

SHP SURGE TANK DIMENSIONING BY THE METHOD OF CHARACTERISTICS (MOC)

**Regina Mambeli Barros¹, Geraldo Lúcio Tiago Filho², Ivan Felipe Silva dos Santos³, and
Fernando das Graças Braga da Silva⁴**

¹ Civil Engineer, Phd. and Masters from PPG-SHS/EESC/USP, Phd. Professor - IRN/ UNIFEI, Researcher from National Reference Center in SHP-CERPCH; Natural Resources Institute – IRN; Federal University of Itajubá, UNIFEI, Av.BPS, 1303, Itajubá-MG, CEP: 37500-903 Av.BPS, 1303, Itajubá-MG, CEP: 37500-903, phone number +55 35 36291224. Corresponding author's e-mail: remambeli@hotmail.com

² Mechanical Engineer, Phd. in Hydraulic Systems from USP and Masters in Mechanical Engineering in Flow Machines from UNIFEI, Director and Phd. Professor - IRN/ UNIFEI, Executive Officer and Researcher from National Reference Center in SHP-CERPCH; Natural Resources Institute – IRN, UNIFEI, phone number +55 35 36291156

³ Water Engineering and Masters' student in Engineering of Energy - UNIFEI, Researcher from National Reference Center in SHP-CERPCH; Natural Resources Institute – IRN; Federal University of Itajubá, UNIFEI, phone number +55 35 36291454

⁴ Civil Engineer, Phd. and Masters from PPG-SHS/EESC/USP, Phd. Professor - IRN/ UNIFEI, Researcher from National Reference Center in SHP-CERPCH; Natural Resources Institute – IRN; Federal University of Itajubá, UNIFEI, phone number +55 35 36291485

ABSTRACT

This study aims to present the hydraulic transitory study as method of characteristics (MOC) applications for solving the *Saint-Venant* equations in a case study, namely: in a penstock of a SHP, as a simple pipeline, in the case of valve-closure in the downstream boundary, with a reservoir in upstream boundary by using the proposed methodology by Chaudry [6] about hydrodynamic models development. The obtained results for first case study showed the simulated values for valve pressure with variation for turning valve between 4 and 12 seconds results in maximums values of pressures that oscillated from 114.93 mca (4s) to 61.64 mca (12s). These results, after Eletrobras methodology application, conveyed to a value of 12 m for the penstock.

Key-words: Method of characteristics, Hydraulic transient, Saint-Venant equations

1. INTRODUCTION

Hydraulic transient events are caused during a change in state from one steady or equilibrium condition to another (Afshar *et al.* [2]) and the main components of these disturbances are pressure and flow changes that bring about propagation of pressure waves throughout the system [1, 2]. Design and operation of any pipeline system entail that head and flow distribution in the system is predicted at different operating conditions, which justify the modeling of these phenomena, as have been purposed in this paper.

Numerous numerical approaches have been introduced for calculation of the pipeline transients, namely: finite volume method (FVM), finite element method (FEM), wave characteristics method (WCM), method of characteristics (MOC), and finite difference method (FDM). Among these methods, MOC is the most commonly used method due to its simplicity and its superior performance as compared with other methods [2].

The water hammer effects caused by closure of spherical valves against the discharge were studied by Karadžić *et al.* [9], and Barros *et al.* [3, 4]. As a result of Karadžić *et al.* [9], it has been found that flow conditions do not have a significant impact on the spherical valve closure time for the cases investigated by them, and the developed numerical models shown reasonable agreement with measured results. Barros *et al.* [4] concluded that the measurement of peak pressure over the valve is seen to be reduced for the valve closure-time 4s to 12s, as increases of the time value as can be seen in Figure 1.

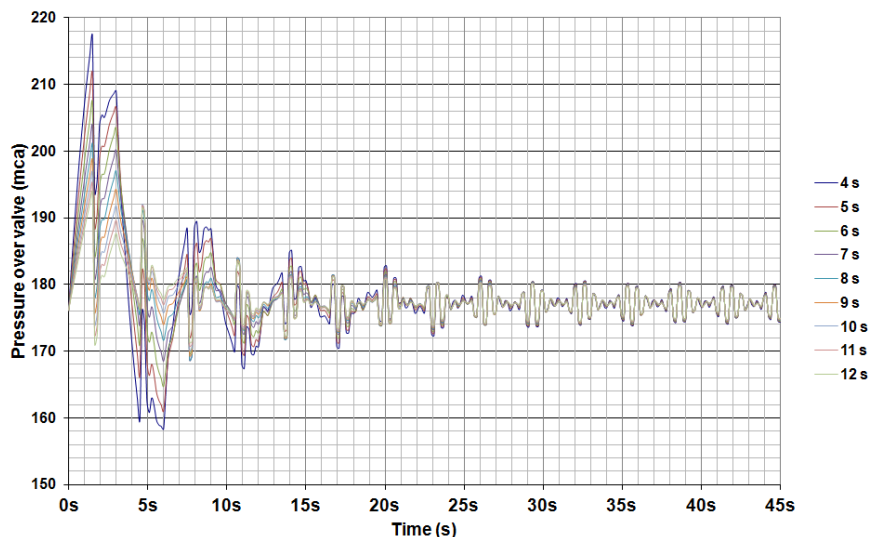


Figure 1: Variation of pressure over the valve. Source: Barros *et al.* [4]

In spite of the lack of experiments for quantitative validation, the purpose of the present paper consists on to present computational results are expected to be instructive for the optimum design of the SHPs to purposes to mitigate the potential damage caused by valve-induced closing-time water hammer, for a SHP case study dimensioning by Barros *et al.* [, 5] in Minas Gerais, Brazil.

2. METHODOLOGY

2.1. The Saint-Venant equations

Methodology proposed by Chaudry [6] concerning the development of hydrodynamic models has been used, in which runoff is regarded as a phenomenon using the physics laws, namely conservation of mass (assuming space), conservation of momentum etc.

Equations (1) and (2), e.g., mass and momentum conservation equations, have been called *Saint-Venant* equations. There are partial differential equations with few explicit solutions, as recommended by Chaudhry [6].

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q'T \quad (1)$$

Where:

T: surface width [L]; A: cross-sectional area [L²]; Q: discharge rate [L³/T], and $q'T = q(m^3 / s / m)$ is the unitary side entrance (meters), as developed by Chaudhry [6].

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = q' \frac{Tv}{A} + g(S_f - S_0) = 0 \quad (2)$$

Where:

v: speed; S₀: slope of the water line (slope of the bottom), and S_f: slope of the energy line.

By assuming one-dimensional flow and based on the continuity and momentum equations which describe the general behavior of fluids in a closed pipe in terms of two variables (y, piezometric head, and v, fluid velocity) the analyses of most hydraulic transients in pressurized systems may be carried out. Wave propagation velocity or celerity, *c*, friction *f*, and pipe diameter *D*, are pipe parameters which may be considered constant during time, despite of they be spatial functions, as recommended by Izquierdo and Iglesias [8].

The alternatives for solving such equations for both the discharge and depth of water variations along both the flow (x) and over time (t), according to are Chaudhry [6], namely: i. simplifying the equations; ii. using numerical methods (by replacement of derived by differences); and iii. making changes.

2.1.1. Methods of characteristics (MOC)

This method aims to transform the two partial differential equations (1) and (2) - there are two independent $\partial v/\partial x$ and $\partial v/\partial t$ -, into ordinary equations which have more convenient properties for numerical calculation at the same time that explicit solutions can be allowed [6]. The particle trajectories within the wave can be observed as follows, according to Chaudhry [6] (Equations 3 to 6).

$$\text{I: } \frac{dx}{dt} = v + \sqrt{\frac{gA}{T}} \quad (3)$$

$$\text{II: } \frac{dv}{dt} + \sqrt{\frac{gT}{A}} \frac{dy}{dt} + \frac{q'T}{A} \left(v - \sqrt{\frac{gA}{T}} \right) + g(S_f - S_0) = 0 \quad (4)$$

$$\text{III: } \frac{dx}{dt} = v - \sqrt{\frac{gA}{T}} \quad (5)$$

$$\text{IV: } \frac{dv}{dt} - \sqrt{\frac{gT}{A}} \frac{dy}{dt} + \frac{q'T}{A} \left(v + \sqrt{\frac{gA}{T}} \right) + g(S_f - S_0) = 0 \quad (6)$$

By observing Equations (3) to (6), the two partial differential equations have become into four ordinary differential equations simplifying the work of resolution (Figure 2, as proposed by Chaudhry [6]).

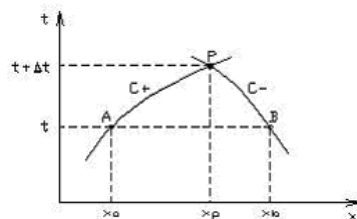


Figure 2: Characteristics equations. Source: elaborated by the authors as based on Chaudhry [6]

If c^+ and c^- intersect in P point, the Equations (4) and (6) can be solved simultaneously (P as intersection of c^+ and c^-).

2.1.2. Solving strategy of the four equations

When the numerical solution of differential equation is gotten, it is possible to replace the derivative by these approaches. Thus, the equations (3) and (5) approach may be as an explicit method or as an implicit method (Chaudhry [6]). The Implicit *MOC* was proposed by Afshar and Rohani [1] aiming to alleviate the shortcomings and limitations of the mostly used conventional *MOC*, allowing for any arbitrary combination of devices in the pipeline system.

The approximation of an implicit differential equation is stable - sometimes unconditionally stable -, while the explicit is unstable, unless Δt very small and consistent with Δx has been chosen.

(E), and according to the *Courant* condition, as recommended by Tucci [11]: $\frac{\Delta x}{\Delta t} \leq \frac{1}{v_0 + c_0}$ and

$$\frac{T.S_0.v_0}{y_0} \geq 171$$

The simultaneous resolution of these equations provides coordinates of P as shown in Figure 3a. The solution to the explicit scheme is straightforward, as recommended by Chaudhry [6]. Equations (3) to (6) can be written as follows in Equations (7) to (10).

$$\text{I: } \frac{dx}{dt} = v + c; \tag{7}$$

$$\text{II: } \frac{dv}{dt} + \frac{g}{c} \frac{dy}{dt} = q.(S_0 - S_f) + \dots \tag{8}$$

$$\text{III: } \frac{dx}{dt} = v - c; \tag{9}$$

$$\text{IV: } \frac{dv}{dt} - \frac{g}{c} \frac{dy}{dt} = q.(S_0 - S_f) + \dots \tag{10}$$

$$\begin{aligned} x_p - x_A &= (v + c)(t_p - t_R) \\ v_p - v_R + \frac{g}{c_R}(y_p - y_A) &= g(S_0 - S_f|_A)(t_p - t_A) \\ x_p - x_B &= (v + c)(t_p - t_B) \\ v_p - v_B + \frac{g}{c_B}(y_p - y_B) &= g(S_0 - S_f|_B)(t_p - t_B) \end{aligned}$$

Since P is known, the equations I and IV (equations 7 and 10) must be searched, with the coordinates x_A and x_B , the equations II and IV (equations 8 and 9) are numerically solved in order to obtain v_p and y_p into the all grid points, as recommended by Chaudhry [6]. The contouring points have no negative feature (c^+ and c^-), so that the channel have been ended. This calculation is applied only to "interior points", but not for points where the contour has only one characteristic curve. At $x=0$ there is only the negative curve (c^-) and $x=L$ there is only positive curve (c^+), according to Chaudhry [6].

In these contouring points, the characteristic equation for v and y (II and IV, i.e., 10 and 12 equations) must be supplemented by another equation from the boundary condition (Figure 2b). Generally, at $x=0$, the hydrograph is used as input boundary condition and the extreme downstream ($x=L$), a relationship between flow and elevation (curve-key) as boundary condition [6]. This sequence of calculations is described in Streeter and Wylie [10] and was recommended by Chaudhry [6].

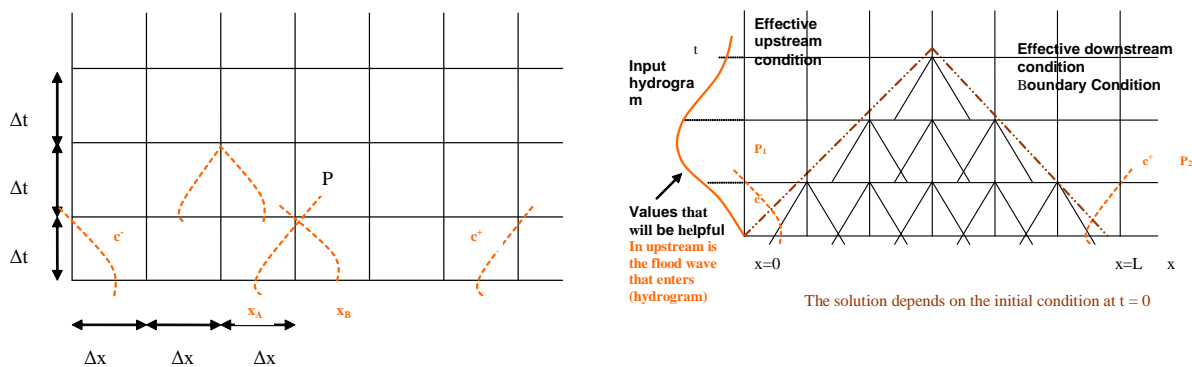


Figure 3: MOC: **a)** Solving strategy of the characteristic equations with a regular grid; and **b)** Boundary conditions in the characteristic equations solving strategy with a regular grid. Source: elaborated by the author as based on Chaudhry [5]

Hydraulic transitory study in the small hydropower by characteristics method was studied also by Barros, Tiago Filho, and Silva [3], as well as Barros *et al.* [4] studied MOC in order to surge tank dimensioning for Small Hydro Power (SHP) plant design, especially regarding its surge tank sizing. For this purpose, the authors (*op. cit.*) the criteria for maximum allowable pressure was used, as studied in the warmer hammer, and to the maximum permissible overspeed, the last one, in case of load rejection, according to the Brazilian Electric Power, Eletrobras [7].

3. DATA OF CASES STUDIES

This study aims to present the hydraulic transitory study as MOC applications for solving the *Saint-Venant* equations a case studies as follows.

3.1. valve closure in a small hydropower

A spreadsheet in *Microsoft® Excel®* for modelling water hammer, as proposed in Chaudhry [5] and presented by Streeter and Wylie [9] was developed in order to conduct a simple case study which has been aimed at valve-closing at the end of downstream, by considering a constant level upstream reservoir (Figure 4). The valve-closing equation was specified by $C_d A_v / (C_d A_v)_0 = (1 - t / t_c)_m$ where t_c was the closure-time, whose value ranged from 4.0 to 12.0 s; $m=3.2$; $L= 131$ meters; $D= 1.3$ meters; $f=0.019$, $2.95 \text{ m}^3/\text{s}$ for turbine discharge and $H_0=41.30$ meters. Equations (4) and (6) have been solved by the MOC using numerical grid, as recommended by Streeter and Wylie [10], and Chaudhry [6].

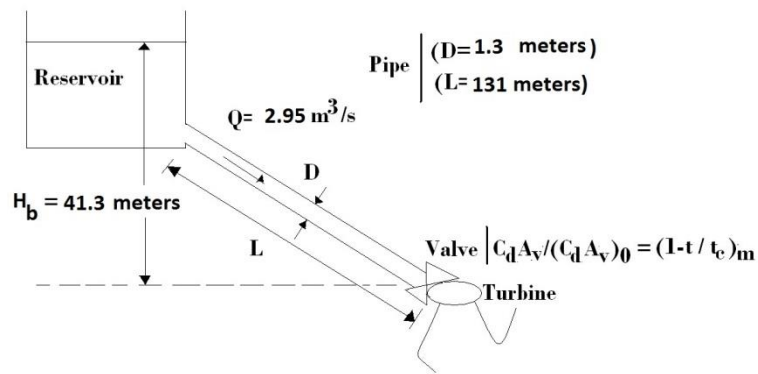


Figure 4: Schematic representation of system of the valve closure in a small hydropower case study [6]

Data for this SHP are shown in Figure 5a, 5b, 6a and 6b, according to Barros *et al.* [6].

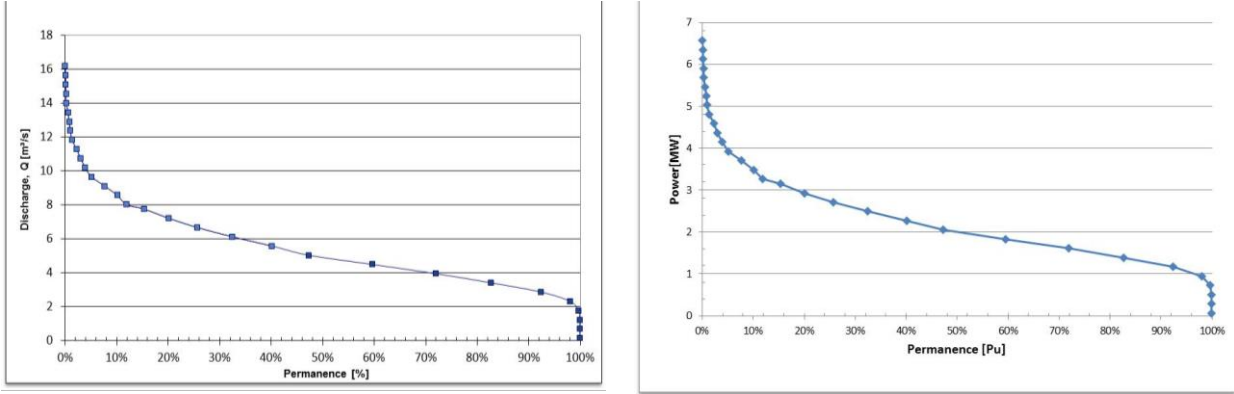


Figure 5: Permanence curves for a) Discharge (left); and b) Power (right), according to Barros *et al.* [6]

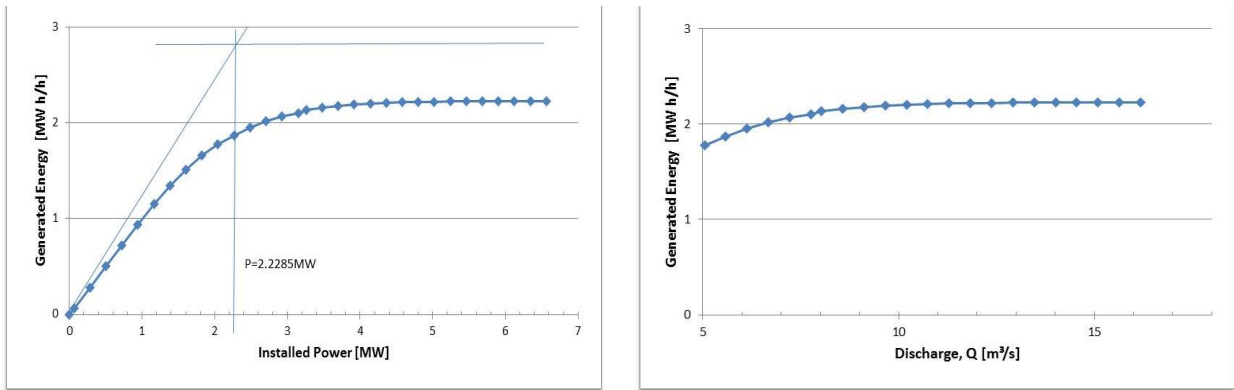


Figure 6: Curves for a) Installed Power (left); and b) Discharge *versus* Generated Energy (right), according to Barros *et al.* [6]

3.2. Surge tank dimensioning

In order to ensure the stability of the oscillations of the water level inside the surge tank, this structure must have a cross section with a minimum internal area, calculated by the *Thoma* formula, as shown by Equation (11), as preconized by Eletrobras [6] and used by Barros *et al.* [4].

$$A_c = \frac{v^2}{2 \cdot g} \cdot \frac{L_{ta} \cdot A_{ta}}{(H_{min} - h_{ta}) \cdot h_{ta}} \quad (11)$$

Where: A_c : is the minimum internal area of the cross section of the surge tank [m²]; v : flow velocity in the adduction pipe [m/s]; g : gravitational acceleration, equal to 9.81 [m/s²]; L_{ta} : length of the

adduction pipe [m]; A_{ta} : internal cross-sectional area of the adduction pipe [m²]; H_{min} : minimum head [m]; h_{ta} : head loss in the adduction system between the water outlet and the surge tank [m].

The height of the standpipe (H_c) is determined according to Eletrobras Eletrobras [6] and used by Barros *et al.* [4] by the oscillation level water inside it, by disregarding the losses in the adduction system or by considering the losses in the adduction system. In the first case, it can be calculated from the elevation (Y_e) of the maximum static water level and the depletion (Y_d) of the minimum static water level by Equation (12); and in the second case it can be calculated by using the Equations (13) to (15), according to Eletrobras [6] and used by Barros *et al.* [4].

$$Y_e = Y_d = v \cdot \sqrt{\frac{A_{ta} \cdot L_{ta}}{g \cdot A_c}} \quad (12)$$

$$Y_e = z_e \cdot Y_e, \text{ where:} \quad (13)$$

$$z_e = 1 - \frac{2}{3}k + \frac{1}{9}k^2 \quad (14)$$

$$k = \frac{h_{ta}}{Y_e} \quad (15)$$

Where: k : is the relative load loss; h_{ta} : is the head loss in the adduction system between the water intake and the surge tank [m], with the load loss due to friction in the pipeline (h_a) calculated for smooth walls: k_a equal to 0.32 (Scobey) or k_a equal to 100 (Strickler), as shown in Eletrobras [6] and used by Barros *et al.* [4].

- Calculation of Y_d :

For the calculation of depletion, it is necessary to determine which of the two cases would be the most unfavorable from the following situations:

- (1) Depletion consecutive at the maximum lift, due to the closing total (100%) of turbine; or
- (2) Depletion resulting from partial opening of 50% to 100% of the turbine.

For the first (1) verification, the procedure is as follows:

Calculation of $Y_d = z_d \cdot Y_d$.

The coefficient is obtained from the graph of the Figure 9, and Table 3, based on graphs from M.M. Calame and Gaden (*apud* Eletrobras, [6]) and was used by Barros *et al.* [4]), by entering with the parameter that is called as k' (Equation 16). For the second verification the table 1 must be used in order to obtain k' .

$$k' = \frac{h'_{ta}}{Y_d} = \frac{h'_{ta}}{Y_e} \quad (16)$$

Where: h'_{ta} : is the head loss in the adduction system between the water intake and the surge tank [m], as a load loss due to friction in the pipeline (h'_a) calculated for the rough walls: k_a equal to 0.40 (Scobey), or k_a equal to 80 (Strickler), as shown in Eletrobras [6] and used by Barros *et al.* [4].

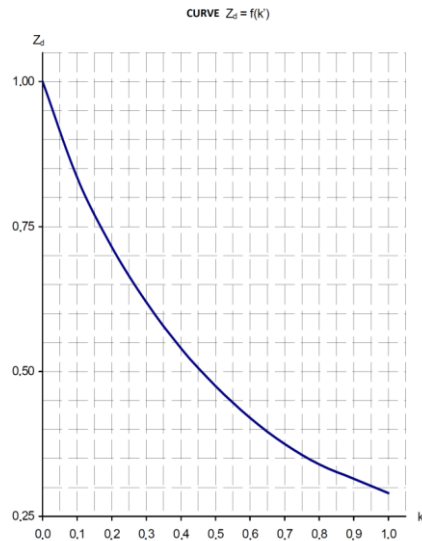


Figure 7: Curve $z_d=f(k')$ as shown in the Eletrobras [6] and used by Barros *et al.* [4]

Table 1: Consecutive depletion at the maximum elevation resulting from turbine total closure - 100%. Determination of the coefficient z_d as a function of k' . Source: Eletrobrás [6] and used by Barros *et al.* [4]

k'	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	1.000	0.982	0.964	0.946	0.928	0.910	0.895	0.881	0.866	0.852
0.10	0.837	0.823	0.809	0.794	0.780	0.766	0.755	0.744	0.734	0.723
0.20	0.712	0.702	0.692	0.683	0.673	0.663	0.654	0.645	0.637	0.628
0.30	0.619	0.611	0.603	0.594	0.586	0.578	0.570	0.562	0.555	0.547
0.40	0.539	0.532	0.526	0.519	0.513	0.506	0.500	0.494	0.487	0.481
0.50	0.475	0.469	0.464	0.458	0.453	0.447	0.442	0.437	0.432	0.427
0.60	0.422	0.417	0.412	0.408	0.403	0.398	0.394	0.390	0.386	0.382
0.70	0.378	0.374	0.371	0.367	0.364	0.360	0.357	0.353	0.350	0.376
0.80	0.343	0.340	0.337	0.334	0.331	0.328	0.325	0.322	0.319	0.316
0.90	0.313	0.310	0.308	0.305	0.303	0.300	0.298	0.296	0.293	0.291
1.00	0.289	-	-	-	-	-	-	-	-	-

Note:

The z_d values listed in the table are negative.

Table 4: Depletion resulting from partial opening of 50% to 100% of the turbine.

Determination of the coefficient z'_d as a function of k' . Source: Eletrobrás [6] and used by Barros *et al.* [4]

k'	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.500	0.504	0.507	0.511	0.514	0.518	0.522	0.525	0.529	0.532
0.10	0.536	0.540	0.544	0.548	0.552	0.556	0.560	0.564	0.569	0.573
0.20	0.577	0.581	0.585	0.589	0.593	0.597	0.601	0.605	0.610	0.614
0.30	0.618	0.622	0.627	0.631	0.636	0.640	0.644	0.649	0.653	0.658
0.40	0.662	0.666	0.671	0.675	0.680	0.684	0.689	0.693	0.698	0.702
0.50	0.707	0.711	0.716	0.720	0.725	0.729	0.734	0.739	0.744	0.749
0.60	0.754	0.759	0.764	0.770	0.775	0.780	0.791	0.791	0.797	0.802
0.70	0.808	0.814	0.819	0.825	0.830	0.836	0.848	0.848	0.854	0.860
0.80	0.866	0.872	0.878	0.885	0.891	0.897	0.910	0.910	0.917	0.923
0.90	0.930	0.937	0.944	0.952	0.959	0.966	0.980	0.980	0.986	0.993
1.00	1.000	-	-	-	-	-	-	-	-	-

Note:
The z_d values listed in the table are negative.

The height of the surge tank is then determined by using the following Equation (17), according to Eletrobras [6] and used by Barros *et al.* [4].

$$H_c = Y_e + y_e + (Y_D \text{ or } Y'_D) + y_D + Y_R \quad (17)$$

Where: y_e and $y_d \approx 1.0$ [m]: is the increasing in the height of elevation and depletion for security; and Y_R : is the maximum depletion of the water level of the reservoir [m]. In this case study, as considered was 0.0 m.

4. RESULTS AND DISCUSSION

4.1. Water hammer results

The pressures over the valve (mca) for the various valve closure-times (between 4s and 12s) have been presented in the graph in Figure 8.

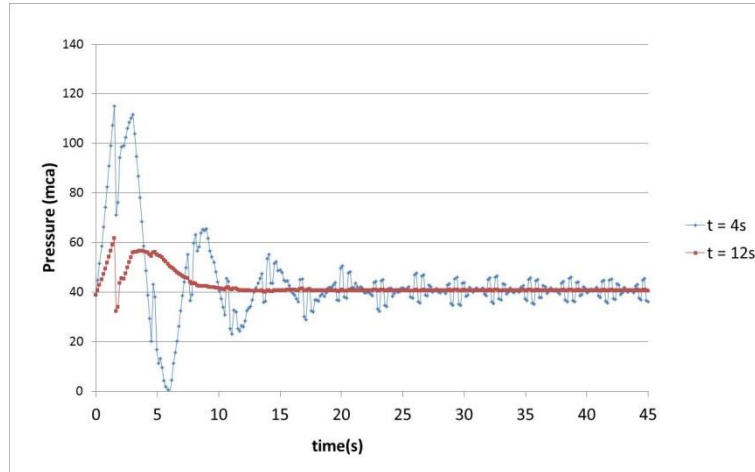


Figure 8: Variation of pressure over the valve

For instance, in the calculation of pressure and depression for valve closure-time, $t = 4\text{s}$:

- Overpressure: ($+h_s$) resulted in a pressure with value of p_i equal to 114.93 mca, in time equal to 1.5 s after valve-closure (elapsed time); and
- Depression: ($-h_s$) resulted in a pressure with value of p_i equal to -0.85 mca, in elapsed time equal to 6 s.

The calculation of pressure and depression for valve closure-time, $t = 12\text{s}$:

- Overpressure: ($+h_s$) resulted in a pressure with value of p_i equal to 61.64 mca, in elapsed time equal to 1.5 s after valve-closure; and
- Depression: ($-h_s$) resulted in a pressure with value of p_i equal to 32.28 mca, in elapsed time equal to 1.65 s.

4.2. discharge propagation into a channel

Figure 9 presents the results values for Equations (11) to (17) for surge tank dimensioning.

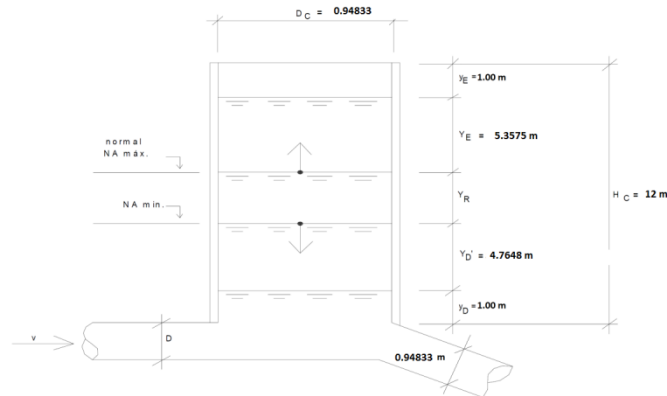


Figure 9: Dimensions of the surge tank obtained for the case study as used by Barros *et al.* [4, 5]

Values considering a larger diameter maybe could still be measured in order to prevent a so slender concrete structure. In this case, the dimensions of the surge tank would be reduced, which would lead also to the SHP total cost being cheaper.

5. CONCLUSIONS

A method of characteristics (MOC) was used in this paper in order to simulate the response of a pipe system upstream of power plants in the case of valve closure. Thi case study was Small Hidro Power (SHP) Plant as designed by Barros *et al.* [5] which has been aimed at valve-closure at the end of downstream, with a reservoir an constant level reservoir at extreme upstream. The valve-closure value ranged from 4.0 to 12.0. As results, the measurement of peak pressure over the valve is seen to be reduced for the valve closure-time 4s to 12s, as increases of the time value.

Values of time after valve-closing, i.e., 1.5 s and 6.00 s, for a closing time of 4s, values such as 114.83 mca and -0.85 mca were been obtained while for a valve closing-time of 12s these values would be 61.64 mca (at 1,50s) and 32.28 mca (at 1.65s). The surge tank dimensioning resulted with a diameter of 0.948 m and total height of 12m.

Therefore, the MOC numerical approaching has confirmed as being very useful for several engineering purposes, including cases of hydraulic transients. As recommendation, the authors suggest a validation with systems both, actual and laboratory scales, which will help to produce more realistic results.

AKNOWLEDGMENTS

We are enormously grateful to Prof. Dr. Fazal Hussain Chaudhry for their valuable contributions. We are grateful to the Federal University of Itajubá (UNIFEI) by PROAP. We are thankful to the National Center for Small Hydro Power Plant (CERPCH, in Portuguese) by providing any information when necessary.

REFERENCES

- [1] AFSHAR, M.H.; ROHANI, M. (2008). Water hammer simulation by implicit method of characteristic. *International Journal of Pressure Vessels and Piping*, v. 85, p. 851–859
- [2] AFSHAR, M.H.; ROHANI, M.; TAHERI, R. (2010). Simulation of transient flow in pipeline systems due to load rejection and load acceptance by hydroelectric power plants. *International Journal of Mechanical Sciences*, v. 52, n. 1, p 103-115. Available on:<<http://www.sciencedirect.com/science/article/pii/S0020740309002161>>. Accessed in jun. 30 2011
- [3] BARROS, R.M.; TIAGO FILHO, G. L.; DA SILVA, F. G. B. (2011) Hydraulic transitory study in the small hydropower by characteristics method. **American Journal of Hydropower, Water and Environment Systems**, aug. 2014, p. 38-47.
- [4] BARROS, R.M.; TIAGO FILHO; DOS SANTOS, I. F. S.; G. L.; DA SILVA, F. G. B. (2014) Hydraulic transitory study in the small hydropower by characteristics method in order to surge tank dimensioning. IN: 27° IAHR Symposium on Hydraulic Machinery and Systems. Montréal, Canadá. *Proceedings...* International Hydraulic Association Research, Montréal, 2014.
- [5] BARROS, R.M.; CARVALHO, B. M. C.; BETHONICO C.; DOS SANTOS, S. J.; AUGUSTO, J. V. S.; CASTEGNARO. L.; FERREIRA, T. V. V. (2012). *PCH Buraco do Tatu: Estudos de Projeto Básico – Relatório Final* (Monografia) Especialização em Pequenas Centrais Hidrelétricas. 149 p+anexos. Centro Nacional de Referência em Pequenas Centrais Hidrelétricas – Universidade Federal de Itajubá (CERPCH-UNIFEI). Itajubá, Minas Gerais, Brasil, 2012.
- [6] CHAUDHRY, F. (2001). *Hydrology: quantitative aspects*. Class notes. São Carlos.
- [7] ELETROBRAS. The Brazilian Electric Power. (2000). *Guidelines for Studies and Projects of SHPs*. 458 p.
- [8] IZQUIERDO, J.; IGLESIAS, P. L. Mathematical Modelling of Hydraulic Transients in Simple Systems. *Mathematical and Computer Modelling*, v. 35, p. 801-812
- [9] KARADŽIĆ, U.; BERGANT, A.; P VUKOSLAVČEVIĆ, P. (2010). Water hammer caused

by closure of turbine safety spherical valves. IN: 25th IAHR Symposium on Hydraulic Machinery and Systems IOP Publishing; 2010. *IOP Conf. Series: Earth and Environmental Science* 12 (2010) 012096 doi:10.1088/1755-1315/12/1/012096.

- [10] STREETER, V. L.; WYLIE, E. B. (1982). *Fluid mechanics*. 7. São Paulo: McGraw-Hill do Brasil. 585 p.T
- [11] TUCCI, C. E. M. (1998). *Hydrological models*. Porto Alegre: Ed University / UFRGS / Brazilian Water Resources Association.