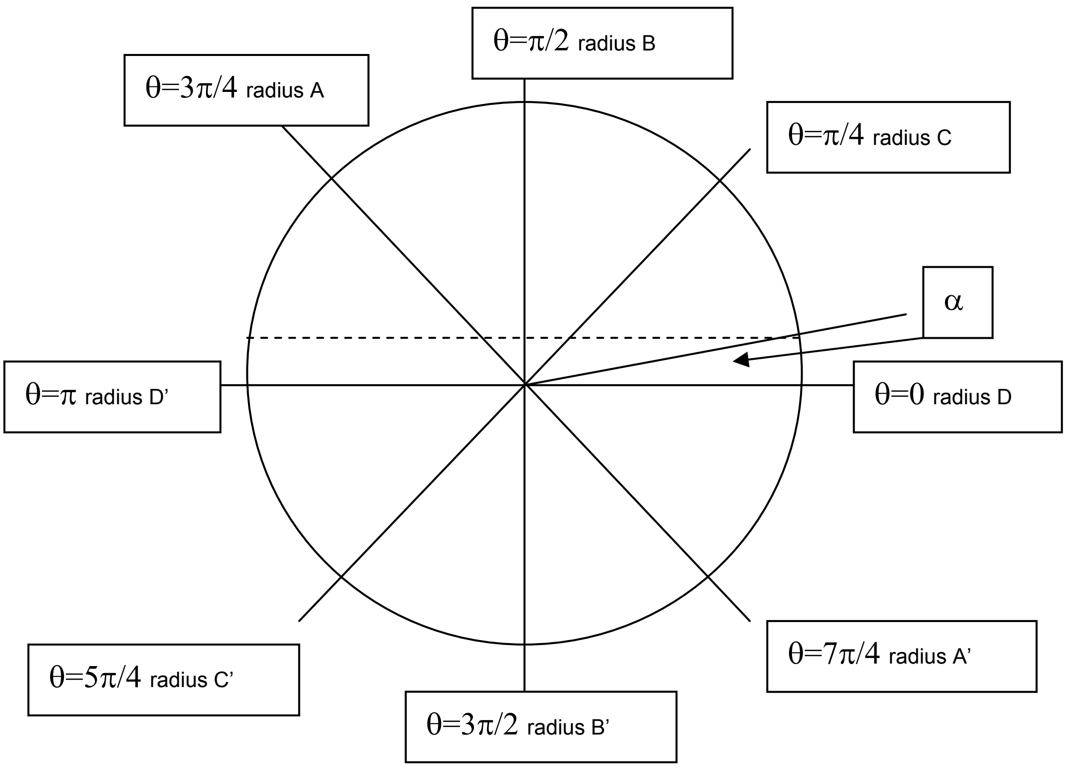
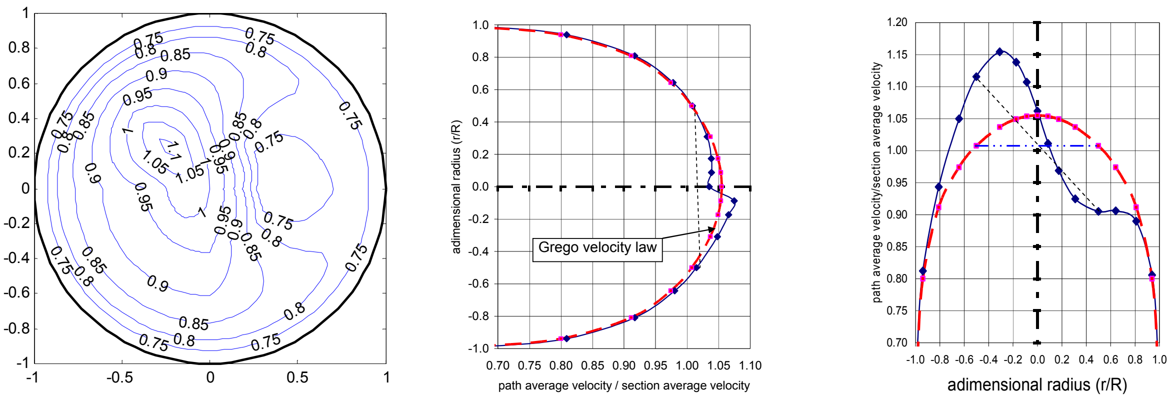


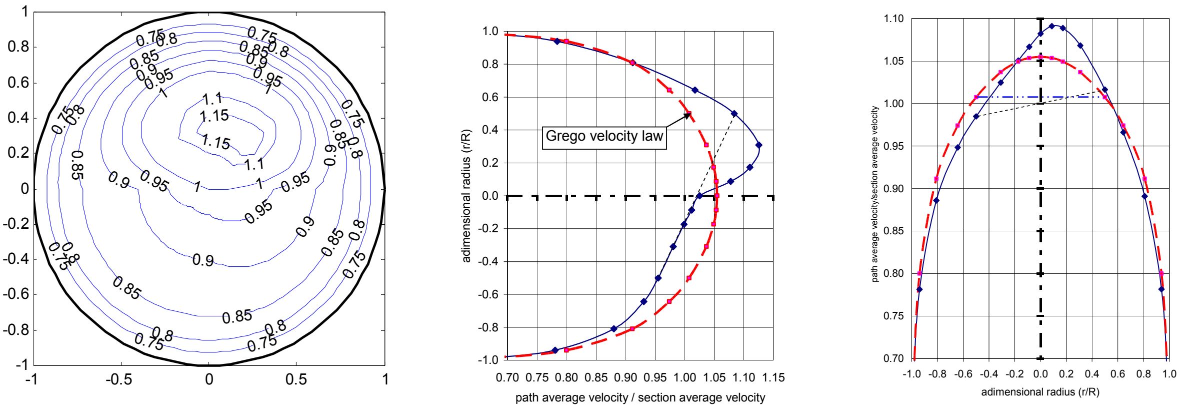
Figure 3 – Scheme for the calculus of diametric velocities and velocities along the acoustic paths

The value of the average velocity along the acoustic paths positioned at ±0.5R coincides with the angle α of 30º. The calculus was then repeated for vertical acoustic paths, i.e. positioned at 90º as to the horizontal axis, and for the same angles α. A graphic description of the average velocities calculated along the acoustic horizontal and vertical paths and expressed in adimensional form is given in Figure 4 for each of the nine theoretical profiles taken into account. In the same figure we reported also the isotachs expressed in adimensional form with reference, in this case, to velocity V_0 in conduit centreline, and not to the average velocity in the section.
The isotachs show highly disturbed flow conditions, for which (as it is pointed out below in Table 1) the values of the accuracy indexes of discharge measurements, if conducted through current meters method (i.e. the difference ε between diametric discharges values and the asymmetry index Y), go beyond the limits suggested by the ISO 3354 Standard.
It should be stressed, however, that also experimental data, obtained through tests in hydroelectric plants, in the conditions of observance and not of the standards, show flow pattern that are sometimes particularly asymmetrical, as reported in Figure 5.
These results suggest that, when acoustic flowmeters are installed downstream of disturbances with insufficient straight lengths, less than 20 diameters, the use of crossed acoustic paths is advisable, because there are transversal flows and, therefore, the measurement of each path can underestimate or overestimate velocity against the actual velocity.
As a consequence, we recommend to use installations with two crossed acoustic paths, positioned at a distance of ±0.5R from the conduit centreline, because, as shown by the results discussed in the Conclusions, the average velocities measured through the acoustic flowmeter in these positions represent with a good degree of reliability (uncertainty lower than 2%) the average section velocity also in case of highly irregular theoretical profiles, when discharge measurements through current meters are very difficult. As shown in Table 1, the coefficients which describe reliability of discharge measurement through the current meters method (i.e. the comparison between diametric discharges ε and the asymmetry index Y) go beyond the limit values prescribed by the Standards, for most of the theoretical profiles considered.

Profile n° 1



Profile n° 2



Profile n° 3

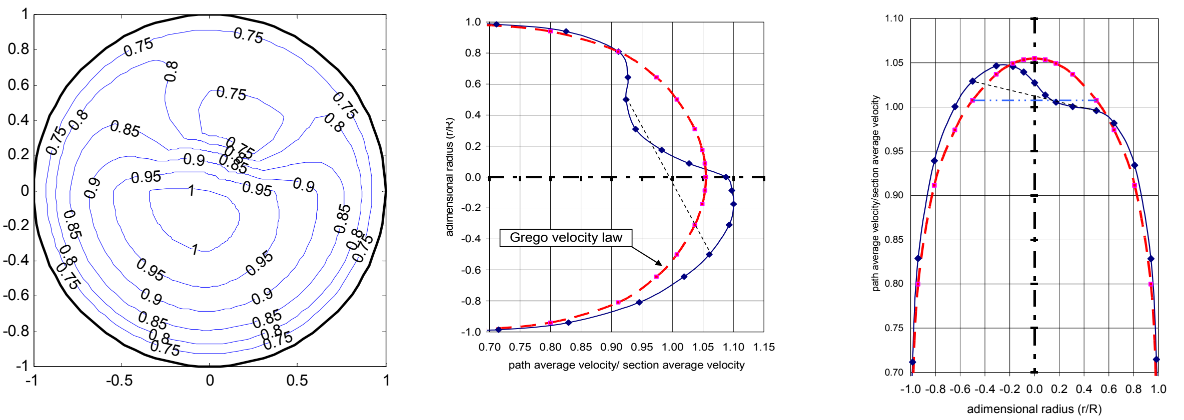
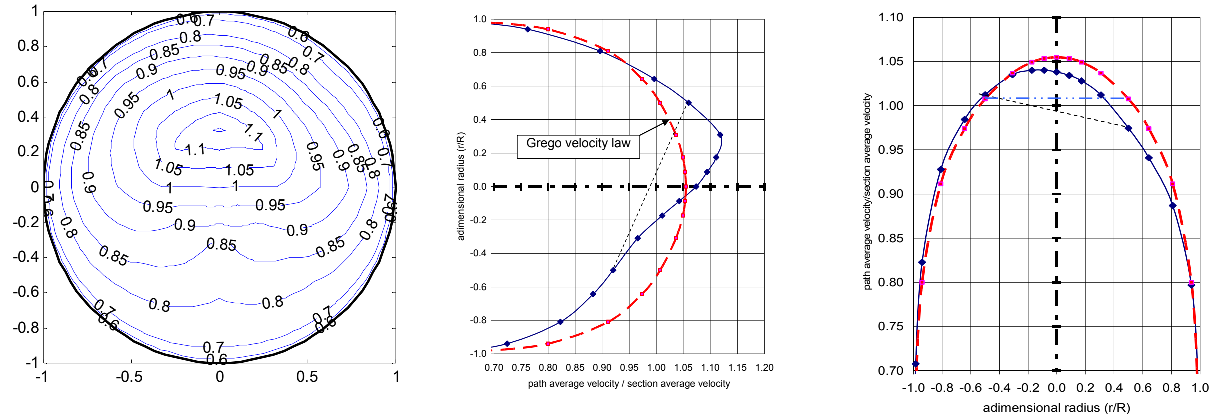
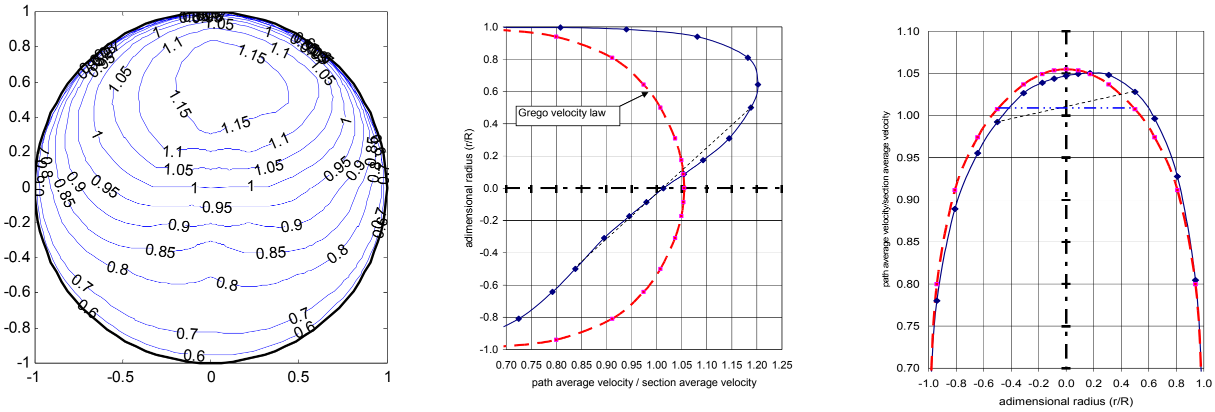


Fig.4a - Isotachs and vertical and horizontal profiles of average velocities along paths

Profile n° 4



Profile n° 5



Profile n° 6

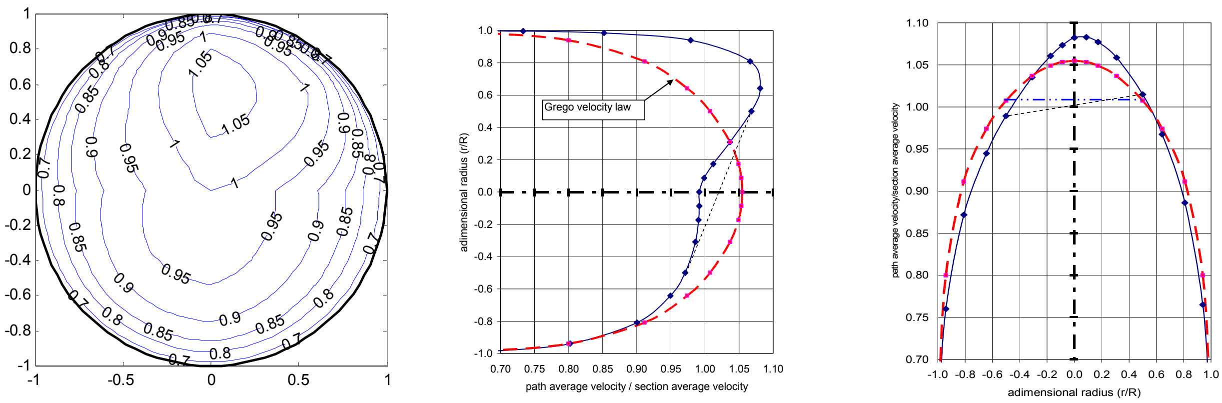
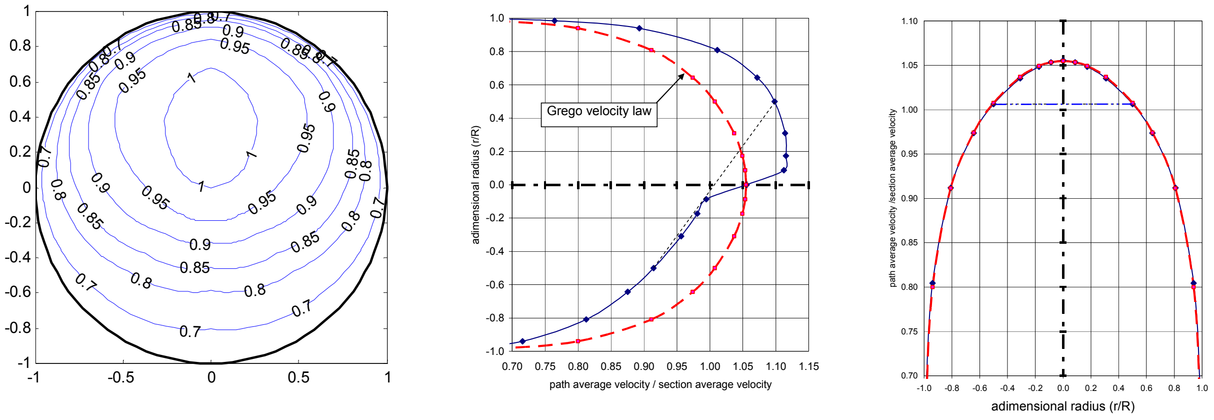
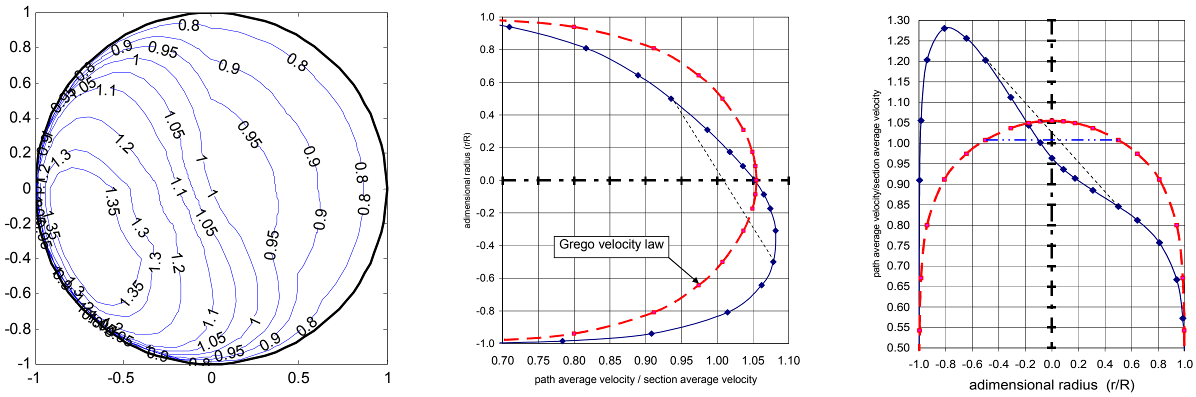


Fig.4b- Isotachs and vertical and horizontal profiles of average velocities along paths

Profile n° 7



Profile n° 8



Profile n° 9

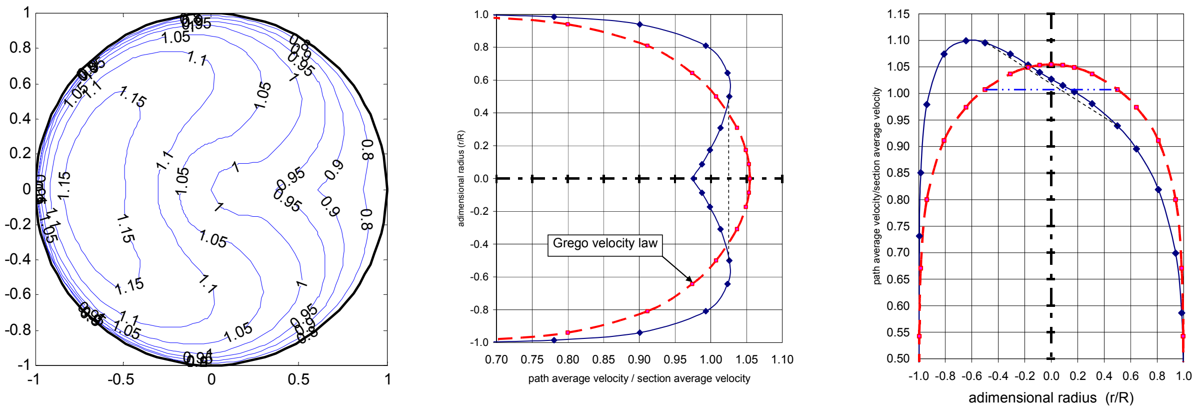
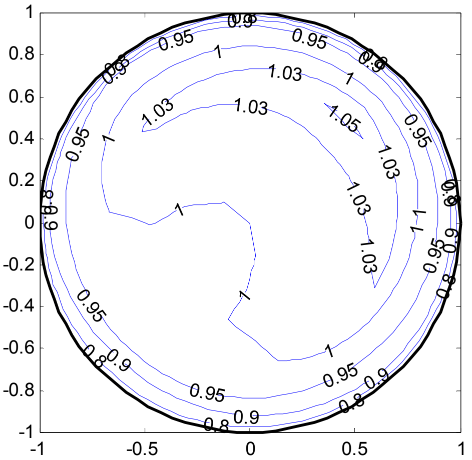
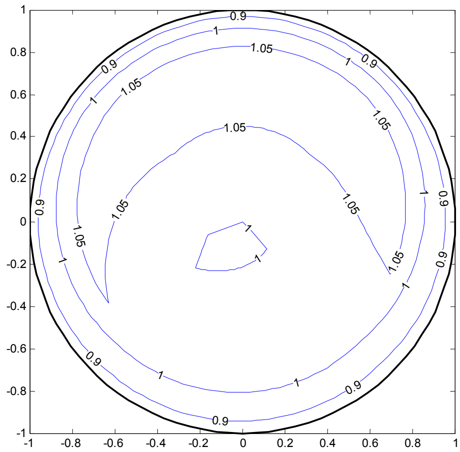


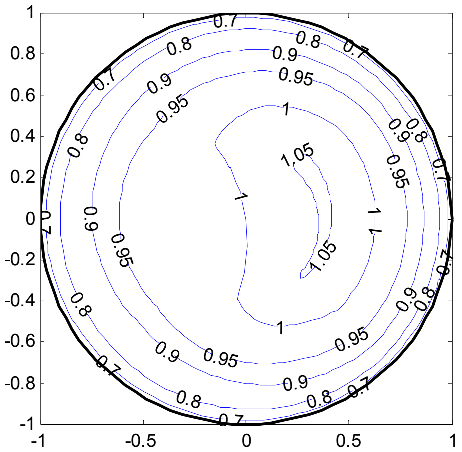
Fig.4c- Isotachs and vertical and horizontal profiles of average velocities along paths



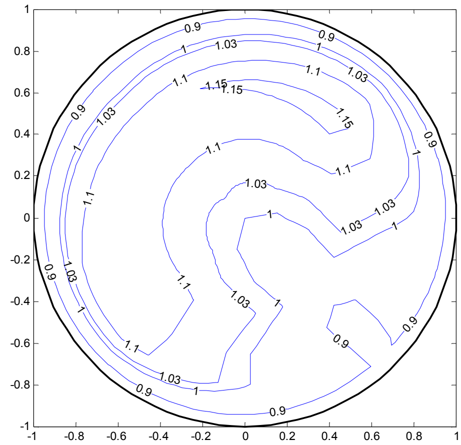
L_u = 3.7 D L_d = 0.8 D



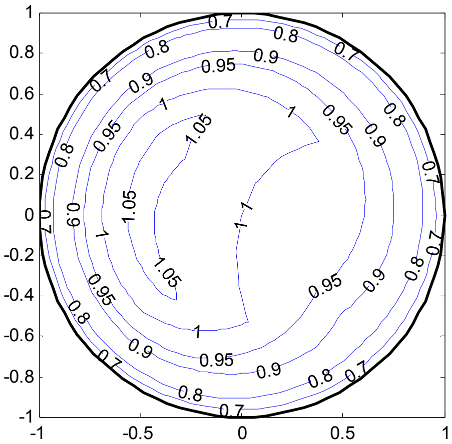
L_u = 10.3 D L_d = 8.4 D



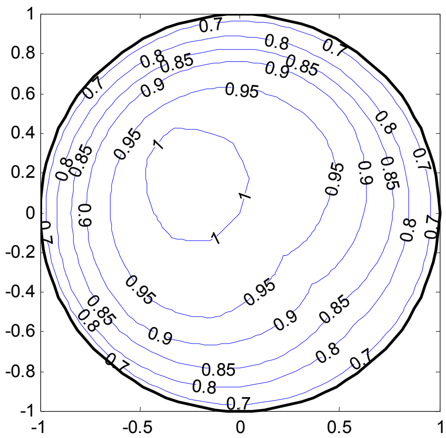
L_u = 6 D L_d = 15D



L_u = 20 D L_d = 1.7 D



L_u = 30D L_d = 5D



L_u = 40 D L_d = 10 D

Fig.5 – Adimensional isotachs for experimental tests with indication of straight length upstream and downstream of the discharge measurement section

Table 1

Profile	Radius	Angle	$V_{a.\,radial}/V_0$	Y	Φ	$V_{a.\,diameter}/V_0$	ε	V_m/V_0	Vpath/$V_{a.}$ section d = ±0.5 R		Difference
(n°)		(rad)	(1)	(2)		(3)	(%)	(4)	horizontal	vertical	(%)
1	D	0	0.7195	0.10	D-D'	0.8346	-1.6	0.7335	1.0117	1.0103	0.14
	C	π/4	0.7656					0.7912			
	B	2π/4	0.8527		C-C'	0.8410	-0.8	0.9001			
	A	3π/4	0.9272					0.9932			
	D'	4π/4	0.9498		B-B'	0.8527	0.6	1.0218			
	C'	5π/4	0.9164					0.9797			
	B'	6π/4	0.8527		A-A'	0.8628	1.8	0.9001			
	A'	7π/4	0.7984					0.8321			
		average	**0.8478**		average	**0.8478**	average	**0.8940**	(4)/(3) =	**1.0545**	

Profile	Radius	Angle	$V_{a.\,radial}/V_0$	Y	Φ	$V_{a.\,diameter}/V_0$	ε	V_m/V_0	Vpath/$V_{a.}$ section d = ±0.5 R		Difference
(n°)		(rad)	(1)	(2)		(3)	(%)	(4)	horizontal	vertical	(%)
2	D	0	0.8527	0.06	D-D'	0.8526	-2.9	0.9001	1.0196	1.0005	1.91
	C	π/4	0.9584					1.0322			
	B	2π/4	0.9536		C-C'	0.8946	1.9	1.0263			
	A	3π/4	0.9009					0.9603			
	D'	4π/4	0.8524		B-B'	0.8927	1.7	0.9001			
	C'	5π/4	0.8308					0.8727			
	B'	6π/4	0.8318		A-A'	0.8718	-0.7	0.8739			
	A'	7π/4	0.8427					0.8876			
		average	**0.8779**		average	**0.8779**	average	**0.9316**	(4)/(3) =	**1.0612**	

Profile	Radius	Angle	$V_{a.\,radial}/V_0$	Y	Φ	$V_{a.\,diameter}/V_0$	ε	V_m/V_0	Vpath/$V_{a.}$ section d = ±0.5 R		Difference
(n°)		(rad)	(1)	(2)		(3)	(%)	(4)	horizontal	vertical	(%)
3	D	0	0.8527	0.06	D-D'	0.8526	3.0	0.9001	0.9922	1.0125	-2.00
	C	π/4	0.7471					0.7680			
	B	2π/4	0.7519		C-C'	0.8109	-2.0	0.7740			
	A	3π/4	0.8046					0.8399			
	D'	4π/4	0.8524		B-B'	0.8128	-1.8	0.9001			
	C'	5π/4	0.8747					0.9276			
	B'	6π/4	0.8737		A-A'	0.8337	0.7	0.9263			
	A'	7π/4	0.8628					0.9126			
		average	**0.8275**		average	**0.8275**	average	**0.8686**	(4)/(3) =	**1.0497**	

Profile	Radius	Angle	$V_{a.\,radial}/V_0$	Y	Φ	$V_{a.\,diameter}/V_0$	ε	V_m/V_0	Vpath/$V_{a.}$ section d = ±0.5 R		Difference
(n°)		(rad)	(1)	(2)		(3)	(%)	(4)	horizontal	vertical	(%)
4	D	0	0.8939	0.09	D-D'	0.8938	3.0	0.9321	0.9903	0.9933	-0.31
	C	π/4	0.9012					0.9461			
	B	2π/4	0.9463		C-C'	0.8520	-1.8	0.9827			
	A	3π/4	0.9558					0.9860			
	D'	4π/4	0.8936		B-B'	0.8515	-1.9	0.9321			
	C'	5π/4	0.8029					0.8569			
	B'	6π/4	0.7567		A-A'	0.8748	0.8	0.8196			
	A'	7π/4	0.7938					0.8503			
		average	**0.8680**		average	**0.8680**	average	**0.9132**	(4)/(3) =	**1.0521**	

Table 1 (following)

Profile	Radius	Angle	$V_{a.\ radial}/V_0$	Y	Φ	$V_{a.\ diameter}/V_0$	ε	V_m/V_0	$V_{path}/V_{a.\ section}$ d = ±0.5 R		Difference
(n°)		(rad)	(1)	(2)		(3)	(%)	(4)	horizontal	vertical	(%)
	D	0	0.8939		D-D'	0.8938	-2.8	0.9321			
	C	π/4	1.1021					1.0928			
	B	2π/4	1.1462		C-C'	0.9358	1.7	1.1287			
5	A	3π/4	1.0474	0.18				1.0529	1.0130	1.0101	0.29
	D'	4π/4	0.8936		B-B'	0.9364	1.8	0.9321			
	C'	5π/4	0.7696					0.8326			
	B'	6π/4	0.7266		A-A'	0.9130	-0.7	0.7976			
	A'	7π/4	0.7786					0.8392			
		average	**0.9198**		average	**0.9198**	average	**0.9510**	(4)/(3) =	**1.0340**	

Profile	Radius	Angle	$V_{a.\ radial}/V_0$	Y	Φ	$V_{a.\ diameter}/V_0$	ε	V_m/V_0	$V_{path}/V_{a.\ section}$ d = ±0.5 R		Difference
(n°)		(rad)	(1)	(2)		(3)	(%)	(4)	horizontal	vertical	(%)
	D	0	0.8527		D-D'	0.8526	-6.1	0.9001			
	C	π/4	0.9823					1.0009			
	B	2π/4	1.0277		C-C'	0.9310	2.6	1.0362			
6	A	3π/4	0.9118	0.07				0.9461	1.0196	1.0021	1.74
	D'	4π/4	0.8524		B-B'	0.9584	5.6	0.9001			
	C'	5π/4	0.8797					0.9211			
	B'	6π/4	0.8891		A-A'	0.8884	-2.1	0.9284			
	A'	7π/4	0.8650					0.9097			
		average	**0.9076**		average	**0.9076**	average	**0.9428**	(4)/(3) =	**1.0388**	

Profile	Radius	Angle	$V_{a.\ radial}/V_0$	Y	Φ	$V_{a.\ diameter}/V_0$	ε	V_m/V_0	$V_{path}/V_{a.\ section}$ d = ±0.5 R		Difference
(n°)		(rad)	(1)	(2)		(3)	(%)	(4)	horizontal	vertical	(%)
	D	0	0.8527		D-D'	0.8526	0.0	0.9001			
	C	π/4	0.9303					0.9604			
	B	2π/4	0.9624		C-C'	0.8527	0.0	0.9854			
7	A	3π/4	0.9303	0.10				0.9604	1.0063	1.0063	0.00
	D'	4π/4	0.8524		B-B'	0.8527	0.0	0.9001			
	C'	5π/4	0.7752					0.8398			
	B'	6π/4	0.7431		A-A'	0.8527	0.0	0.8148			
	A'	7π/4	0.7752					0.8398			
		average	**0.8527**		average	**0.8527**	average	**0.9001**	(4)/(3) =	**1.0556**	

Profile	Radius	Angle	$V_{a.\ radial}/V_0$	Y	Φ	$V_{a.\ diameter}/V_0$	ε	V_m/V_0	$V_{path}/V_{a.\ section}$ d = ±0.5 R		Difference
(n°)		(rad)	(1)	(2)		(3)	(%)	(4)	horizontal	vertical	(%)
	D	0	0.8527		D-D'	1.0873	5.7	0.9001			
	C	π/4	0.8517					0.8993			
	B	2π/4	0.8840		C-C'	1.0860	5.6	0.9245			
8	A	3π/4	1.0694	0.19				1.0687	1.0070	1.0241	-1.67
	D'	4π/4	1.3218		B-B'	0.9701	-5.7	1.2654			
	C'	5π/4	1.3204					1.2639			
	B'	6π/4	1.0563		A-A'	0.9717	-5.6	1.0584			
	A'	7π/4	0.8739					0.9166			
		average	**1.0288**		average	**1.0288**	average	**1.0371**	(4)/(3) =	**1.0081**	

Table 1 (following)

Profile	Radius	Angle	$V_{a.\,radial}/V_0$	Y	Φ	$V_{a.\,diameter}/V_0$	ε	V_m/V_0	Vpath/$V_{a.}$ section d = ±0.5 R		Difference
(n°)		(rad)	(1)	(2)		(3)	(%)	(4)	horizontal	vertical	(%)
9	D	0	0.8527	0.09	D-D'	0.9878	-4.1	0.9001	1.0259	1.0173	0.85
	C	π/4	0.9711					0.9922			
	B	2π/4	1.0556		C-C'	1.0387	0.8	1.0579			
	A	3π/4	1.1064					1.0974			
	D'	4π/4	1.1228		B-B'	1.0556	2.5	1.1106			
	C'	5π/4	1.1064					1.0974			
	B'	6π/4	1.0556		A-A'	1.0387	0.8	1.0579			
	A'	7π/4	0.9711					0.9922			
		average	**1.0302**		average	**1.0302**	average	**1.0382**	(4)/(3) =	**1.0078**	

Profile	Radius	Angle	$V_{a.\,radial}/V_0$	Y	Φ	$V_{a.\,diameter}/V_0$	ε	V_m/V_0	Vpath/$V_{a.}$ section d = ±0.5 R		Difference
(n°)		(rad)	(1)	(2)		(3)	(%)	(4)	horizontal	vertical	(%)
A	alls	alls	0.8526	0.00	alls	0.8526	0.0	0.9000	1.0063	1.0063	0.00
		average	**0.8526**		average	**0.8526**	average	**0.9000**	(4)/(3) =	**1.0556**	

Profile	Radius	Angle	$V_{a.\,radial}/V_0$	Y	Φ	$V_{a.\,diameter}/V_0$	ε	V_m/V_0	Vpath/$V_{a.}$ section d = ±0.5 R		Difference
(n°)		(rad)	(1)	(2)		(3)	(%)	(4)	horizontal	vertical	(%)
B	alls	alls	0.8678	0.00	alls	0.8678	0.0	0.9152	1.0073	1.0073	0.00
		average	**0.8678**		average	**0.8678**	average	**0.9152**	(4)/(3) =	**1.0547**	

6) Conclusions

Starting from theoretical profiles of velocity distribution in circular penstocks, used by Salami to test the integration error of discharge measurements conducted through hydrometric current meters, installed on a fixed crossed frame with 4 or 8 radii, we drew a graphic representation of the adimensional isotachs and calculated the average transversal velocities on the paths positioned at different distances from the conduit centreline, and among them also values ±0.5R, i.e. positions used for installation of acoustic flowmeters when not high accuracy is required. For each theoretical profile we calculated the average velocity in the section, obtained by the velocity integration against the square radius, starting from the velocities along each of the eight radii. From these velocities we calculated the asymmetry index Y and the difference ε between diametric discharges. Furthermore, for each of the eight radii we calculated the average velocity, by the velocity integration against the radius, i.e. the velocity that would be measured by a couple of acoustic transducers set up on the conduit axis.
To sum up, the following results (see Table 2) show that velocity measured by an acoustic flowmeter with 4 acoustic paths, positioned in a configuration with crossed planes at the distance of ±0.5R, represents with an accurate approximation the average velocity in the conduit, even in the case of highly distorted flows, downstream of disturbances when straight lengths are not sufficient. This result, however, can be obtained only by setting up crossed acoustic paths, so that velocity is measured correctly.

Table 2

profile	$V_{a.sect.}/V_0$	$V_{a.\ diameter}/V_0$	(2)/(1)	Vpath/$V_{a.}$ section d = ±0.5 R		ε_o	ε_V
(n°)	(1)	(2)		horizontal	vertical	(%)	(%)
1	0.84779	0.89397	1.0545	1.0117	1.0103	0.43	0.29
2	0.87792	0.93165	1.0612	1.0196	1.0005	1.22	-0.68
3	0.82749	0.86858	1.0497	0.9922	1.0125	-1.50	0.51
4	0.86802	0.91323	1.0521	0.9903	0.9933	-1.69	-1.39
5	0.91977	0.95102	1.0340	1.0130	1.0101	0.57	0.28
6	0.90760	0.94282	1.0388	1.0196	1.0021	1.21	-0.52
7	0.85271	0.90012	1.0556	1.0063	1.0063	-0.10	-0.10
8	1.02878	1.03710	1.0081	1.0070	1.0241	-0.04	1.66
9	1.03022	1.03821	1.0078	1.0259	1.0173	1.85	0.99
A	0.85260	0.90000	1.0556	1.0063	1.0063	-0.10	-0.10
B	**0.86780**	**0.91524**	**1.0547**	**1.0073**	**1.0073**	0.00	0.00

ε_o = horizontal difference between theoretical profile and fully developed flow (profile B)

ε_v = vertical difference between theoretical profile and fully developed flow (profile B)

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