MODERN ERRORS IN WINTER - KENNEDY PIEZOMETERS

by

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ABSTRACT

The Winter - Kennedy piezometers are a reliable, time proven method of measuring relative flow rates. They are usually calibrated in terms of a volumetric flow rate coefficient. However, it is seldom recognized that their function is based on the effects of the corresponding weight flow rate. Therefore, in order to be closely calibrated and used, particularly with modern electronic instrumentation, there are certain correction ratios that may need to be applied. This is especially true in order to provide correct and accurate historical comparisons to other tests.

INTRODUCTION

In order to determine the efficiency, E, of hydraulic turbines, the definition of fluid power, $HP = Q\gamma HE/550$, requires the measurement of power output, HP; volumetric flow rate, Q; specific weight of water, γ; and head, H. The flow rate can be measured in two fundamental ways - absolute and relative.

In the absolute method, turbine discharge is measured in volumetric flow rate terms, such as cubic feet per second. There are a number of different techniques to measure flow rate in absolute terms and the resulting, calculated efficiency is referred to as absolute efficiency.

Due to the expense of absolute flow measurements as well as the physical nature of some water passages, flow rate is more often measured in relative terms by measuring its effect on some other known parameter or "indexing" it against a known parameter. The resulting, calculated efficiency is referred to as relative efficiency.

There are a large number of such indexing parameters. Any place where there is a change in velocity, such as at a reducing station, or anything that causes a head loss, such as a valve or even a trash rack, may be used as an index of the flow rate. Also, any place where the flow is in a curvilinear path, a differential pressure between the inner and outer radii will by developed which may be used as an indexing parameter.

One example of such a curvilinear path is the "elbow taps" used to measure flow in piping systems. Typically, in a 90^0 elbow, a pressure tap may be placed on the inside and outside radius. They need not be at the same elevation, for at zero flow there will be no pressure differential.

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Newton's Second Law stipulates that force is equal to the time rate of change of momentum, $F = d/dt(mv)$. If it is noted that the mass flow rate is ρQ, then an incremental amount of mass for a given time is ρQdt. Substituting this back into the first equation, $F = d/dt(\rho QdtV) = d(\rho QV)$. By canceling the dt's, the differential now becomes a change with respect to anything, temporal or spatial. Therefore, for any constant mass flow rate process, $F = \rho Q\Delta V$. By substituting the Law of Continuity, $Q = AV$, it is noted that the resulting force caused by any change in V is proportional to Q^2. Next, since pressure is force per unit area, it is concluded that any change in pressure between the inner and outer radii is also proportional to Q^2. However, it is rarely noted that this pressure differential is also directly proportional to the density, ρ, of the fluid in the spiral case. The more dense the fluid for a given volumetric flow rate, the larger the resulting pressure differential. The density of any fluid is equal to the specific weight of the fluid divided by gravitational acceleration, $\rho = \gamma/g$. Therefore, the pressure differential across the 90^0 elbow is proportional to both Q^2 and to γ. The specific weight of water is a function of temperature, latitude, and altitude and published values[Ref 2a] vary from 61.712 to 62.473 lbs/ft^3.

PRESSURE DIFFERENTIAL MEASURING SYSTEMS

In 1933, Ireal A. Winter and A. M. Kennedy[Ref 1] extended this principle to hydraulic turbines by installing piezometers on the inner and outer radii of the spiral case of a hydraulic turbine in about the middle of the first quadrant of the spiral. The inner piezometer tap was just above the top ring of the stay ring and the outer piezometer, on the same radial plane from the center of rotation, was at the mid height of the spiral case. Innumerable tests since then have verified that the square root of the pressure differential is indeed proportional to the flow rate. In fact, the present test codes[Refs 2a&b] will only accredit a calibration of the Winter - Kennedy piezometers if the nominal square root exponent is between 0.48 and 0.52.

Another piezometric method to measure relative flow, used much less frequently, is the Joseph Peck method. In this system, one piezometer is placed in the nose of a stay vane and one along the vane's side. Thus, they act like Pitot static tubes and operate under hydraulic principles different from those of the Winter - Kennedy piezometers. However, in the Joseph Peck system flow is still proportional to the square root of the piezometric difference and the following described sources of modern error also apply to this method.

METHODS OF MEASUREMENT

Historically, the pressure differential of the Winter - Kennedy piezometers has been measured by water manometers or differential water manometers. The height of the water column in a manometer tube, h, (or the differential, D, in the heights of the water columns of two manometer tubes) is proportional to the pressure (or pressure difference) at the piezometer tap and inversely to the specific weight of the fluid, $h = P/\gamma$ or $D = \Delta P/\gamma$. When water manometers are used to measure the Winter - Kennedy piezometers, they use the same fluid as in the spiral case and in addition, are normally bled down frequently to prevent gasses from coming out of solution and forming air pockets. Thus, the fluid in the water manometers tends to be at the same temperature as the flow in the spiral case and therefore has the exact same specific weight as the fluid in the spiral case.

Today, there is an increasing trend to use modern electronic instrumentation in testing hydraulic turbines, including pressure transducers to measure piezometric pressures. These transducers are normally calibrated in the laboratory to a reference standard specific weight of fluid. Consequently, the specific weight assumed by the transducers to measure the pressure differential across the Winter - Kennedy piezometers may not be the same as the actual specific weight of water in spiral case which caused the pressure differential in the first place. So long as the results of such an instrumented relative efficiency test are confined to comparison within that test only, this difference in specific weights does not make any difference.

COMPARISONS OF TEST RESULTS

As our hydro plants age, there is an increasing tendency to compare relative efficiency test results taken over a considerable span of time. These comparisons are done with a view to evaluating the extent of any performance degradation as well as the potential for performance improvement by runner replacement. In such cases, attempts have been made to compare the results of earlier relative efficiency tests using water manometers with subsequent relative efficiency tests using pressure transducers. If the readings of the pressure transducers are not converted to an equivalent manometer differential using the same specific weight of water as exists in the spiral case at the time of the test, the relative flow rate and resulting relative efficiency, for comparison, will contain an error due to the incorrect calibration. This error is equal to the square root of the ratio of the specific weights, $(\gamma_2/\gamma_1)^{1/2}$, and will be present in the results of the test using pressure transducers for comparison. On specific tests, the magnitude of this error has been great enough to cause the incorrect conclusion that the turbine has actually increased in efficiency during the time interval between tests.

FIRST MODERN ERROR

This error is actually contained in the Winter - Kennedy calibration coefficient. As previously described, the pressure differential across the Winter - Kennedy's is proportional to γ and Q^2, such that $\Delta P = (1/K_1^2)\gamma_1 Q^2$, where $1/K_1^2$ is a constant of proportionality to the weight flow rate, γQ. This pressure differential is "read" as the height of a water column in a manometer as, $\Delta P = D\gamma_2$. Since the pressure differentials at the interface of the piezometer tap and spiral case must be equal,[2] $\Delta P = D\gamma_2 = (1/K_1^2)\gamma_1 Q^2$. From this, $Q = K_1(\gamma_2/\gamma_1)^{1/2}(D)^{1/2}$. Thus, where $\gamma_1 = \gamma_2$, that is the specific weights in the spiral case and manometer are equal, the ratio is unity and K_1 converts from a weight flow rate calibration coefficient to its customarily recognized form as a volumetric flow rate calibration constant, K. Conversely, where the pressure transducer is calibrated to some laboratory reference specific weight, γ_2, the correct Winter - Kennedy calibration coefficient to eliminate this modern source of error is $K = K_1(\gamma_2/\gamma_1)^{1/2}$.

There are two basic methods to calibrate the Winter - Kennedy piezometers. The first is by an *in situ* independent measurement of the absolute volumetric flow rate. As just described, so long as the device measuring the pressure differential is calibrated to the same specific weight as exists in the spiral case or the correction factor of $(\gamma_2/\gamma_1)^{1/2}$ is included, the resulting volumetric flow rate calibration coefficient does not have a modern source of error.

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[2] by Pascal's Principle which states pressure is transmitted equally and uniformly in all directions.

SECOND MODERN ERROR

With the modern trend to reduce or eliminate field testing, or at least the more expensive absolute testing, and rely entirely on model testing, another method of "calibration" is being used more frequently. This involves relating peak relative efficiency of the field test to peak absolute model efficiency using test data from a homologous turbine model. The usual method, at the same head, is simply to equate the square root of the manometer differential at peak relative efficiency, $D^{1/2}$, with the predicted prototype flow[3] at the peak efficiency of the model, Q. This resulting modern calibration equation, $Q = K(D^{1/2})$, actually has additional sources of error.

As defined at the start, the absolute efficiency from the model, E_{abs}, is a ratio of the power out, HP_{abs}, divided by the fluid power in. The later is composed of the weight flow rate, γQ; times the head, H,

$$E_{abs} = 550HP_{abs}/(\gamma_{abs}QH)$$

The relative efficiency, E_{rel}, is also composed of the power out divided by the fluid power in. For this case, the latter is composed of the weight flow rate in the form of the specific weight, γ; times an "index" of the volumetric flow rate, $K(D^{1/2})$; times the head, H,

$$E_{rel} = 550HP_{rel}/[\gamma_{rel}K(D^{1/2})H]$$

In this form, "K" is the customary volumetric flow rate calibration coefficient.

In order for the efficiencies to be equal (regardless of the power at which each occurs),

$$550HP_{abs}/(\gamma_{abs}QH) = 550HP_{rel}/[\gamma_{rel}K(D^{1/2})H]$$

Assuming this equality is being done at the same head, this equation reduces to,

$$HP_{abs}/(\gamma_{abs}Q) = HP_{rel}/[\gamma_{rel}K(D^{1/2})]$$

or,

$$Q = [K(HP_{abs}/HP_{rel})(\gamma_{rel}/\gamma_{abs})](D^{1/2})$$

Consequently, it is noted that the volumetric flow rate calibration coefficient is modified by the ratio of the specific weights of water, but in this case to the unity exponent. Secondly, the volumetric flow rate calibration coefficient is also modified by the ratio of the powers at which the respective peak efficiencies occur. If these two ratios are not accounted for, this more "modern" method of calibrating the Winter - Kennedy's will contain the corresponding errors. The fact that there is a power shift between the prototype power at which the model predicts that peak efficiency will occur and where it actually occurs has been previously documented.[Refs 3 & 4] This shift is such that the power at which peak efficiency occurs is usually higher than that predicted by the model.

In this instance, it is assumed that during the relative efficiency test the same specific weight, γ_{rel}, exists in the spiral case as is used to measure the Winter - Kennedy differential. Otherwise as previously derived, K must be replaced by K_1 times the square root of the ratio of those particular two specific weights and the equation becomes,

$$Q = [K_1(\gamma_2/\gamma_1)^{1/2}(HP_{abs}/HP_{rel})(\gamma_{rel}/\gamma_{abs})](D^{1/2})$$

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[3] This predicted flow may or may not have an incremental change due to the application of some estimated efficiency step - up to convert model to prototype values. Efficiency step - up is not one of the error sources referenced in this paper.

Since γ_1 and γ_{rel} are both the specific weights in the spiral case, they are equal and the equation simplifies to,

$$Q = [K_1(HP_{abs}/HP_{rel})(\gamma_2\gamma_{rel})^{1/2}/\gamma_{abs}](D^{1/2})$$

CONCLUSIONS

The Winter - Kennedy piezometers have been proven over the past 65 years to be a reliable means of measuring relative flow rates. Although considered to be a volumetric flow rate measuring device, they actually measure the effect of a weight flow rate or lbs/sec. Consequently, with the advent of electronic test instrumentation, the square root of any difference in the ratio of the specific weights of water between the spiral case and the reference value of the measuring device should be included in the Winter - Kennedy calibration coefficient. This is especially important if comparing with the results of other pre - electronic tests. Further, if seeking to calibrate the Winter - Kennedy piezometers by equating the peak relative efficiency with the absolute peak efficiency of a homologous model, the ratios of both the specific weights and the powers at which the peak efficiencies occur need to be included.

REFERENCES

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3. Sheldon, Lee H., *Model to Prototype Efficiency Step - Up for Francis Turbines*, Transactions of the ASME Second Symposium on Small Hydro - Power Fluid Machinery, November 1982.

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NOTATION TABLE

A	area, ft^2
d	derivative, dimensionless
D	piezometric differential, ft
E	efficiency, dimensionless, decimal
F	force, lbs
g	gravitational acceleration, ft/sec^2
h	height of manometer water column, ft
H	head, ft
HP	power, horsepower
K	W - K volumetric flow rate calibration coefficient, $ft^{5/2}/sec$
K_1	W - K weight flow rate calibration coefficient, $ft^{5/2}/sec$
P	pressure, lbs/ft^2
Q	volumetric flow rate, ft^3/sec

t	time, sec
V	velocity, ft/sec
Δ	difference, subtractive, dimensionless
γ	specific weight of water, lbs/ft^3
ρ	density, $slugs/ft^3$
$_{abs}$	absolute
$_{rel}$	relative
$_1$	denotes spiral case side
$_2$	denotes manometer side